

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Space Programs Summary 37-44, Vol. IV

Supporting Research and Advanced Development

For the Period February 1 to March 31, 1967

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JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

April 30, 1967

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SPACE PROGRAMS SUMMARY 37-44, VOL. IV

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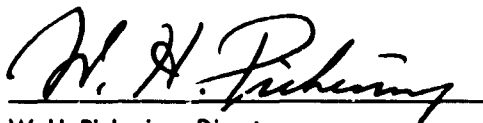
Preface

The Space Programs Summary is a six-volume bimonthly publication designed to report on JPL space exploration programs and related supporting research and advanced development projects. The titles of all volumes of the Space Programs Summary are:

- Vol. I. *The Lunar Program* (Confidential)
- Vol. II. *The Planetary-Interplanetary Program* (Confidential)
- Vol. III. *The Deep Space Network* (Unclassified)
- Vol. IV. *Supporting Research and Advanced Development* (Unclassified)
- Vol. V. *Supporting Research and Advanced Development* (Confidential)
- Vol. VI. *Space Exploration Programs and Space Sciences* (Unclassified)

The Space Programs Summary, Vol. VI, consists of: an unclassified digest of appropriate material from Vols. I, II, and III; an original presentation of the JPL quality assurance and reliability efforts, and the environmental- and dynamic-testing facility-development activities; and a reprint of the space science instrumentation studies of Vols. I and II.

Approved by:



W. H. Pickering, Director

Jet Propulsion Laboratory

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Systems Analysis 4

SYSTEMS DIVISION

NC7-29142

A. Some Periodic Orbits in the Rectilinear Restricted Three-Body Problem, R. A. Broucke

We have described in SPS 37-42, Vol. IV, pp. 31-33, and SPS 37-43, Vol. IV, pp. 20-22, some recent results obtained in the elliptic restricted three-body problem. The present article gives a few additional results relating to a special configuration of the elliptic problem: the case with equal masses and with extreme eccentricity $e = 1$. The two primaries are supposed to be moving in a Keplerian oscillatory motion on a straight line. This is the limit of the elliptic motion when the eccentricity tends to 1. We have first become aware of the interest of this problem when we have seen that some of the periodic orbits of Stromgren's problem can be extended all the way from the eccentricity $e = 0$ to the eccentricity $e = +1$. This leads us to the conclusion that a special study of the case $e = +1$ could be worthwhile. A similar conclusion was drawn in an article by J. Schubart (Ref. 1), in which he proposed to take the eccentricity $e = +1$ as a starting point to a systematic study of the elliptic restricted three-body problem.

We have used in this study a non-rotating inertial barycentric coordinate system. The equations of the motion of the satellite under the gravitational force of the primaries $(1 - \mu)$ and μ are

$$\frac{d^2x}{dt^2} = - (1 - \mu) \frac{x - x_1}{r_1^3} - \mu \frac{x - x_2}{r_2^3},$$

$$\frac{d^2y}{dt^2} = - (1 - \mu) \frac{y - y_1}{r_1^3} - \mu \frac{y - y_2}{r_2^3},$$

where the distances r_1, r_2 from the satellite to the primaries are given by

$$r_1^2 = (x - x_1)^2 + (y - y_1)^2,$$

$$r_2^2 = (x - x_2)^2 + (y - y_2)^2.$$

The coordinates of the two primaries are given by the well-known laws of elliptic motion. We have used them

in the form

$$\begin{aligned}x_1 &= -\mu(-e + \cos E), \\y_1 &= -\mu(1 - e^2)^{1/2} \sin E \\x_2 &= (1 - \mu)(-e + \cos E), \\y_2 &= (1 - \mu)(1 - e^2)^{1/2} \sin E,\end{aligned}$$

where the eccentric anomaly E is related to the time by Kepler's equation

$$t + X = E - e \sin E.$$

The phase constant X , in our application, may be taken equal to 0 or π according to the initial position of the primaries, either at pericenter or apocenter. Actually, all the orbits which will be described below belong to $X = 0$. There is no difficulty in the numerical solution of Kepler's equation even when $e = 1$, in order to describe the Keplerian motion of the two primaries as a function of time t . It is important to notice that the above formulation of the elliptic three-body problem is valid for all eccentricities ($0 \leq e \leq 1$). We shall consider here only the case $e = 1$. When we have the unit eccentricity, there is no sense anymore in using a rotating coordinate system for this problem, since the primaries are permanently moving on a non-rotating straight line. For the same reason the so-called Nechvile transformation cannot be applied when $e = 1$, and the simple form of the equations of motion, which is completely similar to the form for the circular case, cannot be used for high eccentricity.

Because of the straight-line motion of the two primaries, the problem that we are treating here presents some similarity with the well-known two-fixed center problem. Our problem may be considered as a generalization of it, the two "fixed centers" now moving on a straight line, according to Keplerian motion laws. Because of this fact, we have here a problem which seems more interesting than the two-fixed center problem: the problem we are studying is gravitationally consistent while the definition of the two-fixed center problem is somewhat arbitrary (the primaries attract the satellite but do not attract each other!). Among the similarities with the two-fixed center problem one can also mention the absence of the Coriolis force and the characteristic loops in the orbits, which are due to this force.

The numerical integration of the above equations of motion may encounter some difficulties due to the pres-

ence of the singularities $r_1 = 0$ or $r_2 = 0$. In this particular problem we have applied no regularization to the equations of motion.

Because of the simplicity of the equations of motion, we have first decided to integrate some orbits with regularly spaced initial conditions, in order to see if some periodic orbits would eventually exist. We have soon arrived at the rather surprising conclusion that a large number of periodic orbits are existing, although they are all isolated. We have then computed thirteen of these periodic orbits with somewhat higher precision. We have used the classical Runge-Kutta numerical integration procedure, with a variable step, equal to $0.005 r_1 r_2$. Our objective was not mainly in the high precision of the numerical results, and for instance, by using full double precision, our end results have probably only five- or six-place accuracy. The periodic orbits have been obtained by two-dimensional linear differential corrections, once some good approximation had been obtained by the previous computer runs.

We have given in Fig. 1 a diagram which shows the initial conditions that have been explored. The initial conditions being of the form

$$(x_0, 0, 0, \dot{y}_0),$$

we have arbitrarily restricted ourselves to the limits

$$0.1 < x_0 < 1.0, \quad 0.2 < \dot{y}_0 < 2.2.$$

We have also restricted the velocity $\dot{y}_0$ to be below some parabolic or escape velocity defined by

$$\dot{y}_0 = \left(\frac{2}{x_0}\right)^{1/2}.$$

Imposing all the above restrictions, we obtain the shaded area in Fig. 1. We have swept this area with regular intervals of 0.05 for x_0 as well as for $\dot{y}_0$. The thirteen points on this diagram correspond to the isolated periodic orbits which have been found. The eleven orbits three to thirteen form a single sequence: they all have one loop around one primary and an increasing number of loops around the other primary. We have given the initial conditions of the thirteen periodic orbits in Table 1. All thirteen orbits have the period $T = 2\pi$. In Fig. 2 we have shown the actual orbits, referred to the barycentric

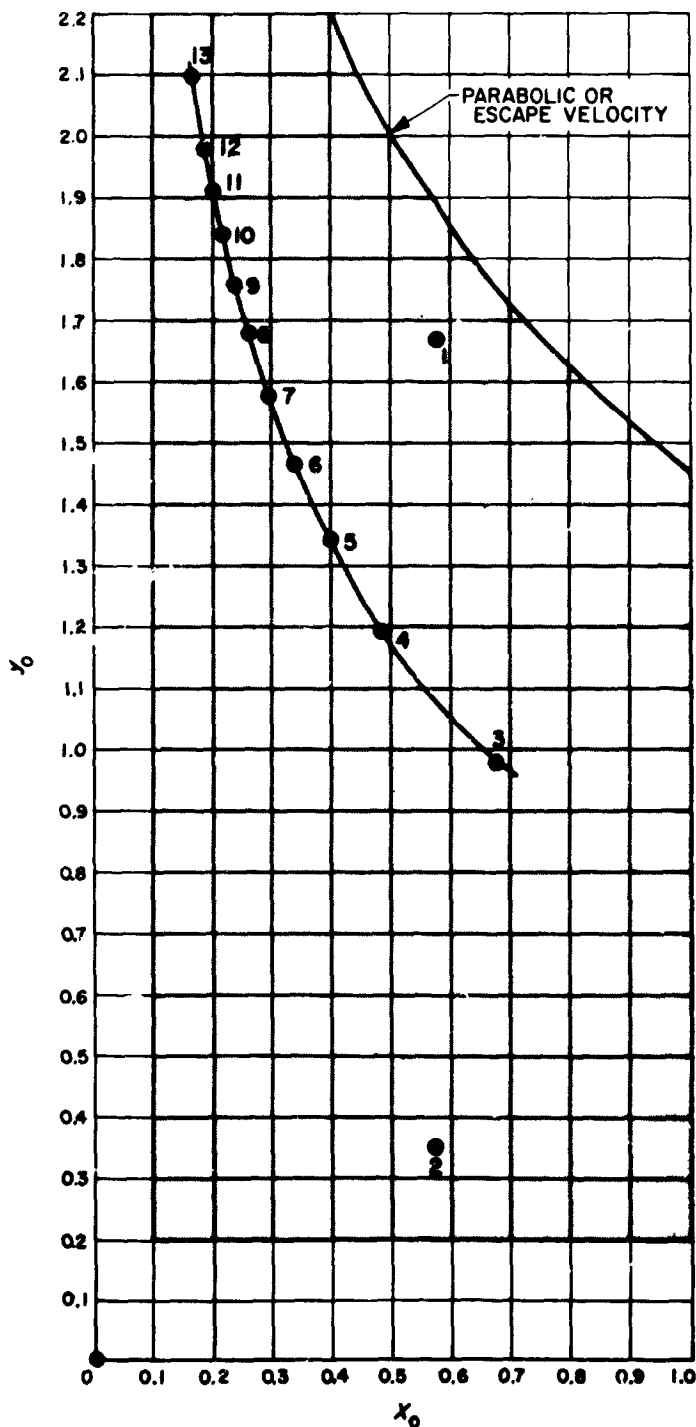


Fig. 1. Diagram of initial conditions for periodic orbits

coordinate system. In Fig. 3 we have shown the six most simple orbits referred to a coordinate system centered at one primary $m_1 = 1 - \mu$ (geocentric) and the other primary $m_2 = \mu$ (selenocentric). In our particular problem we have equal masses $1 - \mu = \mu = 1/2$, and we use the word "geocentric" for the coordinate system centered at the primary which is shown on the left side in our figures.

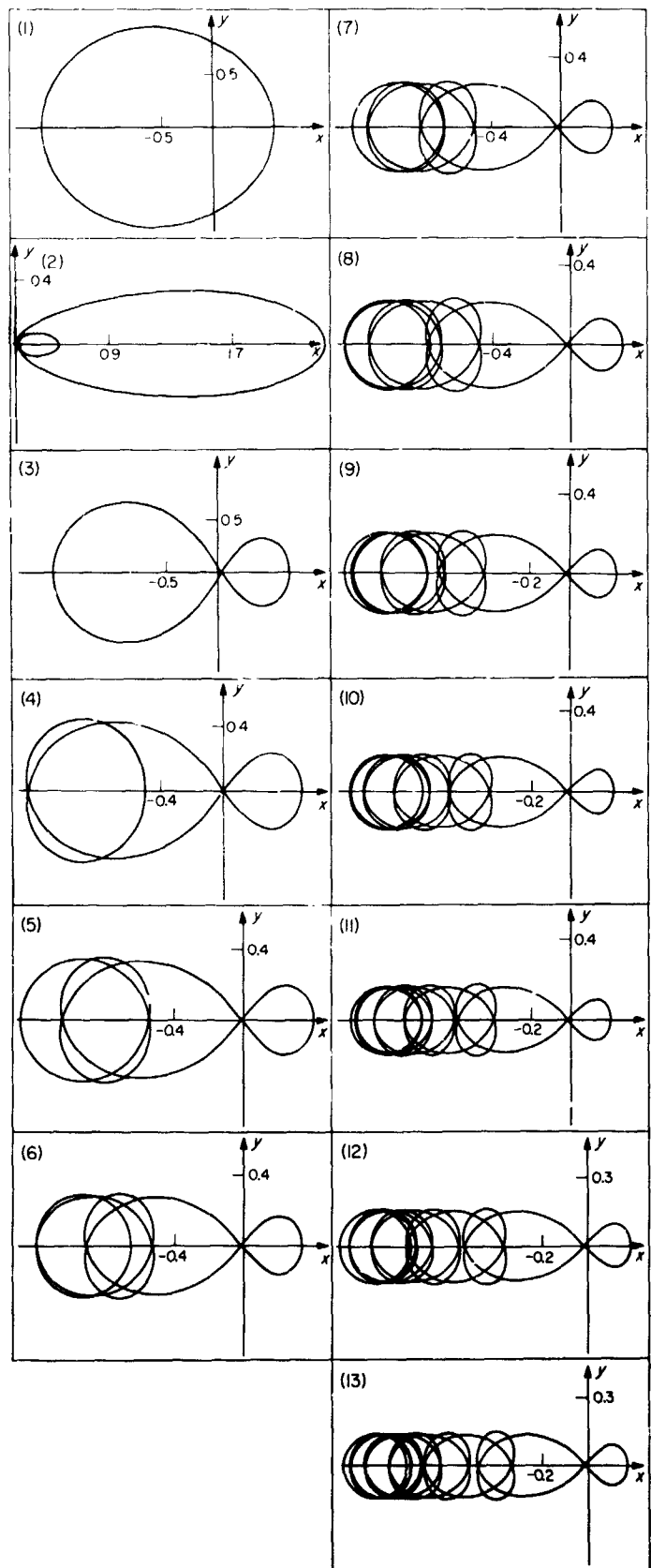


Fig. 2. The 13 periodic orbits in the barycentric (inertial) coordinate system

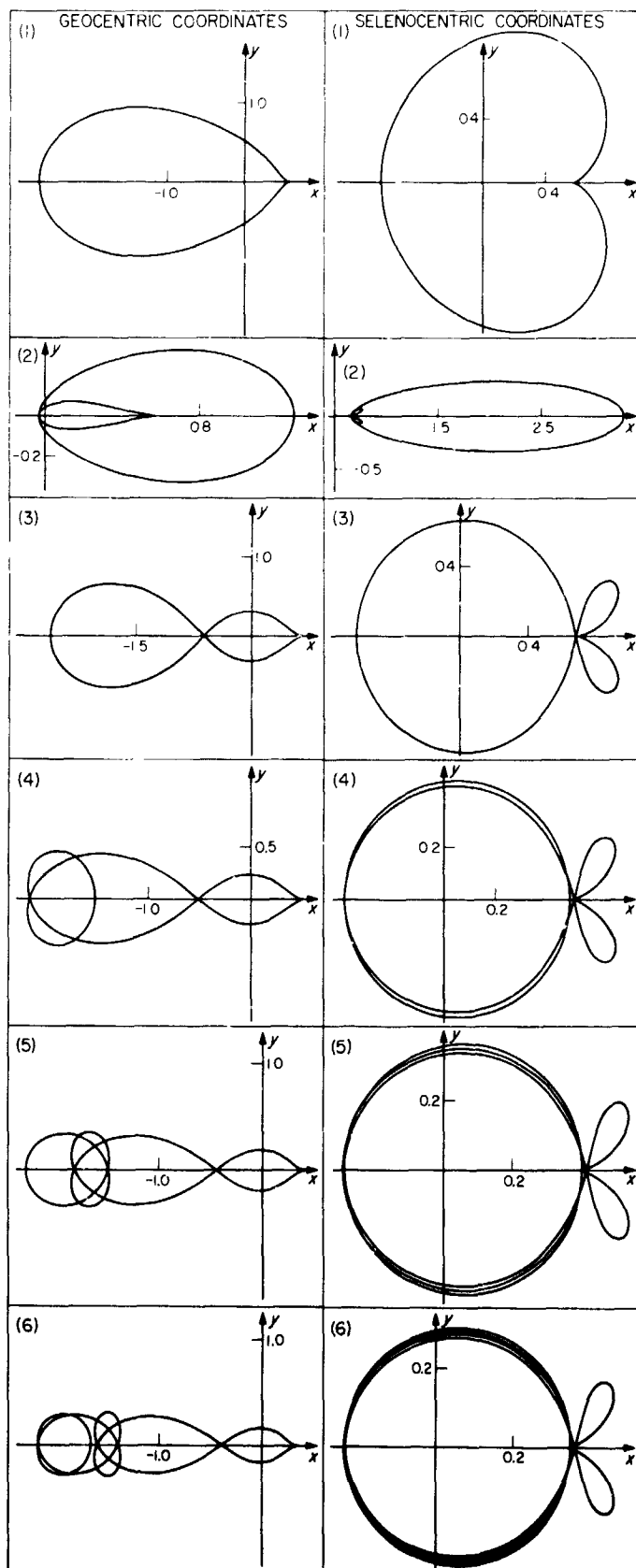


Fig. 3. The first 6 periodic orbits in the geocentric and the selenocentric coordinate systems

Table 1. Initial conditions for 13 periodic orbits

Orbit number	x_0	$\dot{y}_0$
1	0.579152	1.680050
2	0.574506	0.346101
3	0.682309	0.981417
4	0.492110	1.192088
5	0.396211	1.347039
6	0.336523	1.473383
7	0.295148	1.581601
8	0.264484	1.677079
9	0.240690	1.763014
10	0.221596	1.841481
11	0.205876	1.913916
12	0.192667	1.981358
13	0.171617	2.104198

Similarly, "selenocentric" refers to the primary shown on the right side. When we use the geocentric or selenocentric coordinates, we can see that some cusps are present in the form of the orbits. These cusps are due to accelerated translational motion of the frame of reference rather than the motion of the satellite.

As a matter of conclusion let us say that we have here thirteen isolated periodic orbits; however, these orbits are not isolated as far as e and μ are concerned. We have prepared a computer program which should generate new periodic orbits by varying e or μ in a more or less automatic way. For instance, we have been able to make the following computer runs:

- (1) In the neighborhood of orbit 1: 22 periodic orbits with $e = 1.0$ and $\mu = 0.5$ to 0.479.
- (2) In the neighborhood of orbit 2: 12 periodic orbits with $\mu = 1/2$ and $e = 1.0$ to 0.988; 26 periodic orbits with $e = 1.0$ and $\mu = 0.5$ to 0.458.
- (3) In the neighborhood of orbit 3: 23 periodic orbits with $e = 1.0$ and $\mu = 0.5$ to 0.466.

B. Mariner Mars 1969 Celestial Mechanics Experiment, J. D. Anderson

1. Summary

An error analysis has been performed for a set of 15 parameters determinable from the *Mariner Mars 1969* tracking data. Among these 15 parameters are the Earth-Moon mass ratio, the mass of Mars, the astronomical unit, and the position of Mars at spacecraft-Mars encounter.

It is these parameters which are of importance to the celestial mechanics experiment (CME), at least for tracking data taken within the mission duration of launch to encounter plus 3 months.

The motivation for the analysis is to determine the value of adding range data to the spacecraft tracking and to investigate the effect of gas jetting from the infrared spectrometer (IRS) during planetary encounter. The conclusion with respect to range data is that they significantly improve the value of the experiment. With range data it should be possible to determine the astronomical unit and the position of Mars to better than 100 m. Without range data this number is increased to about 1500 m, and realistically the error could be larger because of an inability to calibrate the charged particle contribution to the doppler signal. Both range and doppler data permit this calibration, and provide as well an improved opportunity for data validation.

If the uncertainty in the force imparted by the IRS gas jetting is less than ± 1 dyne over a 90-min period around encounter, then the effect on the CME is insignificant. For an uncertainty greater than ± 1 dyne it is necessary to include a model for the IRS jetting in the solution for the astronomical constants, particularly for the mass of Mars. With the inclusion of this model, we would attempt to reduce the uncertainty in the force from the IRS jetting to the level of ± 1 dyne. In order to accomplish such a reduction the *a priori* uncertainty should certainly not be greater than ± 100 dynes, a number two orders of magnitude larger than the ultimate goal of ± 1 dyne. The *a priori* uncertainty is defined as that obtained from preflight design and testing of the IRS gas jetting system and from any useful telemetry obtained during the jetting period.

2. Effect of Range Data on Celestial Mechanics Experiment

In Subsection 5, errors on the orbital parameters and astronomical constants are computed for doppler tracking data only and for doppler and range data. The results indicate the significance of adding ranging data as far as the celestial mechanics parameters are concerned. In a sense this is an extension of an earlier article (SPS 37-43, Vol. III, pp. 18-24) where the addition of range data to a single pass of tracking was considered. Of course it is an oversimplification to judge the value of ranging data solely on the basis of an error analysis of the sort given here. Other considerations, such as having an independent type of data to compare with the doppler measure-

ments, are equally important. Perhaps more important, D. Trask and L. Efron (SPS 37-41, Vol. III, pp. 3-11) point out that a combination of doppler and range observations can be used to calibrate the total (ionosphere and space plasma) charged particle effect on both types of data. This is especially important for *Mariner 1969*, which does not include a dual frequency experiment. A. Liu and R. Motsch (SPS 37-44, Vol. III, pp. 28-33) have explored further the possibility of a charged particle calibration.

Nevertheless, with the limitations of the present analysis clearly in mind, it is possible to conclude that adding range data to the doppler tracking can reduce the uncertainty on the astronomical unit from a region of ± 150 to ± 1300 m with doppler data only to a region of ± 25 to ± 65 m for range and doppler data. The position of Mars is reduced from a region of ± 150 to ± 1700 m to a region of ± 15 to ± 90 m. The effects on uncertainties in the masses of the Moon and Mars are not as pronounced as for the astronomical unit and the position of Mars.

3. Effect of IRS Gas Jetting on Celestial Mechanics Experiment

In this article it is assumed that the gas jetting from the IRS experiment occurs in a period 1 hr before planetary encounter to $\frac{1}{2}$ hr after encounter, a total period of 90 min. To see what this means in terms of a corruption of doppler and range data, consider the time variation in the geocentric radial velocity and range of the spacecraft during a 12-hr period centered about the encounter time. Fig. 4 shows a plot of the geocentric radial velocity and

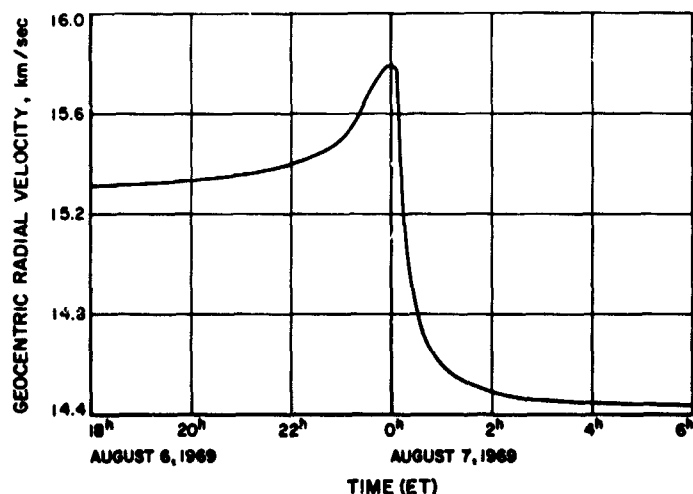


Fig. 4. Geocentric radial velocity vs time during planetary encounter

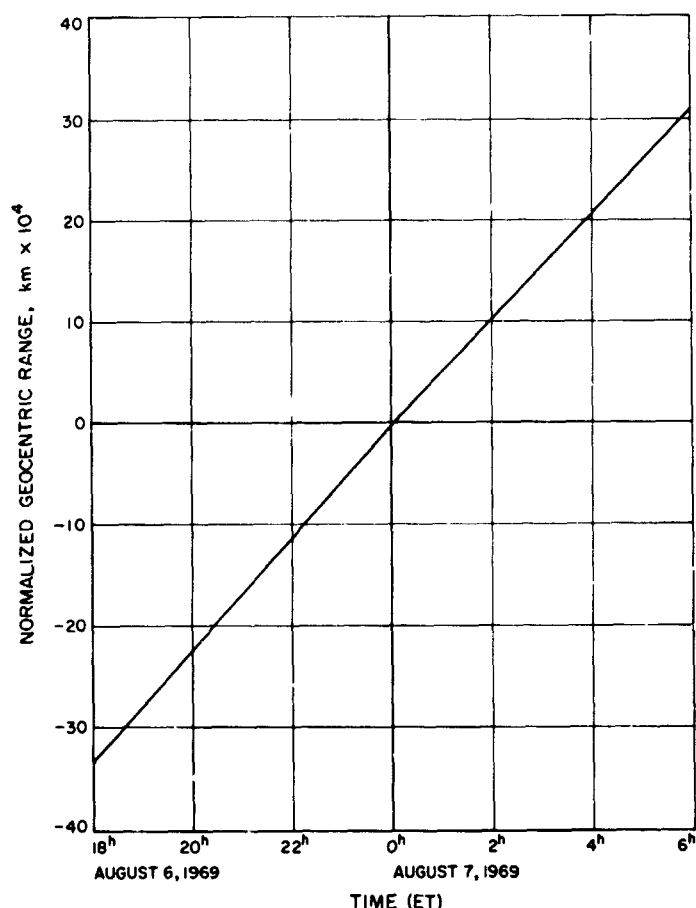


Fig. 5. Difference in geocentric range of date and geocentric range at planetary encounter vs time

Fig. 5 shows the difference between the geocentric range of date and the geocentric range at encounter. Now, as an extreme case, suppose that the tracking data during the 90 min of gas jetting are completely neglected in the determination of the constants. It is clear from Fig. 4 that a significant portion of the doppler curve is lost and that some sort of degradation will occur in the accuracy of constants which depend on the close approach to the planet for their determination. For example, the accuracy of the mass of Mars could easily be degraded. On the other hand, the time variation of the geocentric range during the encounter period does not exhibit any peculiarities during the 90 min of jetting, and the contribution of the range data to the determination of constants should not be significantly affected by the loss of data.

It is important to realize that the gas jetting affects the determination of the constants in two ways. The first and most straightforward is the introduction of a systematic source of error during the 90-min jetting period. This

can be handled, as in Subsection 4, by assuming that the tracking data are corrupted by the jetting and by weighting the corrupted data less in the weighted least squares procedure. The other effect on the determination of the constants is that the orbit of the spacecraft is perturbed during the jetting period, and, as a result, when the gas is depleted, the orbit is not known as well as if the gas jetting had not occurred. This situation can also be handled by properly representing the statistics of the orbital parameters at the end of the gas jetting. Thus, orbital errors which result from the jetting are mapped into the errors in the constants as determined from data taken before, during, and after the jetting period.

Effects of the jetting on the orbital position after a period T can be estimated from $\frac{1}{2} FT^2/m$, which for a force F of 100 dynes amounts to 56 m, while the effect on velocity from FT/m is 0.02 m/sec for the same force. In this article, the inability to compute a perfectly accurate orbit because of an uncertainty in the force model during gas jetting is ignored, and the only effect of jetting on the orbit and constants is presumably to decrease the accuracy of the data during the jetting period.

The numerical results of Subsection 5 show that the only constant significantly affected by the gas jetting is the mass of Mars and that an uncertain force greater than ± 100 dynes effectively destroys the doppler data taken during the jetting period. Even at ± 10 dynes, over 90% of the potential information in the close encounter data is lost. The addition of range data during the jetting period reduces the ± 10 -dyne effect on the mass from a 3σ degradation for doppler data only to a 2σ degradation for range and doppler data. In order that the gas jetting not seriously impair the solution for the mass, it is important that the random uncertainty on the force be considerably less than ± 10 dynes and that range data be taken during a significant portion of the jetting period. Ultimately, the force should be known to ± 1 dyne. A calibration for charged particle effects with range and doppler data is an important part of achieving this ultimate accuracy.

4. Gas Jetting as a Systematic Error Source¹

Let gas escape from a spacecraft of mass m at a force level equal to F in dynes. Then the acceleration a imparted to the spacecraft is $a = F/m$, and if the force is

¹The author wishes to thank D. W. Curkendall for assistance in preparing Subsection 4.

exerted over a time interval T , the deviation in the position of the spacecraft at the end of the interval, compared to a gas-free position, is $\Delta\rho = \frac{1}{2} aT^2$. Alternatively, suppose that counted doppler observations are obtained from which the change in topocentric range over some count time interval T_c is deduced. If the error in this range change is expressed in terms of an error $\sigma_{\dot{\rho}}$ in topocentric range rate, then the range change error over the count interval T_c is given by $T_c \sigma_{\dot{\rho}}$. Further, if the range error is accumulated over N samples of doppler data, then the integrated range error over the N samples is $T_c (N)^{1/2} \sigma_{\dot{\rho}}$, and for an even distribution of samples over the time interval T , such that $T = NT_c$, the integrated range error is $(TT_c)^{1/2} \sigma_{\dot{\rho}}$.

The approach followed here, in representing the error from an unknown gas jetting force, is to absorb the systematic error from the gas jetting into N independent range rate measurements. All that is required is to equate the range error $\frac{1}{2} aT^2$ from the gas jetting over the time interval T to the integrated range error $(TT_c)^{1/2} \sigma_{\dot{\rho}}$ over the same time interval. The resulting range rate error for N independent samples at intervals T_c is

$$\sigma_{\dot{\rho}} = \frac{1}{2} \frac{F}{m} \left(\frac{T^3}{T_c} \right)^{1/2} \quad (1)$$

Let the gas jetting occur over a period of 90 min and set $m = 259$ kg and $T_c = 1$ min. Then for F given in dynes the numerical expression of Eq. (1) is

$$\sigma_{\dot{\rho}} = 0.0989 F \text{ cm/sec} \quad (2)$$

The accuracy of the doppler data is conservatively rated at 0.1 cm/sec when sampled at 1-min intervals, and thus it is apparent from Eq. (2) that if the unknown force F is greater than ± 1 dyne, the gas jetting can be considered a serious source of error. One of the purposes of this article is to investigate fairly rigorously the implications of an unknown force of greater than 1 dyne, at least as far as celestial mechanics parameters are concerned.

5. Numerical Results

A version of the JPL orbit determination program (SPODPA—Mod II, program tape 9493) is used to obtain variances on various constants and parameters of astronomical interest. The orbital parameters for the spacecraft are represented by geocentric equatorial coordinates at the epoch, the position of Mars by heliocentric cartesian

coordinates at the time of the spacecraft's planetary encounter, the mass of the Moon by the selenocentric gravitational constant GM , the Martian gravitational constant by the mass of Mars in solar mass units, and the astronomical unit A is given in metric units. Actually, the fundamental unit is light seconds in the determination of the astronomical units, but error estimates are often given in metric units. The non-gravitational parameters γB , GT and GN are also included in the parameter statistics to allow for uncertainties caused by solar radiation pressure and attitude control forces. The uncertainties for these three parameters are given as a percentage of the total solar radiation pressure. Their inclusion as deterministic parameters results in realistic uncertainties only if the actual *Mariner 1969* data are reduced with an estimator designed to handle random time varying forces.

All error estimates are obtained for two extremes in assumed station location error. The first assumes no error and the second ± 5 m in station location uncertainties. Note that Hamilton, Grimes, and Trask (SPS 37-44, Vol. III, pp. 4-11) have stated that a goal of ± 1 m uncertainty in station locations is reasonable for *Mariner 1969* data reduction. By using ± 0 and ± 5 m, the region of uncertainty for the parameters should be conservatively defined.

Other numerical data which go into a computation of the uncertainties are given in Table 2.

The resulting errors in the 15 parameters of a simulated least squares solution are obtained from the covariance matrix computed in SPODPA—Mod II. In Table 3 the standard deviations are given with no IRS gas jetting during encounter. Four solutions are represented to demonstrate the advantage of including range data. They are:

Solution	Type of data
A	Range and range-rate data, no station location error
B	Range-rate data, no station location error
C	Range and range-rate data, ± 5 -m station location error
D	Range-rate data, ± 5 -m station location error

When IRS gas jetting errors are included, by the method of Subsection 4, there is no appreciable effect on any of the

Table 2. Data used in the computation of uncertainties

Geocentric true equatorial coordinates (March 18, 1969, 11 ^h 52 ^m 43.000 ET)	
x , km	-905058.17
y , km	-1140554.5
z , km	-1118697.0
$\dot{x}$, km/sec	-1.6423604
$\dot{y}$, km/sec	-2.1007967
$\dot{z}$, km/sec	-2.0376762
Heliocentric osculating elements referred to true ecliptic and equinox (August 3, 1969, 13 ^h 30 ^m 48.319 ET)	
a , km	119835290
e	0.22267465
T	February 26, 1969, 00 ^h 03 ^m 4.069 ET
i , deg	1.7830771
Ω , deg	351.34975
ω , deg	164.38212
Aerocentric osculating elements referred to true ecliptic and equinox (August 7, 1969, 00 ^h 00 ^m 53.541 ET)	
a , km	-953.33386
e	6.7704032
T	August 7, 1969, 00 ^h 00 ^m 53.541 ET
i , deg	15.817714
Ω , deg	246.27758
ω , deg	297.13641
Additional parameters	
Astronomical unit, km	149598500
Sun-Mars mass ratio	3088000
Epoch for position of Mars	August 7, 1969, 00 ^h 00 ^m 35.000 ET
Assumed doppler error	± 1 mm/sec random range rate error at 1-min sample interval
Assumed range error	± 367.4 -m random range error at 1-min sample interval (this corresponds to one range measurement per 10-hr tracking pass with an uncertainty of $367.4/(600)^{1/2}$ or 15 m)
Duration of tracking (continuous coverage)	March 19, 1969, 0 ^h 0 to November 7, 1969, 0 ^h 0
A priori error on parameters	
(x, y, z) , km	10 ⁶
$(\dot{x}, \dot{y}, \dot{z})$, km/sec	1.0
$(x_{\odot}, y_{\odot}, z_{\odot})$, km	500
GM , km ³ /sec ²	0.2
$M_{\odot}$, ppm	16.63
A , km	500
(γ_B, GT, GN) , %	1.0

Table 3. Standard deviations in the absence of IRS gas jetting

Parameter	No station location error ± 5 -m station location error			
	ρ and $\dot{\rho}$ (A)	$\dot{\rho}$ only (B)	ρ and $\dot{\rho}$ (C)	$\dot{\rho}$ only (D)
x , m	24	128	188	1054
y , m	31	91	517	2048
z , m	25	77	383	1984
$\dot{x}$, μ /sec	4	27	14	284
$\dot{y}$, μ /sec	3	12	29	188
$\dot{z}$, μ /sec	3	17	23	132
$x_{\odot}$, m	9	42	68	131
$y_{\odot}$, m	14	154	33	1658
$z_{\odot}$, m	7	59	87	647
GM , km ³ /sec ²	0.000056	0.000169	0.000399	0.000255
$M_{\odot}$, ppm	0.24	0.25	1.03	0.48
A , m	26	150	65	1312
γ_B , %	0.000231	0.00142	0.00102	0.0179
GT , %	0.000381	0.00300	0.00565	0.0394
GN , %	0.000268	0.00130	0.00349	0.00518

Table 4. Standard deviation in ppm for the mass of Mars at various levels of IRS gas jetting uncertainty

Uncertainty in IRS force, dyne	No station location error ± 5 -m station location error			
	ρ and $\dot{\rho}$ (A)	$\dot{\rho}$ only (B)	ρ and $\dot{\rho}$ (C)	$\dot{\rho}$ only (D)
No force	0.24	0.25	1.03	0.48
± 10	0.55	0.74	5.96	1.51
± 100	0.57	0.78	6.39	1.72
± 500	0.57	0.78	6.39	1.72

15 parameters except the mass of Mars. In Table 4 the results for the mass are given as standard deviations in parts per million of the total mass. Again the four solutions (A, B, C, D) are represented.

C. A Distribution Free Method Involving the Spacing of Independent Sample Points, H. Lass

Consider a continuous probability density function (PDF), $p(x)$, with $p(x) \geq 0$ and $\int_{-\infty}^{\infty} p(x)dx = 1$. An interesting problem whose solution is well known is the following. What is the probability that from n independent samples the proportion of the population between the minimum and maximum values of the sample data will exceed β , $0 < \beta < 1$?

To solve this problem, we note that if $z = \int_{-\infty}^x p(t)dt$ then z is a random variable whose PDF is uniform on the interval $[0, 1]$, since $0 \leq z \leq 1$ for all x and $q(z)dz = p(x)dx$, which implies that $q(z) = 1$ from $dz/dx = p(x)$ wherever $p(x)$ is continuous. From

$$\int_{x_{n-1}}^{x_{n+2}} p(t)dt = \int_{-\infty}^{x_{n+2}} p(t)dt - \int_{-\infty}^{x_{n-1}} p(t)dt = z_2 - z_1 \quad (1)$$

we recognize that we need only solve the problem for a uniform PDF on $[0, 1]$. Let u and v be the minimum and maximum values, respectively, from n sample values obtained from a uniform PDF. The joint density function on (u, v) is simply

$$p(u, v) = n(n-1)(v-u)^{n-2} \quad (2)$$

since there are n choices for u , $(n-1)$ choices for v , while the remaining $(n-2)$ sample values must lie between u and v with probability $(v-u)^{n-2}$. Hence

$$\begin{aligned} \text{Prob}(z_2 - z_1 \geq \beta) &= \int_0^{1-\beta} \int_{u+\beta}^1 n(n-1)(v-u)^{n-2} dv du \\ &= 1 - \beta^n - n\beta^{n-1}(1-\beta) \end{aligned} \quad (3)$$

If we desire that $\text{Prob}(z_2 - z_1 \geq \beta) = \alpha$, then $1 - \beta^n - n\beta^{n-1}(1-\beta) = \alpha$, which enables one to find the number of sample values, n , for fixed α, β . Thus 93 sample data are needed if we desire that the proportion of the population between the extreme values of the data exceed 95% ($\beta = 0.95$) with probability $\alpha = 0.95$.

Now the above analysis tells us little, if anything, about the spacing of the sample data between the extreme values of the data. Let us impose the condition that

$$\text{Prob} \left[\int_{y_i}^{y_{i+1}} p(t)dt \leq \alpha \right] \quad i = 1, 2, \dots, n-1 \quad (4)$$

with $y_1 < y_2 < \dots < y_i < y_{i+1} < \dots < y_n$, the ordered statistics. Again we need only consider the problem by dealing with the uniform PDF on $[0, 1]$.

The density function for the ordered statistics is

$$p(y_1, y_2, \dots, y_n) = n! \quad 0 \leq y_1 \leq y_2 \leq \dots \leq y_n \leq 1 \quad (5)$$

since there are n choices for y_1 , $(n-1)$ choices for y_2 , etc. We consider the transformation

$$\begin{aligned} y_1 &= z_0 \\ y_2 &= z_0 + z_1 \\ y_3 &= z_0 + z_1 + z_2 \\ &\vdots \\ y_n &= z_0 + z_1 + z_2 + \dots + z_{n-1} \end{aligned} \quad (6)$$

whose Jacobian is identically one, so that the joint density function on $z_0, z_1, \dots, z_{n-1}$ is

$$q(z_0, z_1, z_2, \dots, z_{n-1}) = n! \quad (7)$$

with $z_i \geq 0$, $0 \leq \sum_{i=0}^{n-1} z_i \leq 1$, since $z_0 = y_1$, $z_1 = y_2 - y_1$, $z_2 = y_3 - y_2$, $\dots$, $z_{n-1} = y_n - y_{n-1}$.

Since we are interested in the random variables $z_1, z_2, z_3, \dots, z_{n-1}$, we integrate out the random variable z_0 to obtain

$$h(z_1, z_2, \dots, z_{n-1}) = n! (1 - z_1 - z_2 - \dots - z_{n-1}) \quad (8)$$

with $z_i \geq 0$ and $\sum_{i=1}^{n-1} z_i \leq 1$. Thus

$$\begin{aligned} P_n(\alpha) &\equiv \text{Prob}(z_1 \leq \alpha, z_2 \leq \alpha, \dots, z_{n-1} \leq \alpha) \\ &= n! \int_0^\alpha \int_0^\alpha \dots \int_0^\alpha (1 - z_1 - z_2 - \dots - z_{n-1}) \\ &\quad U(1 - z_1 - z_2 - \dots - z_{n-1}) dz_1 \dots dz_{n-1} \end{aligned} \quad (9)$$

with $U(\xi)$ the unit step function, $U(\xi) = 0$ for $\xi < 0$, $U(\xi) = 1$ for $\xi \geq 0$.

Let $k + \gamma = 1$ with $0 \leq \gamma < \alpha$ so that k is the largest integer for which $k\alpha \leq 1$. Successive integrations of Eq. (9) yield

$$P_n(\alpha) = \sum_{r=0}^k (-1)^r \binom{n-1}{r} (1 - r\alpha)^n \quad (10)$$

If, for example, $n = 4$, $\alpha = 1/4$, we have $k = 3$ and $P_4(1/4) = 15/16$.

The author has been unable to find a closed form for $P_n(\alpha)$. However, a computer should enable one to evaluate $P_n(\alpha)$ for various values of α and n .

D. Techniques for Studying Dispersions of Nonlinear Systems, S. R. McReynolds

Let X be a continuous ensemble of vector objects $\{x^i\}$ with an initial known density function $p_0(x)$. Suppose that each object satisfies a set of nonlinear differential equations

$$\dot{x}_j = f^j(x, t) \quad (j = 1, n) \quad (1)$$

Then an important engineering question is to find the density function $P(x, t)$ of x at a future time t . In general, this task is impossible from an analytic standpoint and laborious from a computational standpoint. In this article we concern ourselves with techniques that may be used to determine quantitative characteristics of $P(x, t)$.

1. The Continuity Equation

Elementary consideration, such as that employed in fluid mechanics, leads us at once to a partial differential equation for $P(x, t)$:

$$\frac{\partial P}{\partial t} + \sum_i \frac{\partial}{\partial x_i} (P f^i) = 0 \quad (2)$$

This equation is referred to as the "continuity equation," and is a special case of the Fokker-Planck equation.

2. Solution by Characteristics

Eq. (2) is a quasi-linear, first-order partial differential equation. Together with the initial density function $P_0(x)$ the solution may be computed numerically. To avoid the problem of computing the partials of P with respect to x by differencing, one may employ the use of characteristics.

In this case, the characteristics consist of the trajectories that the objects follow, namely, solutions to the ordinary set of equations $\dot{x} = f(x, t)$. Along these characteristics the derivative of P , which we denote as $\dot{P}$, is given by

$$\dot{P} = -P \sum_i \frac{\partial f^i}{\partial x_i} \quad (3)$$

The solution to this differential equation is given by

$$P(T) = P_0 \exp \left[- \int_0^T \sum_i \frac{\partial f^i}{\partial x_i} dt \right] \quad (4)$$

The method of solution proceeds as follows. Partition the set of initial conditions by selecting points in the region of interest. Map these initial points forward by integrating the system equations until time T . The probability density of the ensemble at this point is given by Eq. (4), where the integral is along the corresponding trajectory

Eq. (4) indicates the relationship between stability and diffusion of densities. We note that $\sum_i \partial f^i / \partial x_i = \text{trace}(\partial f^i / \partial x_i)$ = the sum of real parts of eigenvalues of $(\partial f^i / \partial x_i)$. If the system is stable, the real parts of the eigenvalues are negative, and hence $\sum_i \partial f^i / \partial x_i < 0$. Thus, as indicated by Eq. (4), the density of objects will increase with the time along such a trajectory. On the other hand, if the system is unstable, then $\sum_i \partial f^i / \partial x_i > 0$, and the density will decrease with time along the trajectory.

Example:

$$\dot{x} = -x, \quad P_0(x) = \begin{cases} 1, & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$

$$P(x, T) = \begin{cases} \frac{1}{2} e^{-x^2 T} & |x| \leq e^{-x^2 T} \\ 0 & |x| > e^{-x^2 T} \end{cases}$$

Some systems have the property that $\sum_i \partial f^i / \partial x_i = 0$. For these systems the probability densities remain constant along a trajectory. An important example of this is Hamiltonian systems. Hamiltonian systems are systems which satisfy

$$\dot{p}_i = \frac{\partial H}{\partial q_i}; \quad \dot{q}_i = -\frac{\partial H}{\partial p_i} \quad (i = 1, n)$$

Obviously $\sum_i \partial f^i / \partial x_i = \sum_i (\partial H / \partial p_i \partial q_i - \partial H / \partial q_i \partial p_i) = 0$. Conservation of probability density for Hamiltonian systems is known in classical mechanics as Liouville's theorem.

3. A Functional Approximation Technique

For many important cases the initial density is Gaussian

$$P_0(x) = K e^{-\frac{1}{2} (x - \bar{x}_0)^T S (x - \bar{x}_0)} \quad (5)$$

Here S is a positive semi-definite matrix, and $\|x - \bar{x}_0\|_S^2$ is the quadratic form $(x - \bar{x}_0)^T S (x - \bar{x}_0)$ (T denotes transposition). From Eq. (3), it is clear that $P(x, t)$ has the form

$$P(x, t) = K e^{-L(x, t)} \quad (6)$$

Now by substituting this into Eq. (2), it is seen that $L(x, t)$ satisfies the partial differential equation

$$\frac{\partial L}{\partial t}(x, t) + \sum_i \frac{\partial L}{\partial x_i}(x, t) f^i(x, t) = r(x) \quad (7)$$

where

$$r(x) = \sum_i \frac{\partial f^i}{\partial x_i}(x, t) \quad (8)$$

The initial boundary condition for $L(x, t)$ is given by

$$L(x, t_0) = -\frac{1}{2} \|x - \bar{x}_0\|_S^2 \quad (9)$$

The functional technique we shall suggest here consists of approximating $L(x, t)$ in the neighborhood of a trajectory (characteristic) with a quadratic form.

Let a nominal trajectory $\bar{x}(t)$ be chosen as follows:

$$\begin{aligned} \bar{x}(0) &= x_0 \\ \dot{\bar{x}}_i &= f^i(\bar{x}, t) \end{aligned} \quad (10)$$

Now along this trajectory, differential equations (characteristic equations) may be derived for the partial derivatives of $L(x, t)$ with respect to x . These are obtained by taking the partial derivatives of Eq. (7) with respect to x . The first partial derivative of Eq. (7) with respect to x_j , evaluated at $\bar{x}$, yields

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial x_j} + \sum_i \frac{\partial}{\partial x_i} \frac{\partial L}{\partial x_j} f^i(\bar{x}, t) + \sum_i \frac{\partial L}{\partial x_i} \frac{\partial f^i}{\partial x_j}(\bar{x}, t) = \frac{\partial r}{\partial x_j}(\bar{x}, t) \quad (11)$$

Now we note that the operator $\frac{\partial}{\partial t} + \sum_i \frac{\partial}{\partial x_i} f^i(\bar{x}, t)$ is the total derivative operator along the characteristic $\bar{x}(t)$. Denoting this operator by $\dot{}$, Eq. (11) becomes

$$\frac{\partial \dot{L}}{\partial x_j} = - \sum_i \frac{\partial L}{\partial x_i} \frac{\partial f^i}{\partial x_j}(\bar{x}, t) + \frac{\partial r}{\partial x_j}(\bar{x}, t) \quad (12)$$

Note that from Eq. (9),

$$\frac{\partial L}{\partial x_j} = -(x - \bar{x}_0)_j \Big|_{x=\bar{x}(0)} = 0 \quad (13)$$

Hence Eqs. (12) and (13) may be used to compute $\partial L / \partial x_j$ along the characteristic x .

To obtain the characteristic equation for $\partial^2 L / \partial x_j \partial x_k$, the partial derivative of Eq. (11) is taken with respect to x_k . This results in

$$\begin{aligned} \frac{\partial}{\partial t} \frac{\partial^2 L}{\partial x_j \partial x_k} + \sum_i \frac{\partial}{\partial x_i} \frac{\partial^2 L}{\partial x_j \partial x_k} f^i(\bar{x}, t) \\ + \sum_i \left[\frac{\partial^2 L}{\partial x_i \partial x_j} \frac{\partial f^i}{\partial x_k} + \frac{\partial^2 L}{\partial x_i \partial x_k} \frac{\partial f^i}{\partial x_j} + \frac{\partial L}{\partial x_i} \frac{\partial^2 f^i}{\partial x_j \partial x_k} \right] \\ = \frac{\partial^2 r}{\partial x_j \partial x_k} \end{aligned} \quad (14)$$

Using $\dot{}$ for the characteristic derivative, this becomes

$$\begin{aligned} \frac{\partial^2 \dot{L}}{\partial x_j \partial x_k} + \sum_i \left[\frac{\partial^2 L}{\partial x_i \partial x_j} \frac{\partial f^i}{\partial x_k} + \frac{\partial^2 L}{\partial x_i \partial x_k} \frac{\partial f^i}{\partial x_j} + \frac{\partial L}{\partial x_i} \frac{\partial^2 f^i}{\partial x_j \partial x_k} \right] \\ = \frac{\partial^2 r}{\partial x_j \partial x_k} \end{aligned} \quad (15)$$

The initial value of $\partial^2 L / \partial x_j \partial x_k$, which is obtained from Eq. (9), is

$$\frac{\partial^2 L}{\partial x_j \partial x_k} = S \quad (16)$$

Note that the terms in Eq. (14) depend upon $\partial L / \partial x_i$, $\partial^2 L / \partial x_i \partial x_j$, $\bar{x}$, and known functions. Hence, the combined initial value problem defined by Eqs. (10), (12), (13), (14), and (15) is complete and well defined. This technique may be used to compute higher derivatives of L .

In each case a complete set of characteristic equations will be obtained.

The method of approximation consists of employing a finite number of terms in the expansion of $L(x, t)$ around $\bar{x}(t)$. If the first two terms are employed, then the approximation to the density function is the Gaussian density function

$$P(x, t) = K \exp \left[-1/2 (x - x^*)^T \frac{\partial^2 L}{\partial x^2} (x - x^*) \right] \quad (17)$$

when

$$x^* = \bar{x} - \left(\frac{\partial^2 L}{\partial x^2} \right)^{-1} \frac{\partial L}{\partial x} \quad (18)$$

This should be a good approximation to the actual solution in the region if $\|x - x^*\|$ is small.

Note that for systems that conserve probability density ($r = 0$), $\partial L / \partial x = 0$. Hence, the approximate mean, or point of maximum density, lies along the original trajectory. In this case, $\partial^2 L / \partial x_i \partial x_j$ satisfies the characteristic equation

$$\frac{\partial^2 \dot{L}}{\partial x_i \partial x_j} = - \sum_k \left[\frac{\partial^2 \dot{L}}{\partial x_i \partial x_j} \frac{\partial f^k}{\partial x_j} + \frac{\partial^2 L}{\partial x_i \partial x_j} \frac{\partial f^k}{\partial x_i} \right]$$

This is the same result that would be obtained by linearizing the equations around the nominal trajectory and then applying the continuity equation to solve the linearized problem exactly.

Reference

1. Schubart, J., "Numerische Aufsuchung Periodischer Losungen im Dreikorperproblem," *Astronomische Nachrichten*, Band 283, pp. 17-22, 1958.

II.3 Computation and Analysis

SYSTEMS DIVISION

A. Computer Technique for Electron and Ion Gun Design, N. C. Adams

The article "Electrostatic Charge Densities on an Electro-optical Star Tracker," by N. C. Adams, E. M. Keberle, and W. Silsby (SPS 37-42, Vol. IV, pp. 10-15) was the first phase of an attempt to evaluate the motion of electrons through an electric field set up inside the star tracker. Several problems arose with this basic approach, forcing further investigation and study. A more computer-oriented method was subsequently discovered which permits the analysis of electron motion through instruments with general-type external boundary conditions. A computer program is currently available for this work.¹

The problem was approached by replacing the continuous Laplace (or Poisson's) equation by a set of finite difference equations. The region of interest was then overlayed with a fine mesh (Fig. 1), and the difference equation was written for each mesh point. The resulting matrix equation and the equations of motion were then solved.

¹The program in its original form was obtained from Dr. V. Hamza, Bellcomm.

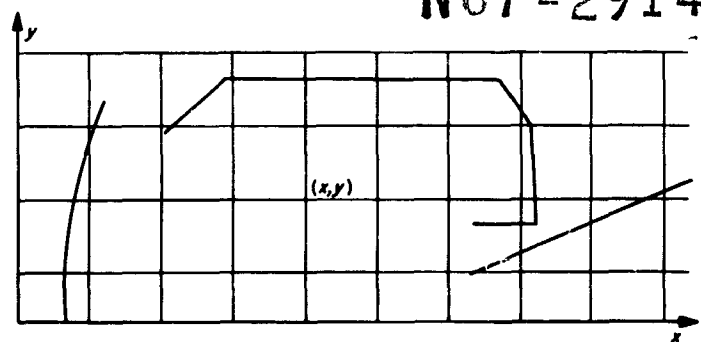


Fig. 1. Mesh overlay on electrodes

Laplace's equation in two dimensions

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

becomes

$$V(x, y) = \frac{1}{4} [V(x+h, y) + V(x, y+h) + V(x-h, y) + V(x, y-h)]$$

for any mesh point (x, y) .

Poisson's equation in two dimensions

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -\frac{\rho}{\epsilon_0}$$

becomes

$$V(x, y) = \frac{1}{4} [V(x + h, y) + V(x, y + h) + V(x - h, y) + V(x, y - h)] + \frac{\rho}{\epsilon_0}$$

By writing the difference equations for each mesh point we obtain N linear algebraic equations with N unknowns. This matrix equation is then solved and numerical differentiation is performed to compute $\partial V/\partial x$ and $\partial V/\partial y$. This yields the force which is required for the equations of motion.

The following is an example of the analysis which can be performed using this program. Given a specific electrode configuration one would like to focus the electrons on an image plate in such a manner as to minimize the dispersion diameter. Different runs can be made varying one design parameter at a time to determine its effect on the focusing. Once a satisfactory dispersion diameter has been obtained in conjunction with an acceptable geometry the design is complete.

At this time the design parameters must be changed manually. Future work will include automating this part of the program. Continued modifications are expected for this program; however, it is available for use in its current state.

B. Perihelion Motion of Mercury, J. D. Mulholland

The perihelion motion of the planet Mercury has been a subject of lively interest ever since the discovery, in the mid-nineteenth century, that Newtonian theory failed to satisfy the observational data. Numerous hypotheses were advanced in attempting to account for the discrepancy, but it was not until the development of the general theory of relativity that a reasonably satisfactory explanation was found. This theory predicts a motion of perihelion in excess of the Newtonian value by the amount (Ref. 1)

$$\frac{\delta \tilde{\omega}}{\phi} = \frac{12 \pi^2 a^2}{C^2 T^2 (1 - e^2)} \quad (1)$$

where

$$\frac{\delta \tilde{\omega}}{\phi} = \text{motion of perihelion per unit of revolution (non-dimensional)}$$

a = semi-major axis

C = speed of light

e = eccentricity

T = orbital period

For the planet Mercury, this amounts to some 43" per century in excess of the Newtonian prediction. The degree to which this figure satisfies the discrepancy noted above has long been regarded as a strong confirmation of general relativity.

Recently, doubt has been expressed in some quarters that the alleged confirmation is really what it seems. R. H. Dicke (Ref. 2), in advocating an alternative gravitational theory, has discussed the implications for general relativity which attend his measured value of the oblateness of Sun. His computations indicate that the measured solar oblateness requires a motion of 3" per century in the perihelion of Mercury, which is incompatible with the published comparisons between observations and general relativity theory (Ref. 1, 3, 4).

While there are still some unanswered questions regarding the effect of solar oblateness, it is nonetheless worthwhile to re-examine the contributions to the perihelion motion according to Newtonian theory and that due to Eq. (1). These values are directly related to the adopted values of the planetary masses, for many of which there have been significant redeterminations recently.

I have taken the data given by Clemence (Ref. 1) and revised them linearly with the changes in the adopted values of the masses and their standard deviations. The revised value of the relativity effect is computed using $C = 299\,792.5$ km/sec and elements of Mercury taken from Ref. 5. Table 1 gives the resulting data.

The application of the revised masses appears to have improved the correspondence between theory and observation to an extreme degree. The discrepancy is indicated as less than one-fourth the formal standard error. If we follow Clemence (Ref. 1) and estimate the oblateness

effect (based on the rotation of the outer parts of Sun) as being $0''.10 \pm 0''.20$, then the discrepancy is reduced to

$$0''.05 \pm 0''.68$$

which is truly overpowering, if one believes it. This seems to indicate that no reconciliation is possible between the existence of a solar oblateness as large as is claimed by Dicke and the soundness of general relativity. I do not, however, regard this as a valid conclusion to be drawn from the present data.

The most obvious of the weak links in the comparison of Table 1 is the value of general precession. The range of values currently in use for this quantity (reduced to 1850.0) is from Newcomb's $5024''.53$ to Oort's $5025''.645$. Newcomb's value is probably too low, but the present condition of the theory of motion of Earth is bad enough that we cannot hope for a great deal of confidence in any compromise value, such as that in Table 1.

On the other hand, the planetary contributions are based on planetary theories, and changes in the theories

will contribute changes to $\delta\omega$. Such changes have occurred both because of analytic refinement and due to revision of mass values. Linear adjustments in $\delta\omega$ due to mass revisions will not entirely account for the true effect. If Dicke's results are confirmed, it will be necessary to recompute the Newtonian components of $\delta\omega$ *ab initio* in order that a reliable comparison of theory and observation can be made.

C. FORTRAN IV Language Extensions: ENCODE/DECODE Statements, H. L. Smith

1. Introduction

This article describes an extension to the FORTRAN IV language that includes the ENCODE and DECODE statements implemented for the IBM FORTRAN IV compiler in the 7094 computer. These statements are conceptually associated with the formatted WRITE and READ statements, respectively. In an ENCODE/DECODE operation, no actual input/output takes place; data conversion and transmission take place between an internal buffer and elements specified by a list.

2. Description

In a number of data processing applications, it is necessary to process data at the character level. While the FORTRAN language is word-oriented, some provision is made to process character data in the formatted input/output statements. The ENCODE and DECODE statements utilize the syntax of the FORMAT statement and the subroutines which are normally in memory to process the FORMAT statements. In addition to the compiler changes, modifications to the FORTRAN library subroutines FCNV and FIOH were made to interface with a new subroutine FRRD.

a. ENCODE statement. The general form of the ENCODE statement is

ENCODE (a, b, c, d) list

where

a is an unsigned integer constant or an integer variable whose value specifies the number of characters per internal record.

b is either the statement number or the array name of the FORMAT statement describing the data being encoded.

Table 1. Contributions to perihelion advance
(epoch 1850.0)

Newtonian theory		
Planet	1/mass	$\delta\omega$
Mercury	$6047\ 000 \pm 17000$	$0''.025 \pm 0$
Venus	$408\ 505 \pm 6$	$277''.489 \pm 0.11$
Earth-Moon	$328\ 900 \pm 1$	$90''.172 \pm 0$
Mars	$3098\ 335 \pm 600$	$2''.527 \pm 0$
Jupiter	1047.404 ± 0.02	$153''.584 \pm 0$
Saturn	3499.6 ± 0.4	$7''.301 \pm 0$
Uranus	22930 ± 6	$0''.141 \pm 0$
Neptune	19070 ± 21	$0''.043 \pm 0$
Total		$531''.282 \pm 0.01$
General relativity and precession		
General relativity		$42''.98 \pm 0.03$
General precession (Duncombe)		$5025''.32 \pm 0.50$
Total value, observed value, and discrepancy		
Total value (Newtonian theory + relativity + precession)		$5599''.59 \pm 0.50$
Observed value		$5599''.74 \pm 0.41$
Discrepancy		$0''.15 \pm 0.65$

c is an array, an array element, or a variable name which specifies the starting location of the internal buffer.

d is an optional integer variable into which will be stored, upon completion of the operation, the number of characters actually generated.

list is as specified for a WRITE statement.

The ENCODE statement causes the data items specified by the *list* to be converted to character strings, according to the FORMAT specified by *b*, and placed in storage beginning at location *c*.

Characters are placed into the buffer, starting with the first character position of the location specified by *c*, in consecutive character positions. When a new record is begun, it starts at the first character position following the previous record; in other words, the second record begins at character $a + 1$. The beginning of logical records, except for the first, can be referenced only if *a* is an integral multiple of the length in bytes of the variable *c*.

If the number of characters generated by the FORMAT statement is greater than the specified size of the record, the extra characters are lost; they are not filled into the following record. If fewer characters are generated than are necessary to fill the record, it is filled out with blanks. The ENCODE operation of filling the unused record with blanks is done after generating the characters. This differs from the formatted (BCD) WRITE operation of only blanking the first three words of the output buffer prior to generating the characters.

If *d* is specified, it will be set to the number of characters generated for all of the records processed, excluding any trailing blanks used to fill out the record.

b. DECODE statement. The general form of the DECODE statement is

DECODE (*a, b, c, d*) *list*

where

a is an unsigned integer constant or an integer variable whose value specifies the number of characters per internal record.

b is either the statement number or the array name of the FORMAT statement describing the data being decoded.

c is an array, an array element, or a variable name which specifies the starting location of the internal buffer.

d is an optional integer variable into which will be stored, upon completion of the operation, the number of characters actually scanned.

list is as specified for a READ statement.

The DECODE statement causes the character string beginning at location *c* to be converted to data items, according to the FORMAT specified by *b*, and stored in the elements specified by the *list*.

Characters are obtained from the buffer starting with the first character position of the location specified by *c*, from consecutive character positions. When a new record is begun, it starts at the first character position following the previous record; in other words, the second record begins at character $a + 1$. The beginning of logical records, except for the first, can be referenced only if *a* is an integral multiple of the length in bytes of the variable *c*.

As with formatted READ operations, if the FORMAT statement requires more characters from a record than are specified by the count *a*, the extra characters are taken from increasing storage addresses following the specified record. The extra characters should not be assumed to be blanks.

If *d* is specified, it will be set to the number of characters scanned for all the records processed, excluding the characters passed over when skipping to the next record.

3. Examples

The following is a method of packing the partial contents of two words into one word. Information is stored in memory as follows:

LOCX SSSSxx (6 BCD characters/word)

LOCY xxxxTT

To form SSSSTT in storage location LOCZ use the following:

DECODE(6,1,LOCY) TEMP

1 FORMAT(4X,A2)

ENCODE(6,2,LOCZ) LOCX, TEMP

2 FORMAT(A4,A2)

The DECODE statement places the last 2 BCD characters of LOCY into the first 2 characters of TEMP. The ENCODE statement packs the first 4 characters of LOCX and the first 2 characters of TEMP into LOCZ.

The ENCODE statement may be used to compute a field width specification for a FORMAT definition at object time. Assume that in the statement `FORMAT(2A6,Ij)` the programmer wishes to specify `j` at some point in the program. The following permits `j` to vary.

```

REAL FMT' (2)
.
.
ENCODE (12,1,FMT) J
1 FORMAT (6H(2A6,I1,1H))
.
.
WRITE (6,FMT) A,B,M

```

ENCODE packs the value of `J` (converted to BCD) with the characters: `(2A6,I)`. This packed FORMAT is stored in locations `FMT(1)` and `FMT(2)`. The WRITE statement will output `A` and `B` under the specification `A6` and the quantity `M` under the specification `Ij`.

4. Status

Implementation of the ENCODE/DECODE statements is currently operational. The coding is being forwarded to IBM for inclusion in the SHARE Program Library, thus making it available for all 7090/7094 computer installations.

D. Syntactic Processors, D. A. Germann

1. Introduction

Computer-program language processors have traditionally developed along the lines of embedding analysis and code generation as an integral part of the processor. Although this generally produces the fastest processor, modifications to the processor are usually difficult and time consuming. Consequently, as a research tool in language development, the traditional processor has very limited use.

To provide the tools needed for the investigation of languages, syntax-directed processors are used, since modifications to the processor are relatively simple. This

article will present the fundamentals of syntactic processors, and in particular, discuss two such processors which have proven quite useful.

2. Syntax Notation

The syntax of a programming language is the specification of the structure of the language. In fact, the syntactics of programming languages are quite analogous to that of natural languages. For example, a paragraph could be compared to a procedure or block and a common sentence could be compared to a declaration or assignment statement in a programming language. Just as diagramming natural languages aids in understanding their structure, similar diagramming of programming languages will clarify their structure. One popular form for such diagramming is known as Backus Normal Form (BNF) (Ref. 6). The principal features of a dialect of this form are:

- (1) A definition has the form

$$\text{syntactic-type-being-defined} := \text{definition-1} / \text{definition-2} / \dots$$
- (2) The symbol `:=` separates the element being defined from its definition.
- (3) The symbol `/` separates alternates.
- (4) Terminal symbols are enclosed by quote marks.
- (5) Non-terminal symbols stand by themselves without quotes.
- (6) Definitions may be grouped by the use of parentheses.
- (7) Iteration of a definition is indicated by enclosing it in parentheses preceded by a dollar sign.
- (8) A null definition is indicated by the symbol `.NULL`.

As a simple example, consider the following:

`person := man / woman`

This simply defines a `person` as being either a man or a woman. Now, assume we wish to define a crowd as being an arbitrary number of persons. Using rule 7 we could write:

`crowd := $(person)`

As another example, consider the following algebraic expressions:

$$C$$

$$B + C$$

$$A * (B + C)$$

The syntactic equations which define these expressions are as follows:

$$\text{SUM} := \text{TERM} \$ ('+' \text{ TERM})$$

$$\text{TERM} := \text{PRIME} \$ ('*' \text{ PRIME})$$

$$\text{PRIME} := \text{VAR} / ('(\text{SUM})'$$

$$\text{VAR} := 'A' / 'B' / 'C'$$

As a final example, the syntax for a subset of FORTRAN IV is shown in Fig. 2.

3. Development of Syntactic Processors

Before discussing syntactic processors, a few definitions need to be introduced (Ref. 7):

- (1) *Source language*: The language being described. For example, in Fig. 2 the source language is a subset of FORTRAN IV.

```

.SYNTAX.
FORTRAN-PROGRAM..= $( ARRAY-STATEMENT ) $( STATEMENT ) END-CARD
ARRAY-STATEMENT..= 'DIMENSION' ARRAY-LST EOC
ARRAY-LST..= SIM-VARIABLE '(' INTEGER ')' $( ',' SIM-VARIABLE '('
                                         INTEGER ')' )
STATEMENT..= REMARK / CODE-STATEMENT
REMARK..= 'C'
CODE-STATEMENT..= LABEL/.NULL ( FORMAT-STATEMENT / UNLABELED-STATEMENT )
LABEL..= INTEGER
FORMAT-STATEMENT..= 'FORMAT' '(' FORMAT-SPECS ')' EOC
UNLABELED-STATEMENT..= IF / GOTO / STOP / READ / WRITE / ASSIGNMENT
IF..= 'IF' '(' BOOLEAN-EXPRES ')' UNLABELED-STATEMENT EOC
BOOLEAN-EXPRES..= SUM ( REL-OP SUM )/.NULL
REL-OP..= '.EQ.' / '.NE.' / '.GT.' / '.LT.'
GOTO..= 'GO' 'TO' LABEL EOC
STOP..= 'STOP'
READ..= 'READ' UNIT-FORMAT IO-VAR-LST EOC
WRITE..= 'WRITE' UNIT-FORMAT IO-VAR-LST EOC
UNIT-FORMAT..= '(' INTEGER ',' LABEL ')'
IO-VAR-LST..= VARIABLE $( ',' VARIABLE )
ASSIGNMENT..= VARIABLE '=' SUM EOC
SUM..= TERM $( AOS TERM )
TERM..= PRIME $( MOD PRIME )
PRIME..= AOS/.NULL ( VARIABLE / CONSTANT / FUNCTION /
                    '(' SUM ')' )
AOS..= '+' / '-'
MOD..= '*' / '/'
VARIABLE..= SIM-VARIABLE ( '(' SUM ')' )/.NULL
SIM-VARIABLE..= ALPHA $( ALPHA-NUM )
CONSTANT..= INTEGER ( '.' INTEGER/.NULL )/.NULL
FUNCTION..= NAME ( '(' SUM $( ',' SUM ) ')' )/.NULL
NAME..= ALPHA $( ALPHA-NUM )
ALPHA..= 'A' / 'B' / 'C' / 'D' / 'E' / 'F' / 'G' / 'H' /
        'I' / 'J' / 'K' / 'L' / 'M' / 'N' / 'O' / 'P' /
        'Q' / 'R' / 'S' / 'T' / 'U' / 'V' / 'W' / 'X' /
        'Y' / 'Z'
DIGIT..= '1' / '2' / '3' / '4' / '5' / '6' / '7' / '8' /
        '9' / '0'
INTEGER..= $( DIGIT )
ALPHA-NUM..= ALPHA / DIGIT
END-CARD..= 'END' EOC
.END

```

Fig. 2. BNF description of a subset of FORTRAN IV

- (2) *Metalinguage*: The describing language; that is, the language used to describe the source language. In Fig. 2 the metalanguage might be considered that dialect of BNF described in Subsection 2.
- (3) *Target language*: The language produced from the source language by the language processor. Usually, this is an assembly language or actual machine code.

Now, having been introduced to the terminology, assume we wish to create a language processor to translate source language A to target language B for machine X. The classical steps to accomplish this would be:

- (1) Completely specify source language A, perhaps using the BNF notation or another convenient metalanguage.
- (2) Determine the relation between source language A and target language B.
- (3) Hand-write the processor to accomplish this translation using an existing language on machine X.

As may be expected, this is not a trivial task; at best, it is both time consuming and quite prone to errors. A more desirable method would be to have a "universal" processor which will accept a description of the source language and proceed to parse and generate code according to this description. This is exactly the philosophy of syntax-directed processors. To further clarify the differences between a conventional language processor and a syntax-directed processor, the processors are diagrammed in Fig. 3 (Ref. 8).

At this time this mystical "universal" processor, along with its metalanguage input, will be developed. This processor may be considered as a simulator of a special

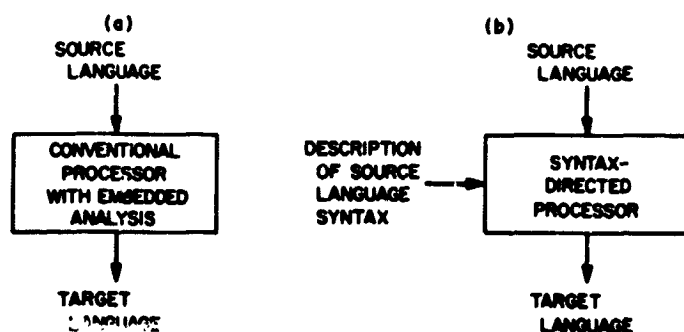


Fig. 3. Processors: (a) conventional; (b) syntax-directed

machine. This meta machine (Ref. 9), described in Tables 2 and 3, contains only those instructions useful in translating a source language into its target language. In fact, using these instructions, converting from the BNF notation to an assembly code is almost immediate. For

Table 2. Description of meta machine

Item:	Description
INBUF	An input buffer containing a continuous string of input characters
OUTBUF	An output buffer to be punched and printed
STACK	A push-down stack
FLAGn	Logical flags (0-n) used for program control

Table 3. Subset of meta machine instructions

Operation	Argument	Instruction
TST	STRING, FLAGn	Compare the first non-blank string in the input stream to the string given as an argument. If the strings are identical, delete the string from the input and set FLAGn true. Otherwise, set FLAGn false.
ID	FLAGn	Test if the next non-blank input string is an identifier. If so, delete it from the input, placing it as the top element of STACK, and set FLAGn true. If not an identifier, set FLAGn false.
CALL	AAA	Enter the procedure at location AAA.
RET	AAA	Return to the procedure which last called AAA.
SET	FLAGn	Set FLAGn true.
RESET	FLAGn	Set FLAGn false.
B	AA	Branch unconditionally to location AA.
BT	AA, FLAGn	Branch to location AA only if FLAGn is true.
BF	AA, FLAGn	Branch to location AA only if FLAGn is false.
PRINT	STRING	Print out the string.
TEMP	n	Create a symbol of form 'TMPn' and place it in STACK.
INC	N, X	Increment N by X.
POP		Pop the top element in STACK.
LAB		Reset the output pointer to column 1.
OUT	(STRING/'n)	Output either the literal string or the nth element in the STACK. Output pointer is reset to column 8.
STOP		Program stop.
END		Denotes end of program.

example, the four syntactic equations presented in Subsection 1 describing a class of algebraic equations are coded below.

SUM	CALL	TERM
	TST	'+',FLAG1
	BT	SUM,FLAG1
	RET	SUM
TERM	CALL	PRIME
	TST	'*',FLAG1
	BT	TERM,FLAG1
	RET	TERM
PRIME	ID	FLAG1
	BT	PRIM1,FLAG1
	TST	('',FLAG1
	BF	BADERR,FLAG1
	CALL	SUM
	TST	'',FLAG1
	BT	PRIM1,FLAG1
	PRINT	'MISSING RIGHT PAREN'
PRIM1	RET	PRIME
BADERR	PRINT	'UNRECOVERABLE ERROR'
	STOP	

Note that although the source language is completely described, no consideration for the target language (step 2, above) has been made. The exact details of this translation are much too involved to be included in this article; hence only the results are shown below.

SUM	CALL	TERM
	TST	'+',FLAG1
	BF	SUM2,FLAG1
	BF	SUM1,FLAG3
	CALL	SAVE
SUM1	CALL	SUM
	CALL	OUTADD
SUM2	RET	SUM
TERM	CALL	PRIME
	TST	'*',FLAG1
	BF	TERM2,FLAG1
	SET	FLAG3
	CALL	TERM
	CALL	OUTMPY
TERM2	RET	TERM

PRIME	ID	FLAG1
	BT	PRIME2,FLAG1
	TST	('',FLAG1
	BF	BADERR,FLAG1
	CALL	SAVE
	CALL	SUM
	TST	'',FLAG1
	BT	PRIME2,FLAG1
	PRINT	'MISSING RIGHT PAREN'
PRIME2	RET	PRIME
BADERR	PRINT	'UNRECOVERABLE ERROR'
	STOP	
SAVE	BF	SAVE1,FLAG2
	INC	N,1
	TEMP	N
	OUT	('STO ',*1)
	RESET	FLAG2
	RESET	FLAG3
	RET	SAVE
SAVE1	RET	SAVE
OUTADD	BF	OUTAD2,FLAG2
OUTAD1	OUT	('FAD ',*1)
	POP	
	RET	OUTADD
OUTAD2	OUT	('CLA ',*1)
	POP	
	SET	FLAG2
	B	OUTAD1
	RET	
OUTMPY	BF	OUTMP2,FLAG 2
	OUT	('XCA')
OUTMP1	OUT	('FMP ',*1)
	POP	
	RET	OUTMPY
	RET	
OUTMP2	OUT	('LDQ ',*1)
	POP	
	B	OUTMP1
	END	

At this point, let us review briefly what we have accomplished. Basically, all we did was to create a simulator for a machine designed for language translation. Next we encoded the source language description (and its relation to the target language) into a form acceptable to the simulator. The next logical step is to allow the input to be in a form familiar to the programmer, namely in Backus Normal Form. This is easily accomplished since, due to its simplicity, the recognition syntax may be given in only five statements.

BNF-PROGRAM ::= 'SYNTAX.' \$(STAT) 'END'

STAT ::= IDENT ' ' EXP1

EXP1 ::= EXP2 \$('/' EXP2)

EXP2 ::= EXP3 \$(EXP3)

EXP3 ::= IDENT / STRING /

'(' EXP1 ')' / '\$' ')' EXP1 ')'

4. Practical Considerations of Syntactic Processors

As stated in the introduction, two syntactic processors have been used extensively. These two, TMG and META, differ primarily in the following aspects: (1) backup capability, (2) output flexibility, and (3) dictionary and classification ability.

By backup capability, I mean the ability to rescan an input stream. For example, consider the following assignment statement:

IF(J) = I + K

Using the syntax of Fig. 2 and the meta machine discussed in Subsection 3, the statement would have been erroneously recognized as an "IF" statement. Thus, when

the equal sign is found, an error message would be issued. To avoid this difficulty, we could resort to a keyword language requiring that certain key words not be used as variables. A better solution would be to use backup; that is, after finding that the statement is not correct, the input pointer is backed up and the alternate is called.

The differences in output flexibility are more subtle. In META the output occurs at the same time as the scanning or parsing, whereas in TMG the output is accumulated in an output tree and all output occurs when the parsing is complete. The primary value in postponing the output is the ability to pass arguments between the various output definitions and the ability to perform a post analysis before the actual output.

The third difference, the dictionary and classification ability, can be best illustrated by the following example:

J = WHAT(I)

Using the meta system, it is unknown whether "WHAT" is an array variable or a function reference. Clearly it is desirable to retain certain attributes about the variables. For this reason, TMG has provided for retaining up to 18 bits of information for each variable found.

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III. Spacecraft Power

GUIDANCE AND CONTROL DIVISION

N67-29144

A. Solar Power System Definition Studies, W. R. Yeane

1. Introduction

The overall objective of this effort is to investigate the problems associated with developing spacecraft power systems for unmanned planetary missions. The effort stresses development of the technology required to solve system design problems associated with meeting JPL mission requirements. One task which has been undertaken is the investigation and development of computer programs for power system design and analysis.

2. Present Activities

The investigation of computer programs available for the spacecraft solar power system analysis and design was approached in a three phase program. The first and continuing phase is to develop in-house capability to meet the immediate program needs.

a. JPL-developed computer programs. A computer program has been developed to show in a parametric manner the interaction of the power profile, the power source size and weight, and the orbit characteristics in a Mars-orbiting *Voyager*-class spacecraft. A series of computer plotting programs demonstrate the possibility of a preliminary

power source analysis, given a representative power profile and mission constraints. The programs accommodate most of the missions requirements, e.g., planetary orbiter, planetary fly-by, and planetary lander. The computations have been made using energy balance equations which consider: (1) battery capacity, energy density, discharge depth, and charge efficiency, (2) solar array area, over-capacity, and energy density, (3) orbit time and eclipse duration, and (4) load profile variations. Computer plots give comparisons of the various parameters in order to make system optimization possible

An additional computer program has been developed for reflecting arbitrary load profiles back to the power sources. This program keeps an inventory of the various power requirements for the science and engineering experiments included in a *Mariner*-class spacecraft. As these loads vary with mission function, the total user load profile is calculated. The user load profile is then reflected back through the conversion equipment, accounting for various device efficiencies, and the effect on the power source is calculated.

Further in-house programs are being developed to predict array electrical characteristics. The programs calculate panel short-circuit current, open-circuit voltage,

maximum power and temperature with variation of the solar cell type, the number of cells in series and parallel, the panel absorptivity and emissivity, and the solar intensity.

b. Search for power system analysis programs. During the second phase, a JPL literature search compiled information on the sources of computerized power system analysis programs and techniques. The information from this literature search provided the basis for a NASA center and industry survey (phase three).

c. Integration of other programs into JPL computer system. During phase three, discussions have been held with NASA Mission Analysis Division (Moffet Field), Goddard, Manned Spacecraft Center, Ames, Langley, and the Electronic Research Center in order to obtain a better understanding of their present and planned efforts. In addition, the major aerospace companies have been surveyed to obtain information on additional computer programs. From the survey, several computer programs have been obtained and integrated into the JPL computer system. The first computer program simulates the solar array voltage and current relation for a given intensity and temperature. The second computer program is a thermal model for determining the solar array temperature as a function of solar intensity and panel absorptivity and emissivity properties. The third program sizes a complete satellite electrical system consisting of solar panels, battery, converter, and inverter from existing empirical data. In this program, the system requirements are:

- (1) Solar array weight, area, and series and parallel connections.
- (2) Battery weight, volume, capacity, and cell number.
- (3) Conversion efficiency and energy density.
- (4) Temperature profile.
- (5) Load profile.

With these selections, the program will be used to "fly" the various missions and check energy balance, solar array design, reliability and redundancy provisions, and thermal control requirements.

3. Future Activities

In the future, additional computer programs will be investigated for adaptation or modification to meet JPL/

NASA requirements. The in-house computer program effort will also be continued.

B. Shielding Analysis for Radioisotope Power Systems, P. Gingo

A portion of the activities being pursued under the Nuclear Power System Definition Studies involves the evaluation of the shielding problems associated with the use of a radioisotope thermoelectric generator (RTG) in a spacecraft power system. Early in this program it became evident that the development of a computer capability for the analysis of this problem would be required because of the complexity of the problem. These computer programs will calculate the neutron and gamma fluxes generated by the radioisotope which will serve as a model for the experimental tests to completely specify the radioisotope radiation spectrum. Three tasks were undertaken to achieve this capability.

1. Available Computation Methods

The first task undertaken was a review of available nuclear computer codes. Among the sources investigated for nuclear codes were the libraries of the Radiation Shielding Information Center (RSIC), Los Alamos Scientific Laboratory, Livermore Radiation Laboratory, Martin Company, Atomics International and the Valley Forge Division of the General Electric Company. After this initial review, it became evident that the required nuclear codes should include the following three methods of calculation:

a. The Kernel integration method. This rapidly convergent program requires a very low execution time. The inherent difficulty with this method is that it is essentially a ray tracing technique which will not account for multiple scattering and geometry effects. The accuracy of these calculations is limited to the accuracy of the buildup factors and the cross sections used.

b. The Boltzmann transport equation. This requires more execution time than the Kernel integration method but is considerably more accurate. Geometry effects are limited to the sphere, slab and the cylinder.

c. Monte Carlo scattering. Execution time is 0.5 hr or more and is very sensitive to the starting constants used. This method includes multiple scattering and is currently the most accurate method used. Because execution time is very long, the Monte Carlo scattering method is generally used once the RTG design is fixed.

In addition to the nuclear computing codes, neutron and gamma cross sections had to be obtained which would be consistent with each nuclear code and its method of calculation.

The work to date has been to collect and integrate the nuclear codes with the necessary cross sections to utilize these nuclear codes. The following has been accomplished to achieve this RTG analysis capability.

2. Introduction of Nuclear Computing Codes to JPL

The Los Alamos DTF-IV Neutron Transport Code was received by the JPL Computation Center directly from Los Alamos and also from the Martin Company. The version given by the Martin Company is a 3-link overlay which would fit into the JPL 7094 Direct Couple Operating System (DCOS) Storage. To date, numerous changes have been made to the Martin version of the DTF-IV Transport Code. These changes include:

- (1) Resizing of "COMMON" to fit the program into our 32-K machine.
- (2) Going to alternate input and output modes in order to gain more available storage for the COMMON arrays.
- (3) Extension of dosage calculations from spherical to cylindrical and slab geometries.
- (4) Correction of specific linearizing indexing conventions to permit the inclusion of higher order terms into the transport equation, which was of the zeroth order in the Martin Baltimore version of the code (simple diffusion equation), thus greatly extending the potential accuracy available to the user.
- (5) Running various check problems from straight card input. The DTF IV program is now fully operational in this respect.

The "QUAD" Kernel Integration Code available from the Los Alamos Scientific Laboratory was also received and integrated into the JPL DCOS 7094.

The current difficulty with this program is the problem of specifying the correct buildup factors which essentially are for single element media. To avoid this, buildup factors can be used which correspond to a mixture of elements taken from the General Electric 14-0 Nuclear Codes program.

The GE 14-0 Code is also a Kernel integration program which allows for buildup factors directly due to a mixture

of elements and is not necessarily averaged to correspond to one element as required by the "QUAD" program. The GE 14-0 program is available on tape at JPL but has not been made operational as yet.

No Monte Carlo Scattering programs have been requested since these represent a future requirement. A literature search will continue for rapidly convergent Monte Carlo programs.

The "MESNE" program was received from the Livermore Radiation Laboratory. This program will accept any basic neutron cross section library tape and convert this into flux averaged neutron cross sections for any specified group structure (limit is 20 groups). The flux averaged neutron cross sections would be the total cross sections, fission cross sections and the neutron transfer matrices which account for the neutron up-and-down scattering.

The "MESNE" program, originally written in Fortran II, has been subjected to a "SIFT" routine to eliminate any JPL 7094 DCOS inconsistencies. The program has been simultaneously converted to Fortran IV. At present, several subroutines are missing and have been requested from Dr. Howerton of the Livermore Radiation Laboratory. These subroutines, "ONMON," "OFFMON," and "DECODE," are used to generate the source tape and interrupt the program execution.

3. Introduction of Nuclear Cross Section Libraries to JPL

Neutron and gamma cross sections have been received to use with the nuclear codes presently operating at JPL. These basic cross sections presently available are the following:

- (1) Effective Broad Beam Neutron Removal Cross Sections, reference AECD 3978, 1955.
- (2) Gamma Ray Absorption Coefficients for elements and mixtures, reference XDC-59-10-19.
- (3) 18-Group Hanson-Roach Neutron Cross Sections, reference GEMP-283.
- (4) The complete Carroll Mills Los Alamos 18, 24, and 25 group neutron cross sections which include down-scattering matrices. Several isotopes, among them Pu 238 and Pu 241, are not included in this library.
- (5) The Howerton evaluated neutron cross section library. This is the most current and complete library available. The "MESNE" program will be

used to reduce the Howerton calculated library into any convenient neutron group structure (to a upper limit of 20 groups) which may be required for the RTG studies.

4. Calculations

A mockup of the modified RTG Headlight configuration generated by the General Electric Company as part of

the *Voyager* follow-on work has been submitted for analysis into the DTF IV computer program utilizing the standard Hanson-Roach neutron 16 group. The gamma dose rates have been calculated at 28 detector points by the "QUAD" program with buildup factors taken from mixtures normally used in the GE 14-0 nuclear code. These calculations are used principally to further check the nuclear codes and will be resubmitted with more accurate assumptions, such as including silicon and the correct isotope specification of Haynes 25.

Table 1. Radioisotope thermal generator, total gamma dose rate versus time of exposure

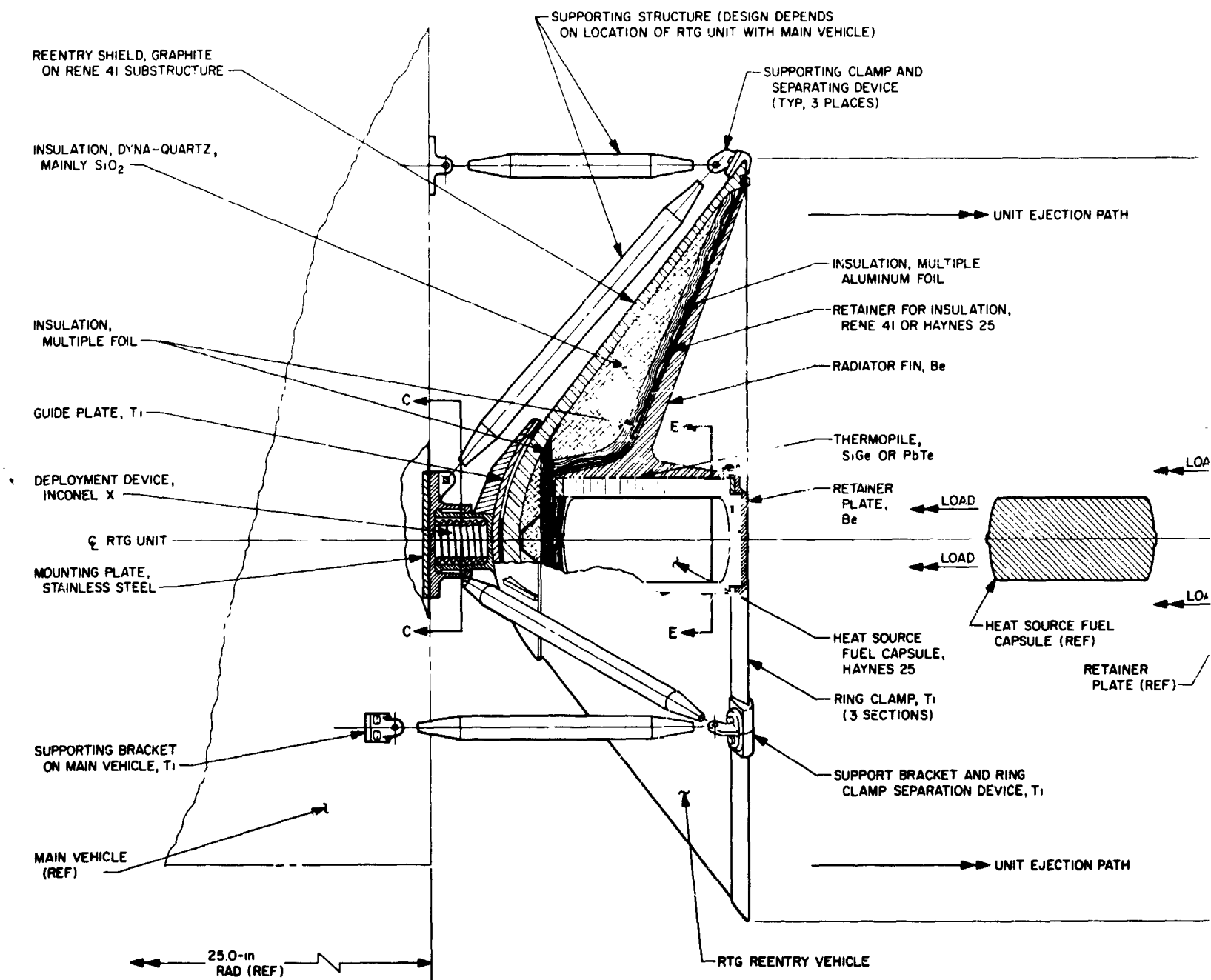
Detector point coordinates, (R cm, θ deg, Z cm)	Total dose rate, (Mrad/thermal-watt-hr)	
	After 1 day	After 5 years
1. (0, 0, 0)	4.1638×10^{-2}	13.30084×10^{-2}
2. (5, 0, 0)	3.2413×10^{-2}	10.6310×10^{-2}
3. (10, 0, 0)	1.5521×10^{-2}	5.8136×10^{-2}
4. (15, 0, 0)	1.0913×10^{-2}	4.3400×10^{-2}
5. (20, 0, 0)	8.8339×10^{-3}	3.4873×10^{-2}
6. (25, 0, 0)	7.3049×10^{-3}	2.8225×10^{-2}
7. (30, 0, 0)	6.1535×10^{-3}	2.3226×10^{-2}
8. (35, 0, 0)	5.1769×10^{-3}	1.9281×10^{-2}
9. (40, 0, 0)	4.3609×10^{-3}	1.6068×10^{-2}
10. (45, 0, 0)	3.7061×10^{-3}	1.3541×10^{-2}
11. (50.8, 0, 0)	3.10003×10^{-3}	1.1246×10^{-2}
12. (50.8, 0, 5)	3.4743×10^{-3}	1.2441×10^{-2}
13. (50.8, 0, 10)	3.7869×10^{-3}	1.3448×10^{-2}
14. (50.8, 0, 15.0)	3.9938×10^{-3}	1.4110×10^{-2}
15. (50.8, 0, 20)	4.0675×10^{-3}	1.4346×10^{-2}
16. (50.8, 0, 25)	3.9978×10^{-3}	1.4123×10^{-2}
17. (50.8, 0, 30)	3.7943×10^{-3}	1.3471×10^{-2}
18. (50.8, 0, 34.54)	3.5162×10^{-3}	1.2576×10^{-2}
19. (45, 0, 34.54)	4.3222×10^{-3}	1.5508×10^{-2}
20. (40, 0, 34.54)	5.2365×10^{-3}	1.8865×10^{-2}
21. (35, 0, 34.54)	6.4515×10^{-3}	2.3306×10^{-2}
22. (30, 0, 34.54)	8.0054×10^{-3}	2.9269×10^{-2}
23. (25, 0, 34.54)	1.0077×10^{-2}	3.7375×10^{-2}
24. (20, 0, 34.54)	1.2818×10^{-2}	4.8603×10^{-2}
25. (15, 0, 34.54)	1.6708×10^{-2}	6.5082×10^{-2}
26. (10, 0, 34.54)	2.2729×10^{-2}	9.2944×10^{-2}
27. (5, 0, 34.54)	6.1841×10^{-2}	22.024×10^{-2}
28. (0, 0, 34.54)	1.0376×10^{-1}	35.4671×10^{-2}

The RTG reentry vehicle configuration used in the present analysis is shown in Fig. 1. (See Sect. "A-A" of attached drawing.) This vehicle has been mocked-up for the neutron and gamma radiation analyses in the form depicted in Fig. 2. The calculations considered the modified RTG as a cylinder in the radial direction and a slab in the axial direction. The results of these calculations are given in Tables 1 and 2.

**Table 2. Radioisotope neutron flux spectrum^a
(subcritical, $K = 0.96$)**

Neutron energy group	Neutron energy (E_L)	Neutron current fraction detector coordinates		
		(R, θ , Z) (10.54 cm, 0, 10 cm)	(R, θ , Z) (0, 0, 34.54 cm)	(R, θ , Z) (0, 0, 0)
1	3.0 Mev	1.196×10^{-1}	1.218×10^{-1}	1.31×10^{-1}
2	1.4 Mev	2.67×10^{-1}	2.59×10^{-1}	2.62×10^{-1}
3	0.9 Mev	1.33×10^{-1}	1.33×10^{-1}	1.32×10^{-1}
4	0.4 Mev	1.96×10^{-1}	2.00×10^{-1}	2.00×10^{-1}
5	0.1 Mev	1.825×10^{-1}	1.895×10^{-1}	1.82×10^{-1}
6	17 kev	7.23×10^{-2}	7.2×10^{-2}	7.1×10^{-2}
7	3 kev	2.21×10^{-2}	1.94×10^{-2}	1.75×10^{-2}
8	0.55 kev	6.21×10^{-2}	4.34×10^{-2}	3.62×10^{-2}
9	0.1 kev	1.465×10^{-2}	6.89×10^{-3}	4.79×10^{-3}
10	30 ev	3.39×10^{-4}	1.12×10^{-4}	6.67×10^{-5}
11	10 ev	1.008×10^{-4}	2.36×10^{-5}	1.19×10^{-5}
12	3 ev	3.59×10^{-5}	5.77×10^{-6}	2.46×10^{-6}
13	1 ev	1.089×10^{-5}	1.24×10^{-6}	4.58×10^{-7}
14	0.4 ev	3.4×10^{-6}	2.76×10^{-7}	8.93×10^{-8}
15	0.1 ev	1.44×10^{-6}	7.97×10^{-8}	2.23×10^{-8}
16	thermal	5.37×10^{-7}	1.79×10^{-8}	4.20×10^{-9}

^aTotal neutron flux = neutron generated at detector times the neutron current fraction.
^bThe fuel density 10.7 gm/cm³ does not account for fuel void.



SECTION A-A

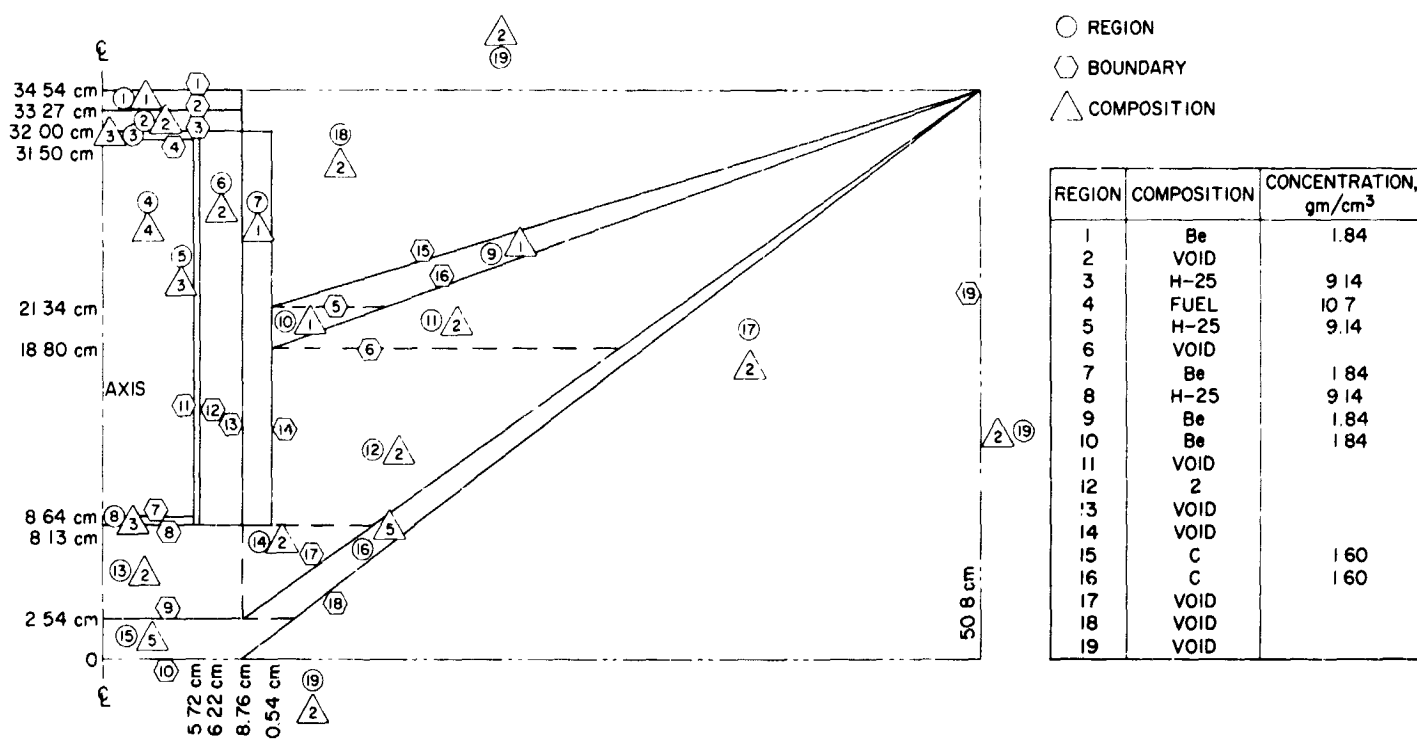


Fig. 2. Radiation model of the RTG Headlight configuration

C. Applied Thermionic Research, O. S. Merrill

1. Introduction

The results presented here are a summary of work performed between June 1965 and June 1966 under JPL Contract 951262 with Thermo Electron Engineering Corporation (TEEC), Waltham, Massachusetts. This effort is a continuation of a research program which has been in progress for several years. It consists of applied research in thermionic energy conversion and includes both experimental and analytical tasks.

The basic objectives of this effort have been to generate knowledge and to devise techniques for improving the performance of thermionic converters. At the time this contract was initiated, the capabilities of cesium-vapor diodes were well defined, and it was obvious that any step increases in performance could only come from significant changes in the nature of the conversion process. As a result, this program undertook the study of two new regimes of operation.

The first was the use of CsF vapor in addition to metallic cesium in the converter. The presence of CsF causes the Cs vapor to be more tightly bound to the emitter surface, so that the cesium pressure required to maintain a given coverage is greatly reduced. This reduction of Cs

pressure results in less scattering of electrons in the plasma, and therefore improved performance. Evidence of these effects had been obtained in the previous year, and an analytical framework forming the basis for further investigation was devised. Questions of stability, however, still remained to be answered before these improvements could be put to effective use.

The second main task of the program was a study of the effects of inert gas additives on the characteristics of the converter. There were three reasons for undertaking this study. Because of the very low cross section of inert gas atoms for low-energy electron collisions, it was hypothesized that inert gases could act as diffusion barriers to Cs ions, thus conserving them, while they remained transparent to electrons. It was reasoned that, if Cs ions were indeed conserved, fewer would be required and the internal voltage loss necessary for ion production in the converter would be reduced. The second reason for undertaking this study was that the presence of an additional gas in the interelectrode space could be used as a tool in the study of the converter plasma. The third reason for this study was the fact that inert gases are major constituents of the fission products of thermionic reactor systems. As such, they may be present in the interelectrode space. It is therefore desirable to know their effects on converter performance.

In the inert gas studies, an etched rhenium emitter was used which, when tested with Cs only, provided a performance reference prior to the introduction of inert gas. It also provided data for comparison with previously-tested rhenium surfaces not electro-etched.

2. Test Converter and Instrumentation

The test converters used in this program are variable parameter research-type devices (Fig. 3). They are of planar geometry and use an active collector guard ring. The parameters whose values can be varied and accurately controlled include the emitter, collector and guard ring temperatures, the interelectrode spacing, the cesium and additive reservoir temperatures and, consequently, the cesium and additive gas pressures.

The emitter is heated by electron bombardment, and its temperature is measured with an optical pyrometer in a

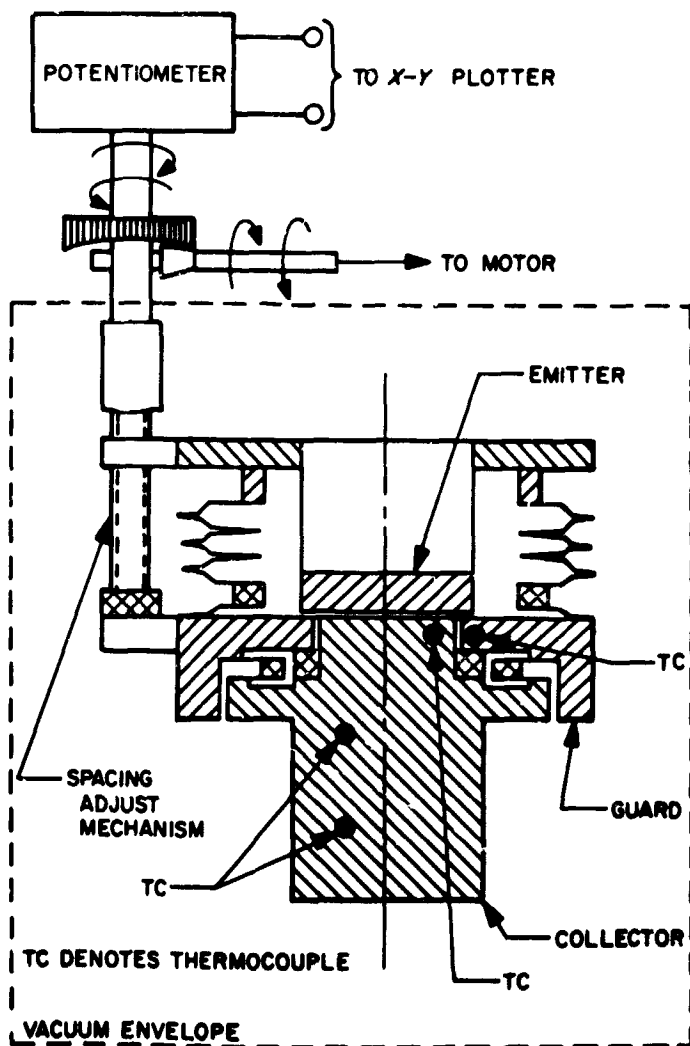


Fig. 3. Schematic of the cesium diode

black-body cavity located on its back surface. The collector and guard-ring temperatures are maintained by balancing the heat input to their respective heaters against the heat rejected to individual water-cooled coils. The variation of spacing is accomplished by flexing a bellows joining the emitter and collector structures by means of a mechanism operated externally to the belljar. Three micrometer spindles are driven by shafts which are geared together above the top plate. The interelectrode spacing is measured directly on three dial indicators which are set to measure changes in the relative position of the emitter and collector structures.

A tubulation leads from the emitter support structure to the cesium reservoir. A similar reservoir is used for the cesium fluoride. The only difference between the two is that the entrance to the tubulation leading to the cesium reservoir is restricted by a 10-mil orifice. This orifice prevents cesium fluoride vapor from condensing in the cesium reservoir at a rapid rate. In the case of the plasma-additive converter, inert gas is introduced by means of a tubulation leading from the cesium reservoir to the exterior of the vacuum envelope of the converter and to the inert-gas injection system.

3. Experimental Procedure

The two main types of experiments performed under this program were quasi-static, low-power work function and dynamic, high-power performance tests. In each of these, the output was obtained in the form of current density-voltage plots on an x-y recorder. Valid data required that there be no electrode temperature variation during a particular run.

The emitter work function, ϕ_E , was determined from the emitter saturation current, J_s , obtained at a given emitter temperature, T_E , by substituting these values into the Richardson equation,

$$\phi_E = -kT_E \ln J_s / AT_E^2$$

where k is the Boltzmann constant, and A is the Richardson constant. The validity of the results is strongly dependent upon the measurement of the true value of saturation current density. A sufficiently large electron-accelerating voltage must be employed to ensure that the emitter work function is the only barrier controlling the current. Ion-rich temperature conditions must be established to eliminate any space-charge barrier formation. A further requirement introduced by back-scattering of electrons in the plasma is that spacing be held to small values, on the order of 1 mil.

The most useful method of measuring collector work function is the retarding plot. The J - V characteristic for an idealized diode, shown in Fig. 4, is made up of two curves; the first is given by

$$J = J_s \quad \text{for } V < V_c$$

and the second by:

$$J = AT_E^2 \exp(-\phi_c + V/kT_E) \quad \text{for } V > V_c$$

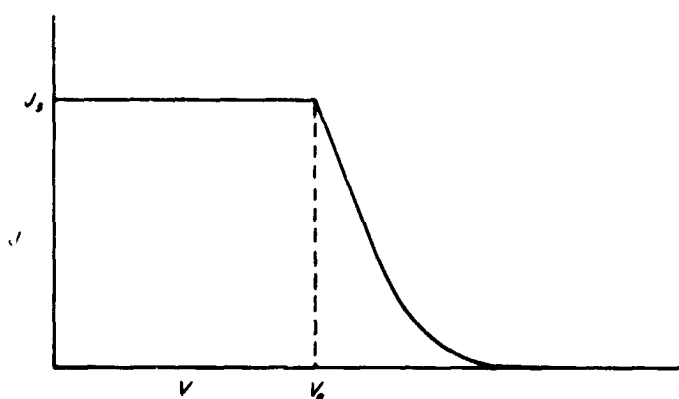


Fig. 4. Ideal J - V diode characteristics

Any point on the exponential part of the characteristic can be used to compute collector work function by substituting J , V and T_E in the second equation. Satisfactory testing with this technique requires low collector temperature (for low back emission), low emitter temperature (for low ion currents), and small spacing (because of scattering effects). The low current levels associated with these measurements allow the use of the quasi-static equipment which, at the same time, provides the accurate guard potential control necessary to eliminate leakage effects.

The high-power performance tests were those wherein "parametric data" or families of current-voltage characteristics are generated through the systematic variation of parameters. Such families may be generated by using as the variable parameter either the emitter temperature, the spacing, the collector temperature, or the cesium reservoir temperature, all other parameters being held constant. In general, the range of variation covered included the region of interest for power production. Therefore the emitter temperature range extends from 1600 to 2050°K, the spacing from 0.2 to 60 mils, and the Cs reservoir temperature from 540 to 700°K. Since this is the high-current region of the characteristics, the dynamic test equipment must be used.

4. Emitter Preparation

The experimental work using cesium fluoride additive was conducted in a tungsten-emitter converter. The inert-gas additive converter used a rhenium emitter.

Two polycrystalline tungsten emitters were prepared with electro-polished and heat-treated surfaces. The changes on the emitter surface which were observed to occur during preparation were grain growth and grain boundary grooving. Each of these changes was expected on the basis of prior experience. Since the emitter is exposed to much lower temperatures during diode operation, and the rate of surface change decreases with time, no further changes were expected to occur during testing.

One polycrystalline rhenium emitter was prepared with an electro-etched and heat-treated surface. Following this preparation, the emitter surface showed rounding and smoothing of the etched surface and faint traces of grain movement as expected on the basis of prior experience. As with the tungsten emitter, the rhenium emitter was expected to be completely stable during diode operation.

5. Additive Studies

a. Cesium fluoride. Under a previous JPL Contract 950671, an impressive diode performance was achieved using CsF as an additive (Fig. 5). However, the improved performance was not stable and after approximately 30 hr the output would revert to a value such as would be obtained without the additive. The first converter tested with CsF exhibited the "additive effects" anticipated but, just as with previous devices, this effect disappeared after 50 hr of testing. A series of diagnostic testing with this device supplemented with glass tube experiments, mass spectrometry and an extensive literature study of related work indicated that water contained in the CsF was responsible for the "additive effects" observed in this work and that reported in the literature (Fig. 6). Furthermore, the literature survey pointed out that almost all past investigations used CsF not sufficiently out-gassed to remove the water. It also became obvious that extreme care must be used in any of these investigations to exclude oxygen, since as little as 10^{-8} torr was sufficient to cause a pronounced work function shift. In normal cesiated converters, these effects are not observed since the cesium quickly "getters" the limited amounts of oxygen present. The significant results obtained from these converter studies may be summarized as follows: (1) There is an apparent threshold in the additive effect; stable effects do not appear until a certain additive reservoir temperature is reached, usually about 700°K. (2) Only two stable work

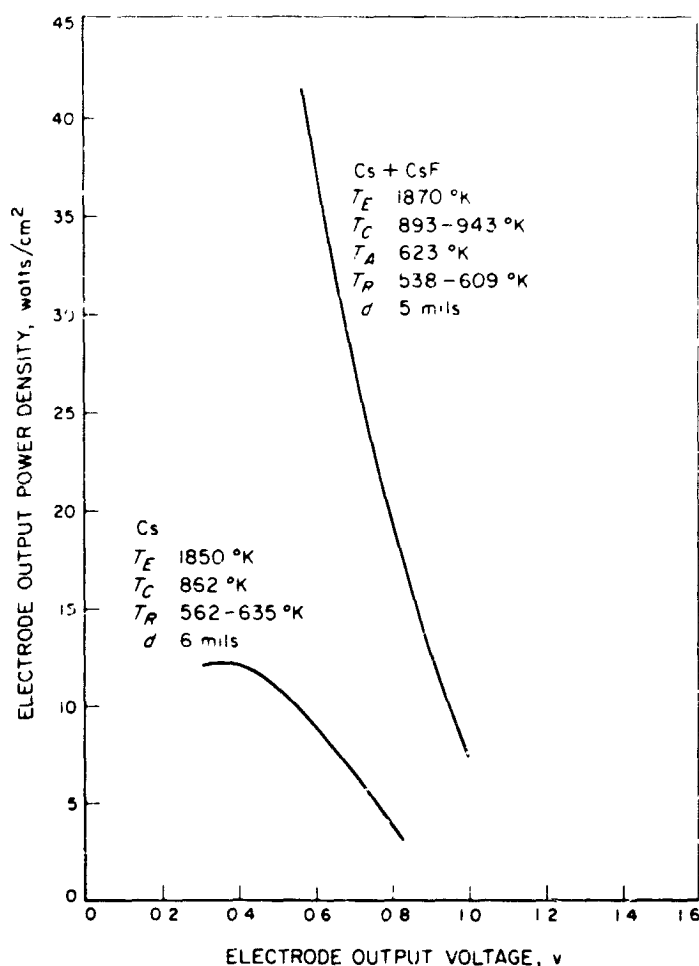


Fig. 5. Comparison of power output with and without additive

function levels have been observed for tungsten, a bare work function at about 4.8 ev and an "additive" work function at about 5.3 to 5.5 ev. (3) When a high CsF reservoir temperature is used together with a high emitter temperature, the work function values soon decay to the bare level and a permanent degradation occurs. (4) After degradation, CsF is still present in the device with a pressure responsive to its reservoir temperature. (5) After degradation, conditions favoring abnormally high additive coverage, such as low emitter temperature, soaking or raising of collector temperature, produce a short transient increase in work function. (6) After degradation, very high additive reservoir temperatures do not produce significant changes in work function beyond a short transient.

Two possibilities for the loss of additive effect were proposed: either a contaminating substance had been introduced along with the CsF or else the test vehicle emitter had become "poisoned" and responsive to CsF

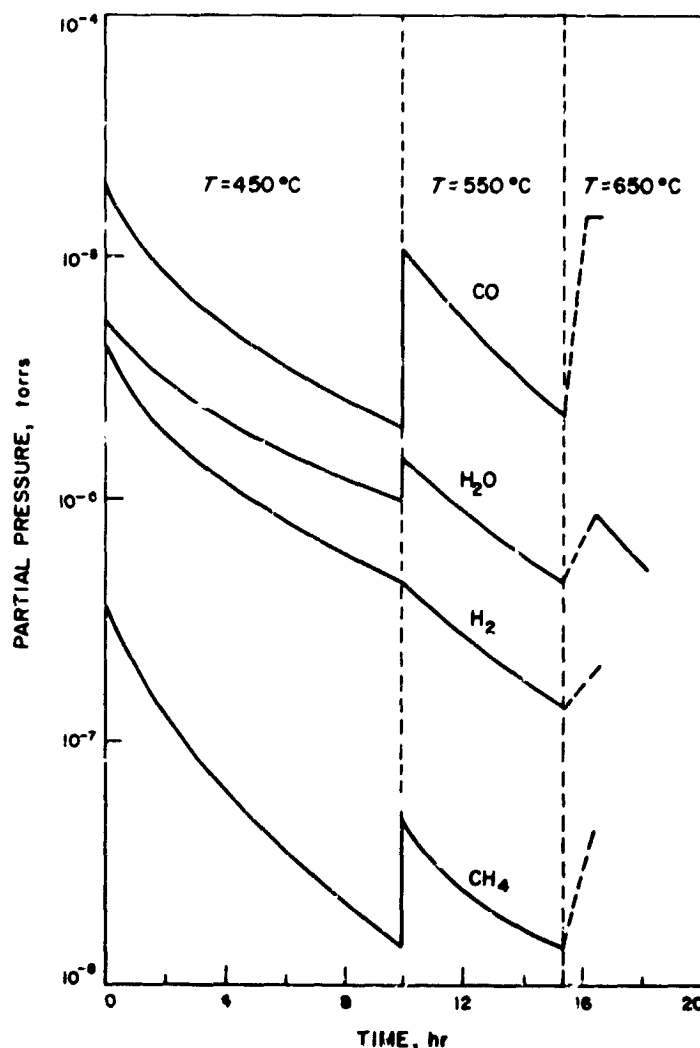


Fig. 6. Mass spectrometer analysis of CsF outgassing

vapor. It was determined that it was the water contamination contained in the fluoride which was responsible for the observed results

In view of these findings, the surface additive task was divided into two parallel efforts. The first was devoted to the investigation of the effects of pure CsF on the work function of tungsten at thermionic temperatures. The second was devoted to the investigation of the effects of oxygen.

To accomplish the first effort, a special metal-ceramic tube was designed and fabricated (Fig. 7). In this vehicle it was possible to exercise better control of the test vehicle atmosphere in order to isolate and measure the effects of pure CsF on the emitter surface. The new vehicle employed fabrication and testing techniques capable of reducing residual gas pressure to extremely low values,

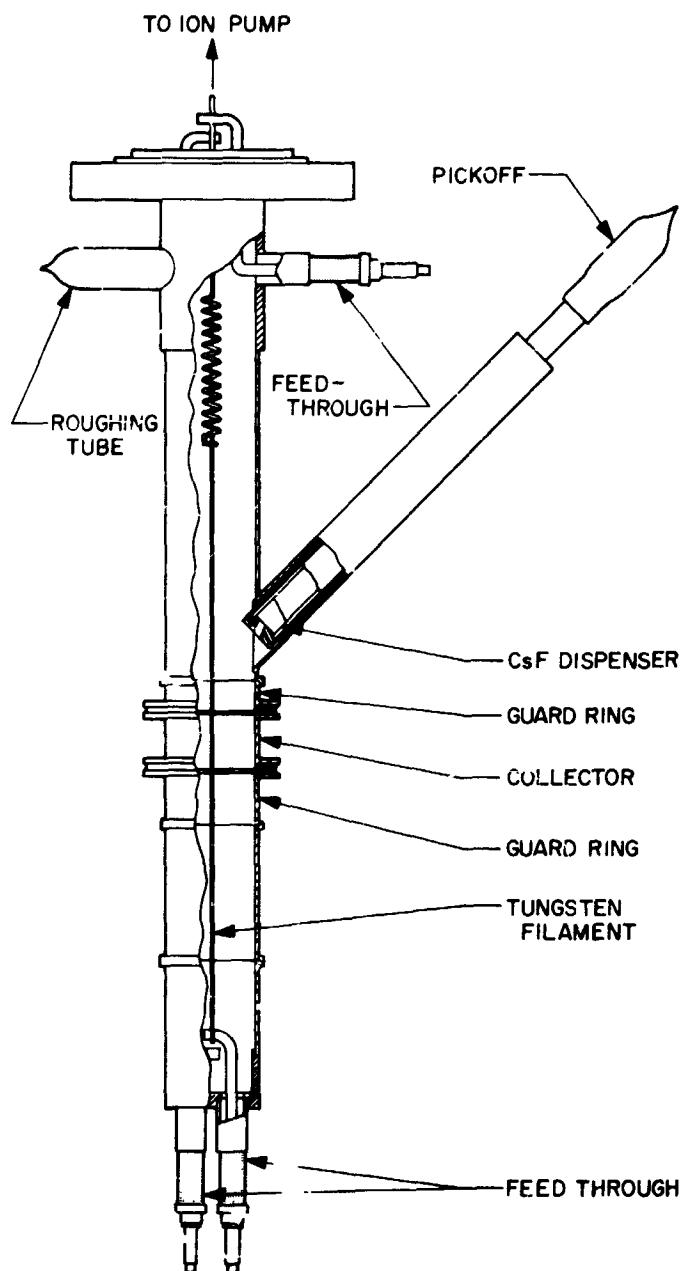


Fig. 7. Surface studies device

i.e., 10^{-11} torr. Results obtained using this device show a maximum increase in work function of the tungsten with CsF additive of 0.3 ev, from 4.6 to 4.9 ev. The arrival rate for cesium ions onto the tungsten filament was $0.3 \mu\text{a}/\text{cm}^2$ and was evaluated by measuring the cesium ion current produced by the dissociation of the CsF on the hot filament.

The energy for desorption was derived from the same experiments and was about 3.3 ev. With the relatively small change in work function and the high arrival rates required, it appears that the fluorine additives hold little promise for improving converter performance.

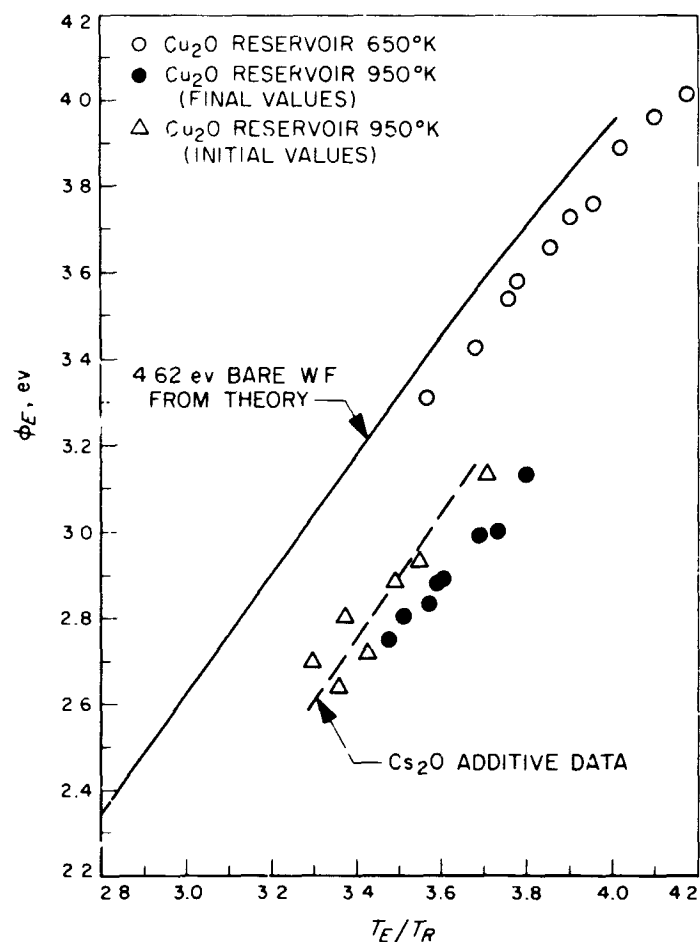


Fig. 8. Tungsten emitter work function comparison

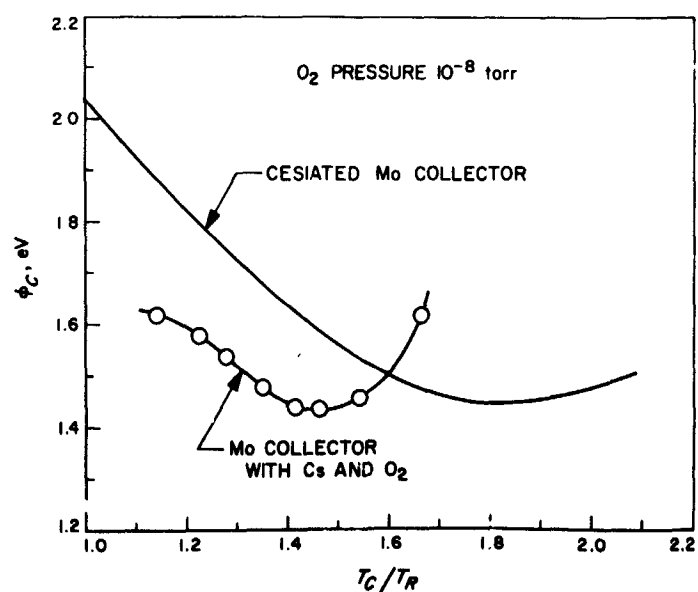


Fig. 9. Cesium molybdenum collector work function comparison

b. Oxygen. The thermodynamic equilibrium of oxygen in the converter environment was analyzed and the conclusions reached that the oxygen pressure required for coverage was chemically compatible with this environment in the vicinity of the collector. A critical experiment was conducted using Cs_2O as the O_2 source. The emitter work function and power output of the device closely resembled those of the CsF charged devices. This converter was tested for stability for 130 hr without discernable change. A second converter specifically designed for oxygen additive using a Cu_2O oxygen source was tested for 300 hr, producing consistent and reproducible data.

Experimental results indicate that the addition of oxygen to the test converter with a tungsten emitter and a molybdenum collector has resulted in a reduction of approximately 0.45 eV in the emitter work function for the same ratio of emitter to cesium reservoir temperatures (Fig. 8). The effect of the oxygen additive on the collector work function is shown in Fig. 9.

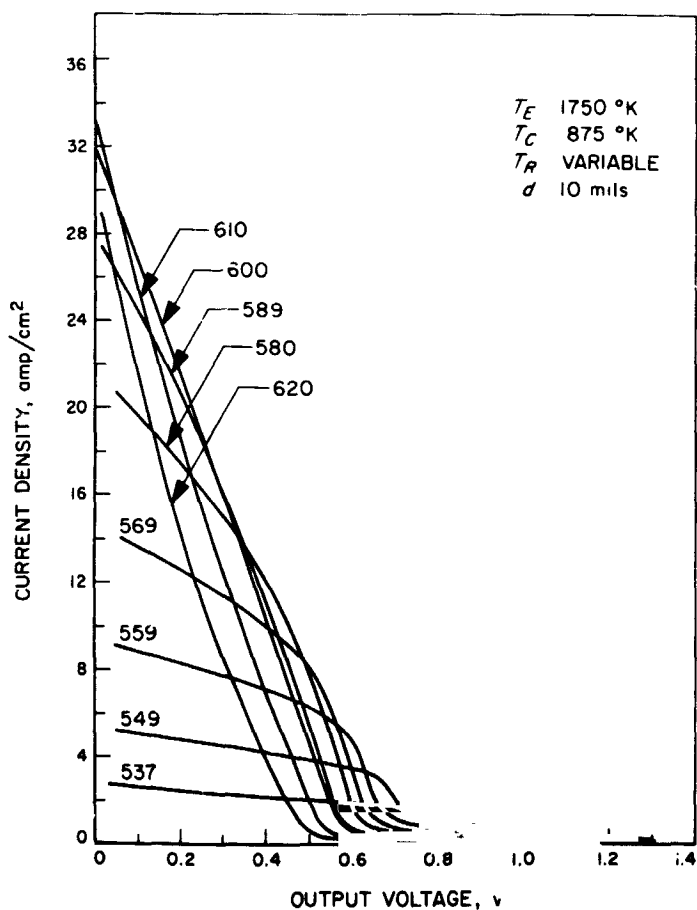


Fig. 10. J-V family with cesium only converter; W-emitter, Mo-collector

The results of two mappings of performance parametric data, at similar operating conditions, are shown with cesium only in Fig. 10 and with cesium plus Cu_2O additive in Fig. 11. Both converters have a tungsten emitter and a molybdenum collector. The envelopes of these families are shown together in Fig. 12 for the purpose of comparison. A significant feature of the oxygen converter charac-

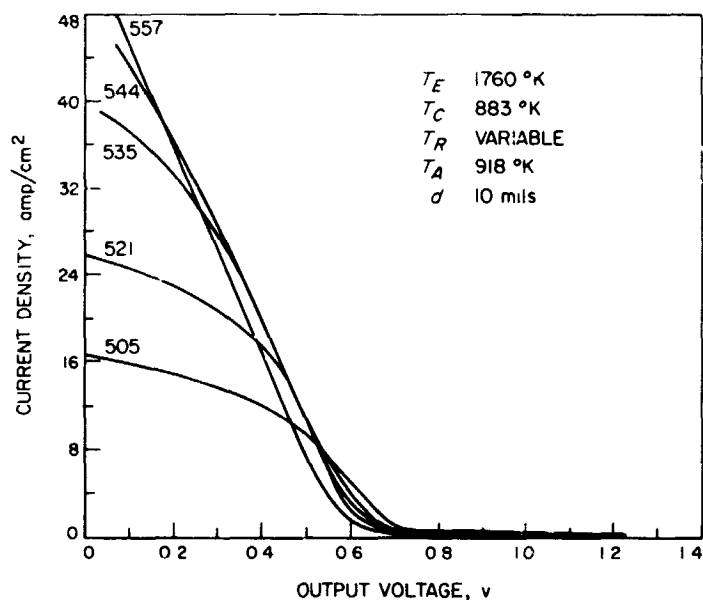


Fig. 11. J-V family with cesium-copper oxide converter; W-emitter, Mo-collector

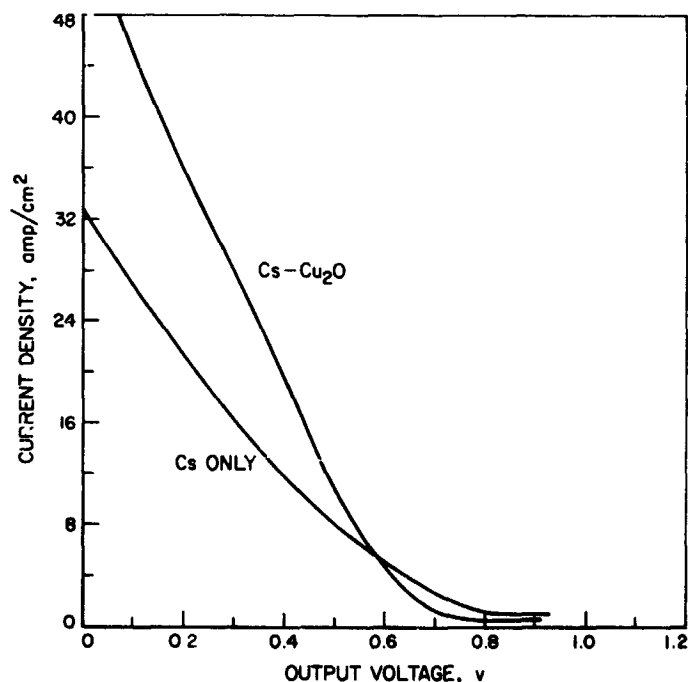


Fig. 12. Comparison of envelopes of J-V family curves

teristics when compared to the Cs only data is the steeper slope of the envelope in the oxygen case. The slope change is a direct result of the lowered cesium pressure through the reduction in scattering.

The oxygen additive converter experiments have demonstrated that the enhanced power output first associated with CsF can be maintained through the use of oxygen for at least 300 hr of operation with no degradation. A typical performance curve showing the comparison of power output versus voltage for converters with polycrystalline tungsten emitters and molybdenum collectors for Cs-only versus Cs plus Cu_2O is shown in Fig. 13.

c. Inert gas. The philosophy of the use of an inert gas additive in a thermionic diode may be summarized as follows: electron currents in thermionic diodes are attenuated by two processes, the formation of electron space charge in the interelectrode spacing which results in buildup of potential barriers for electrons, and electron scattering. Electron space charge in cesium diodes can be neutralized by the cesium ions that are produced in the discharge. These ions, however, are short-lived in the interelectrode spacing due to leakage out of the boundaries; they must therefore be continuously generated to maintain a steady density in the plasma. Loss of the ions due to leakage can be reduced appreciably if the diffusivity of the ions is decreased. This can be achieved by addition of a gas to provide scattering centers for the ions. Most of the gases that have a high scattering cross section for cesium ions also attenuate electron current. Inert gases, however, are an exception to this rule. These gases have a high scattering cross section for ions, but at operating electron temperatures their scattering cross section for electrons is very small. Hence addition of an inert gas to the plasma was expected to preserve the ions while leaving electron current virtually unaffected. A comparison between three inert gases, argon, krypton and xenon, revealed that argon would attenuate the ion current and the electron current by significantly smaller amounts than either krypton or xenon as shown in Table 3. Hence, early in the study, argon was chosen as the inert gas to be used in the subsequent experimental investigations.

Table 3. Effect of addition of 1 torr of inert gases in a 10-mil-spacing plasma at 1000°K

Inert gas	Ion current attenuation, %	Electron current attenuation, %
Argon	64	0.7
Krypton	72	1.4
Xenon	78	3.2

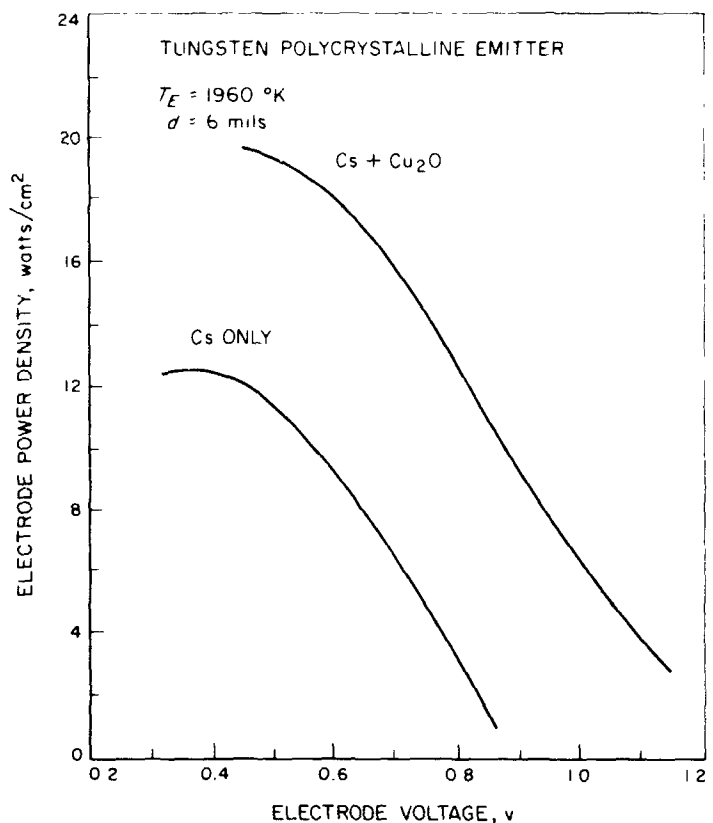


Fig. 13. Power density comparison of converters with Cs only and with Cs plus Cu_2O

The experimental apparatus consists of a variable-parameter diode (previously described) and a gas injection system shown in Fig. 14. The high-purity argon is stored in a cylinder equipped with a two-stage regulator. The gas is introduced into a 75-cc stainless-steel cylinder at a pressure of about two atmospheres. A micro-flow valve allows the rest of the system to be filled to the desired pressure (in the range of 0.2 to 200 torr). The inert gas passes through a cold trap and a hot zirconium trap to minimize the impurities. The gas pressure in the converter is monitored by a diaphragm pressure gage. The pumping system consists of an ion pump, which can be isolated by an all-metal on-off valve, and a mechanical pump used for roughing the entire system.

Several hundred variable-spacing families of volt-ampere characteristics were obtained. The data covers the cesium temperature range of 533 to 563°K at emitter temperatures of 1675 and 1800°K, and the argon pressure range of 0 to 100 torr. These data conclusively show that there is no enhancement of performance under any conditions in the presence of argon. A gradual decrease in output was observed with increasing inert gas pressure.

However, argon appears to have little effect on the characteristics at pressures below 20 torr. This result is of particular interest to nuclear thermionics, since inert gas fission products may enter the interelectrode space of converters located in a reactor core. Typically, 10 torr of argon would result in a 10% reduction in the output of the converter. Two torr of krypton or one torr of xenon would have the same effect.

6. Rhenium Emitter Preparation

To provide a reference for comparison in the evaluation of the ion conservation capabilities of argon, the performance of an electro-etched rhenium emitter was documented through a parametric mapping using Cs only. The parametric mapping of the etched rhenium emitter re-

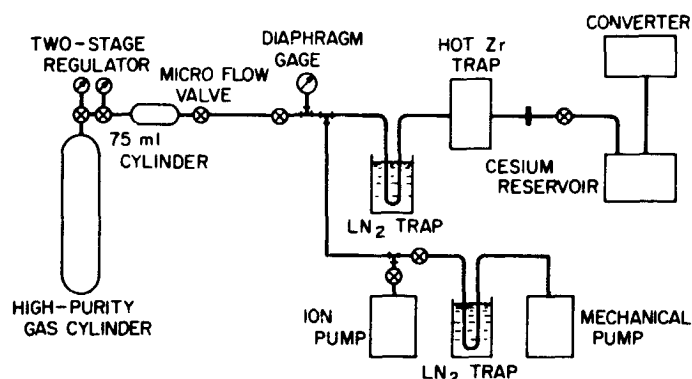


Fig. 14. Inert gas injection system

vealed that the etching preparation resulted in a two-fold improvement in performance over a polished rhenium emitter. Alternatively, the etched rhenium emitter gives the same performance at twice the spacing required by a polished rhenium emitter. A comparison of the work functions of electroetched and electropolished rhenium is given in Fig. 15.

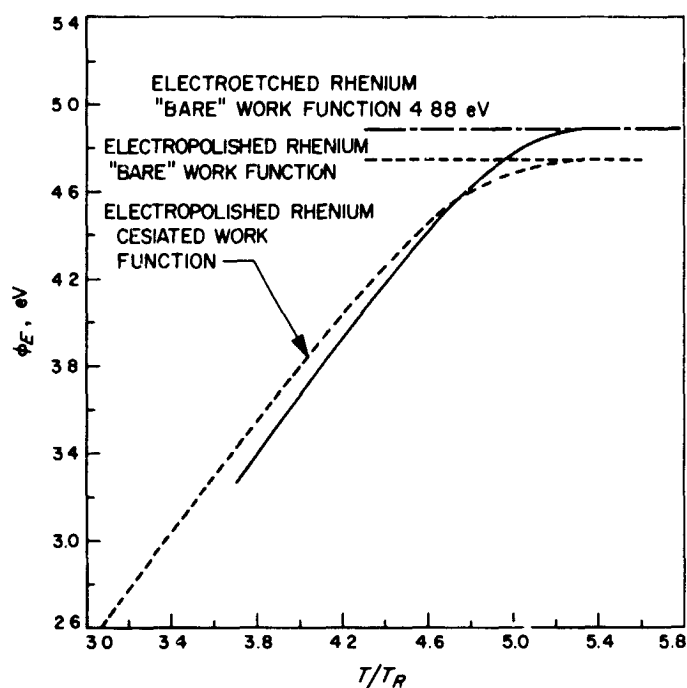


Fig. 15. Comparison of the work functions of electroetched and electropolished rhenium

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IV. Spacecraft Control

GUIDANCE AND CONTROL DIVISION

N67-29145

A. Advanced Horizon Scan Platform, H. H. Horiuchi

1. Introduction

The horizon scanner, mounted on the servo-controlled platform of a spacecraft, senses the local vertical of the target planet in view. The angle between the optical axis of the platform and the local vertical constitutes an error signal to rotate the platform toward the local vertical.

The scan platform consists of a planet sensor, actuator electronics, a gear train and the spacecraft dynamics (Fig. 1). The spacecraft dynamics includes the effect of platform supporting boom and the motor mounting structure. The content of this study includes a survey of basic performance requirements, system analysis, computer simulation, breadboard design and testing. The advanced planet sensor is being developed, and should be available in fiscal 1968. The planet sensor was simulated with analog circuits.

2. Survey of System Requirements

The basic system requirements have been summarized below from the past advanced studies, the technical reports, and the requirements for the current project. The following specifications have been assumed:

- (1) The minimum slewing rate of the platform is 1 deg/sec.

- (2) The platform tracks the local vertical of the target planet with an accuracy of ± 0.5 deg.
- (3) The platform follows the velocity input up to 0.2 deg/sec.
- (4) The planet angular diameter varies from 4 to 60 deg.

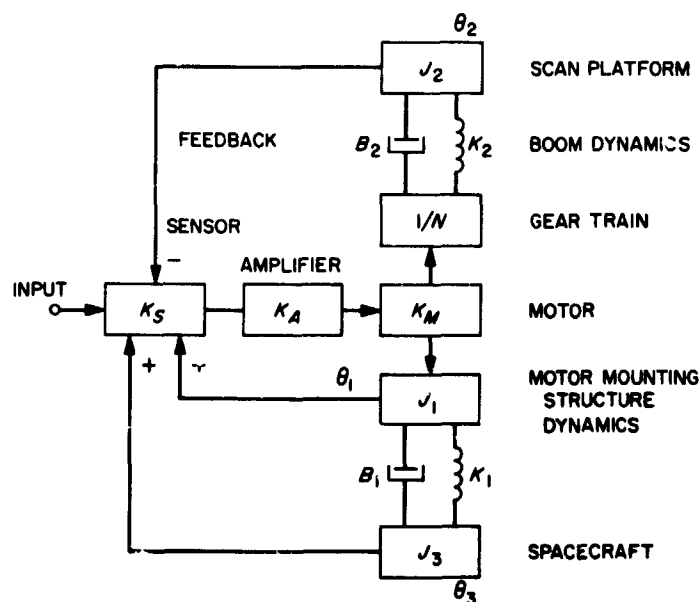


Fig. 1. Scan platform system block diagram

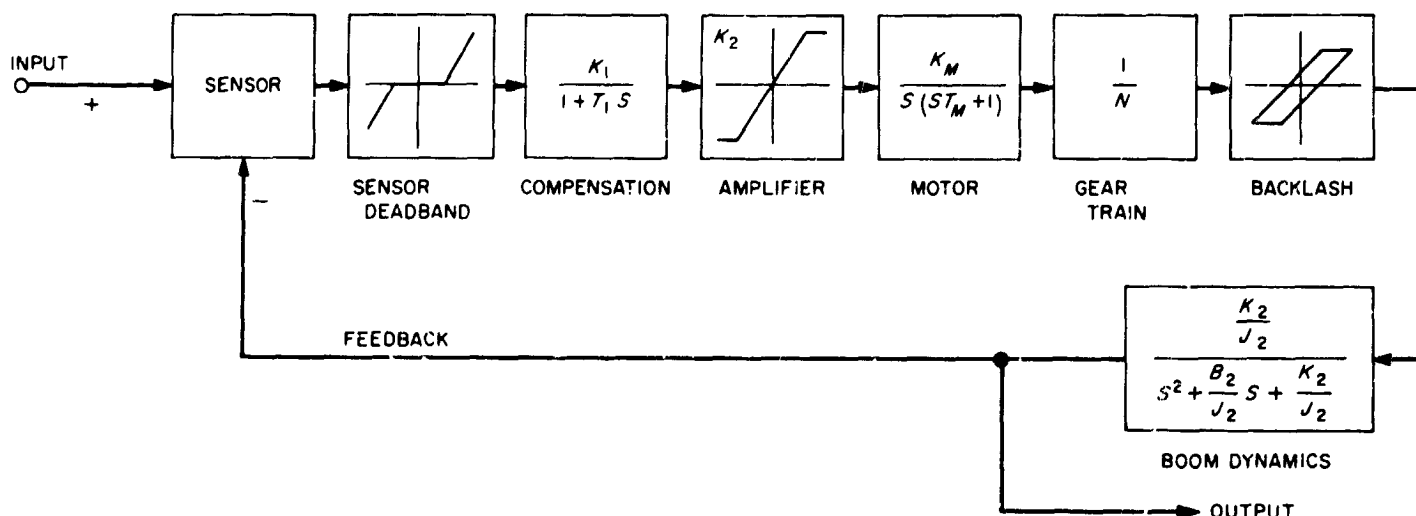


Fig. 2. Platform functional block diagram

- (5) The platform jitter is below 500 μ rad in track modes of operation.

3. System Description

The system functional block diagram is shown in Fig. 2. The planet sensor is shown as the summing point of the feedback system. The pulse-width modulated error signal from the sensor is integrated, amplified and drives the servo system. The boom structural dynamics is shown as a second-order resonance in the feedback loop.

The sensor consists of two halves of semicircular scanning device, each of which has a view angle of 60 deg, and is made of discrete thermopile elements. The maximum sensor resolution then is determined by the angular separation between the two consecutive elements. As an electronic sweep scans each element at 10 cps, the illuminated element (planet in view) produces a pulse.

The first pulse produced in each sweep closes a summing gate, which in turn produces a step output. This step output remains constant until the end of each sweep.

The step outputs from the two sensors are summed in the summing amplifier to produce a pulse whose width is proportional to the difference between the two step outputs. Hence the pulse width is proportional to the error angle between the platform and the planet local vertical.

The sensor design approach with the discrete thermopile elements seems to provide several advantages over the linear optical sensors. With this approach, the sensor

gain may be kept constant over the wide range of input light intensity and also virtually eliminates the thermal drift from the sensor.

4. System Analysis

The Bode diagram of the system shows two structural resonances (Fig. 3). The lower frequency resonance (15 rad/sec, damping factor of 0.07) represents the boom dynamics, and the higher frequency resonance (94 rad/sec, damping factor of 0.01) represents the motor base structural dynamics.

The preliminary system stability analysis was performed using a modified Bode diagram in which the normal loop gain was adjusted to accommodate the gain

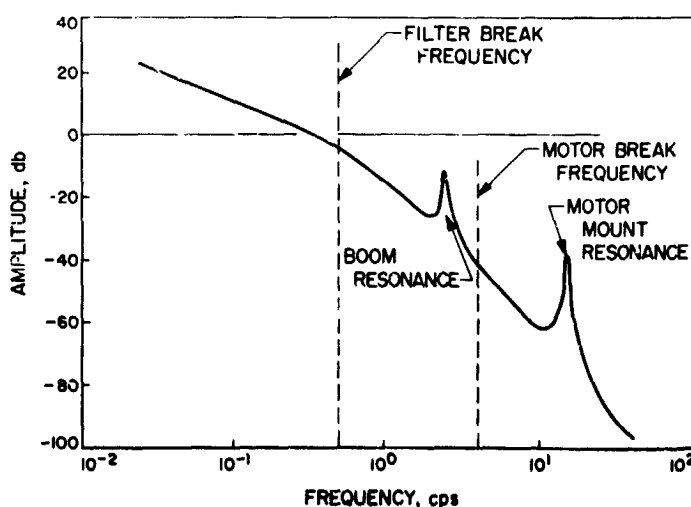


Fig. 3. Scan platform system Bode plot

variation as a function of signal level. The motor break frequency was approximately 4 cps, and the filter break frequency was set for 0.5 cps.

a. Initial simulation. The control system shown in Fig. 2 was simulated for dynamic response using the DSL-90 program on the IBM 7094 digital computer. In this simulation, stiction was not included assuming that the motor delivers sufficient torque to the load.

A permissible range of backlash was investigated to meet the noise requirements. The effect of the backlash to the system noise is summarized in Table 1. The noise level is proportional to the amount of backlash introduced in the range of backlash considered, and the frequency of oscillation is the boom resonant frequency.

Table 1. Effect of backlash to scan system noise

Backlash, rad	±0.0087	±0.0017	±0.00017	±0.000017
Noise, rad	±0.02	±0.004	±0.00035	±0.00004
Noise frequency, rad/sec	15	15	15	15

The tracking capability of the system was investigated by applying a ramp input of 0.0034 rad/sec. The system responds to the ramp input with the average error of 0.0024 rad and with the maximum oscillating amplitude of ±100 μrad. The frequency of oscillation is the boom resonant frequency.

b. System analysis with structural dynamics. The structural dynamics was included in the scan platform control system simulation. The modified section is shown in Fig. 4. The stiction, rolling friction, and backlash are also included in the diagram. The boom dynamics block is placed in series with the feedback loop, while the base dynamics block is placed in parallel with the loop.

The structural dynamics was investigated by using a model which consists of three inertial wheels constrained by springs and dash pots (Fig. 1). The transfer functions of the dynamics blocks were found as follows:

Boom dynamics transfer function.

$$\frac{\theta_2}{T} = \left(\frac{\alpha_1 S + \alpha_0}{S^2 + \beta_1 S + \beta_0} \right) \frac{1}{S^2}$$

Base dynamics transfer function.

$$\frac{\theta_1}{T} = \left(\frac{S^2 + \gamma_1 S + \gamma_0}{S^2 + \delta_1 S + \delta_0} \right) \frac{1}{S^2}$$

and T is the applied torque. The values of the constants, i.e., the α 's, β 's, γ 's and δ 's, are determined by the I 's, K 's and B 's in the diagram. Because of a comparatively large assumed inertia of the spacecraft (170 slug-ft²), the effect of the spacecraft motion was neglected in the analysis.

By using the RTL-90 program on the IBM 7094 digital computer, the system root-locus plots were obtained to study the transient and stability characteristics of the system.

5. System Test

a. Initial breadboard test. The primary purpose of this breadboard test was to determine the maximum jitter rate during the slew and null mode when the control system is driven by a pulse-width modulated error signal. The test circuit is shown in Fig. 5. It was found that the tachometer feedback alone was not sufficient to smooth out the input pulses through the motor due to a relatively small motor time constant.

A filter stage was provided for this purpose. The jitter amplitude referred to the platform was approximately ±160 μrad/sec at the motor threshold voltage. This indicated that it might be possible to achieve a low jitter operation using pulsewidth modulation.

b. Transient test on the breadboard. The characteristics of the planet sensor were simulated using analog circuits as shown in Fig. 6. The planet sensor consists of a saw-tooth generator, two Schmitt triggers and a summing amplifier.

The error signal is generated by manually rotating the potentiometer shaft for a desired error angle. The unbalance in the potentiometer position causes the Schmitt triggers to trigger at a different input voltage level, thus generating the error signal. The servo loop will move the potentiometer shaft back to its original position. The sensor characteristics as a function of potentiometer position are shown in Fig. 7. In this model, the sensor gain is approximately 10 V/deg within ±1 deg range from the null. The transient error response of the system was observed at test point TP-1. The boom structural resonance was simulated by a second-order resonant circuit. The general patterns of the transient response are shown in Fig. 8.

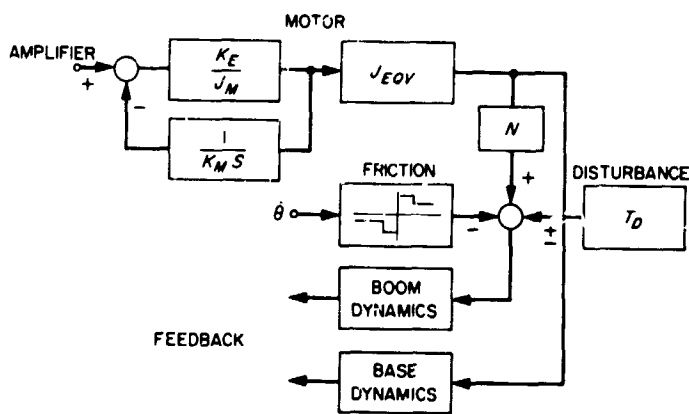


Fig. 4. Scan platform model with dynamics

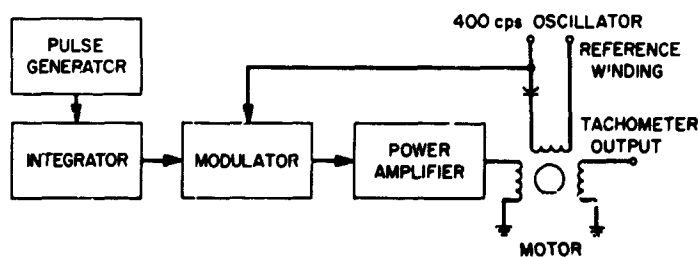


Fig. 5. Pulsewidth modulation test circuit

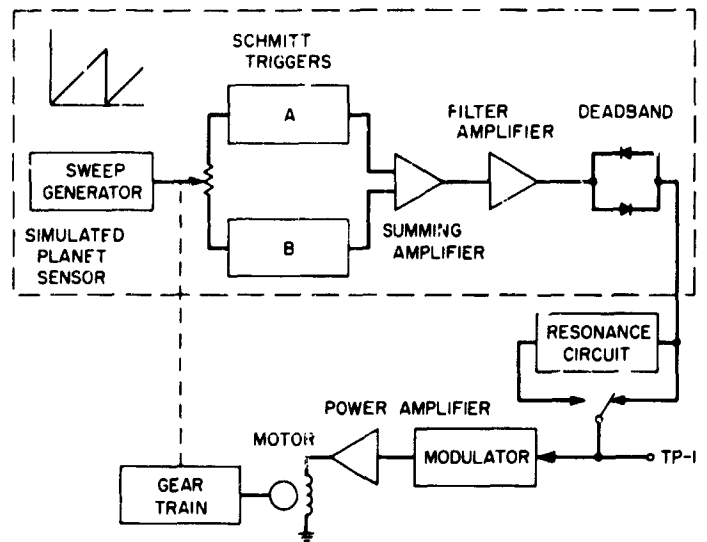


Fig. 6. Scan platform system simulation

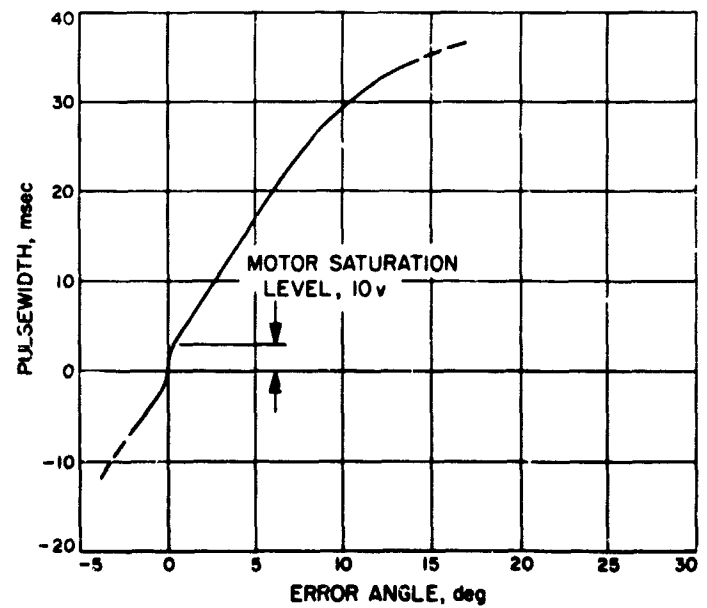
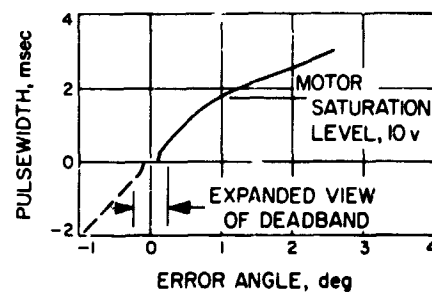


Fig. 7. Characteristics of simulated planet sensor

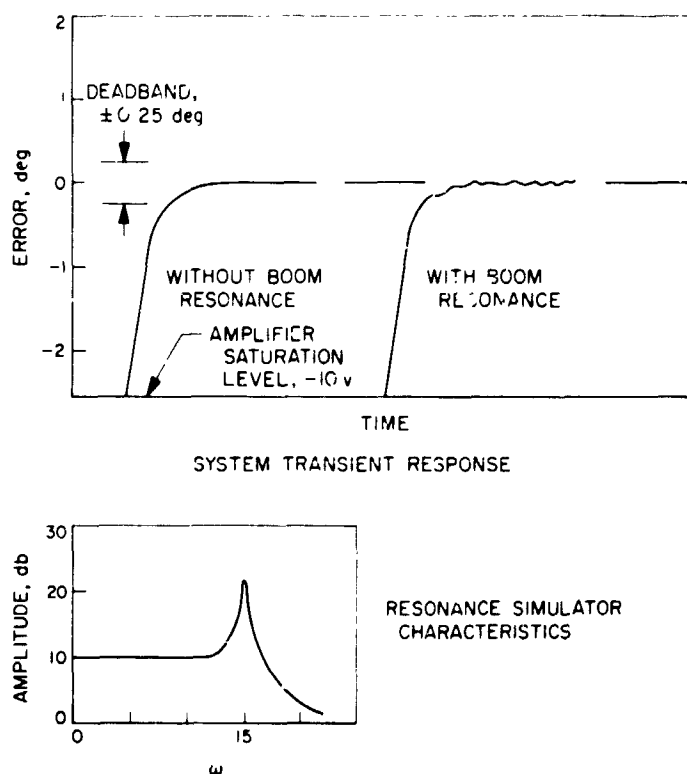


Fig. 8. Platform breadboard test results

The major source of error (other than stiction) in pointing accuracy comes from the planet sensor itself. The selected deadband of ± 0.25 deg corresponds to ± 2.5 volts at the input to the servo-amplifier. Therefore, all drift and offset voltages referred to the amplifier input (± 8.5 mv maximum) becomes negligible. The circuit design was considerably simplified by using integrated circuit amplifiers.

B. Calibration Procedure Used in the Attitude Control Nozzle Thrust Measuring Technique, J. D. Ferrara

The intent of this report is to update a previous report entitled "Attitude Control Thrust-Nozzle Measuring Techniques" by J. C. Randall (SPS 37-39, Vol. IV, pp. 40-42). The cantilever beam and strain gage approach, as detailed in Randall's report, has been attempted. A flattened rod immersed in a pool of mercury has been added behind the valve nozzle combination in order that the amount of damping in the system can be easily varied (see Fig. 9). In using the strain gage approach, considerable damping is required to measure steady-state thrust levels; a minimum amount of damping is required in determining the impulse bit as outlined in Randall's report.



Fig. 9. Micrometer mounted in front of thrust nozzle

The strain gage approach presented the following three problems:

- (1) The gages were found to be very sensitive to 60-cycle noise and vibrations from the mechanical pump used in the vacuum station.
- (2) The gages were found to be sensitive to various temperature and pressure changes in and out of the vacuum jar which resulted in a zero shift, making any calibration procedure difficult.
- (3) The gages were found to be slightly sensitive to the flow of nitrogen gas in the cantilever beam feed tube.

Because of the problems encountered with the strain gages, a photoelectric pickoff was installed. A square-edged plate was mounted to the thruster body such that motion of the beam results in a varying amount of light passing through a slot immediately above the photo-sensitive cell. This technique appears to work very well and is insensitive to the problems encountered with the strain gage. The photoelectric pickoff that is currently being used, however, is not shown in either Figs. 9 or 10.

We are presently calibrating the system using the following method: By mounting a micrometer to the frame immediately in front of the nozzle cone (see Fig. 9) and by advancing the micrometer in known increments and recording the voltage output of the photocell, a curve of thruster displacement versus output voltage can be generated. A stainless steel ball is then suspended in front of the nozzle from a micrometer attachment. This is shown in Figs. 10, 11. When the micrometer is moved a distance X_m , the beam, in equilibrium with the applied force, will

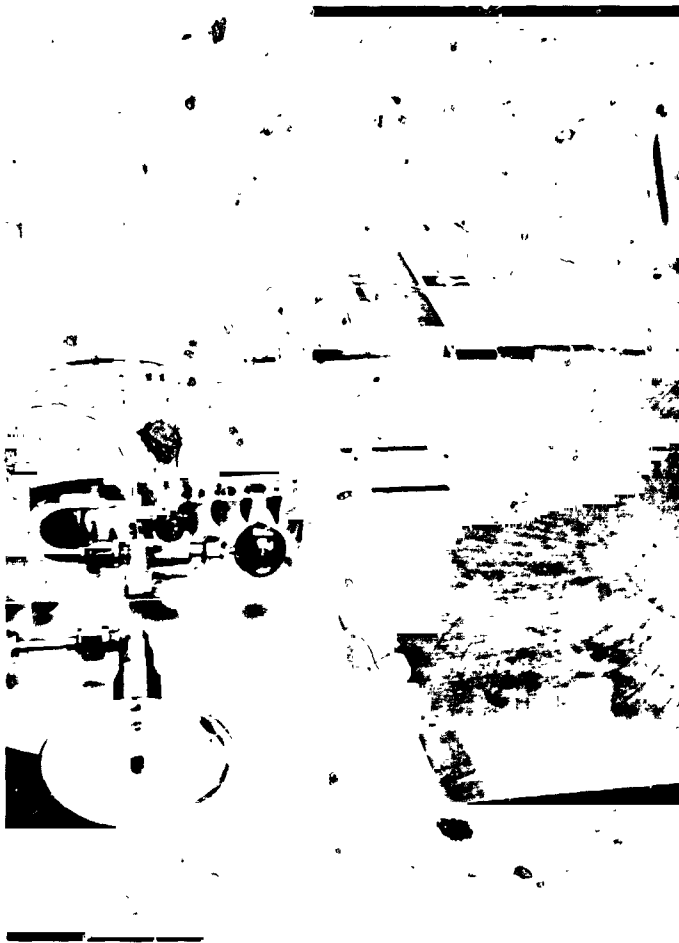


Fig. 10. Steel ball suspended in front of thrust nozzle

deflect an amount X_b ; X_b is determined from the voltage output versus thruster displacement curve described above. A study of Fig. 11 indicates that:

$$f = (m)(\tan \theta) = \frac{(m)(x)}{(l^2 - x^2)^{1/2}} \quad (1)$$

m = weight of ball, gm

$x = |X_m - X_b|$, in.

l = length of wire plus one-half the diameter of the ball, in.

f = applied force, gm.

Since l is from 2 to 3 in. and x is on the order of 0.001 to 0.003 in., f can be accurately determined by a simplification of Eq. (1):

$$f = \frac{(m)(x)}{(l)} \quad (2)$$

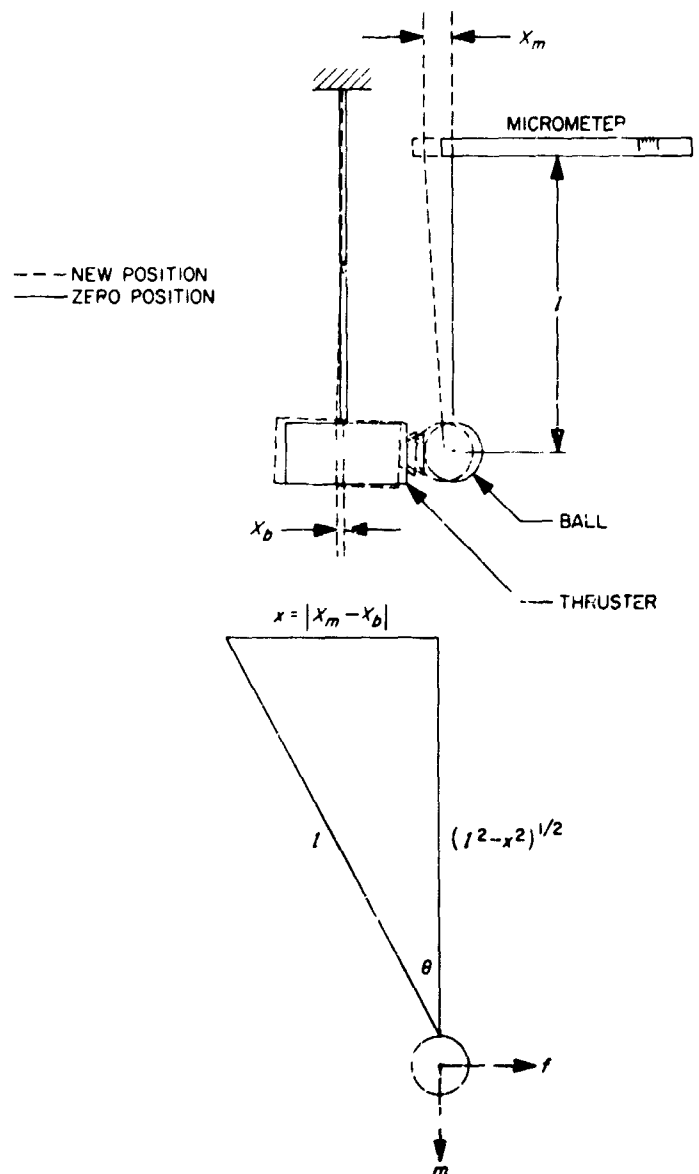


Fig. 11. Beam displacement calibration procedure for measuring nozzle thrust

Using Eq. (2) and the curve mentioned above, a curve can be plotted of the photocell voltage output versus the applied force (f), which is equivalent to the force produced by a firing of the thruster. With this method, the thrust level can be very accurately determined. The impulse bit is determined as outlined in Randall's report (SPS 37-39, Vol. IV, pp. 40-42).

When this calibration procedure is perfected, a test program will begin using *Mariner* type thrusters to verify the results of a computer program described in the SPS report, "Parametric Analysis for Steady-State Performance of Attitude Control Thrusters," by J. D. Ferrera and P. M. McKown (SPS 37-42, Vol. IV, pp. 47-48).

C. Generalized Cruise Gas Requirement Study,

G. Paine

1. Introduction

The aim of this study is to provide an estimate of the cruise gas requirement for a general spacecraft as a function of spacecraft weight. If a better estimate is required, then the craft configuration must be specified since cruise gas consumption depends heavily on both the spacecraft weight and configuration. This study can be used in an evaluation of the tradeoffs between various gas systems as well as of the tradeoffs between communication rates and powers and antenna pointing accuracy.

The general spacecraft chosen has a cylindrical geometry (Fig. 12), a uniform mass density, and no solar panels. The addition of solar panels can be allowed for easily; however, their addition at this stage complicates the design unnecessarily as their size depends on the power requirements and the mission trajectory.

There are several types of gas attitude control systems that can be considered: cold gas, warmed gas, and hot gas. The use of a dual thrust level system can save considerable gas in a large spacecraft by providing a low I_{sp} during cruise and a high I_{sp} during spacecraft maneuvers. The use of heaters in a cold gas system fulfills these requirements as the heaters may be turned on for spacecraft maneuvers and left off during cruise. Such a system would be relatively simple and fail-safe.

The attitude control system analyzed is a cold gas system in which the jets are sized for some angular acceleration constant. The constant is determined by the maximum allowable initial acquisition time and the mid-course maneuver times. A dual level system was not analyzed as they are not employed on current interplanetary

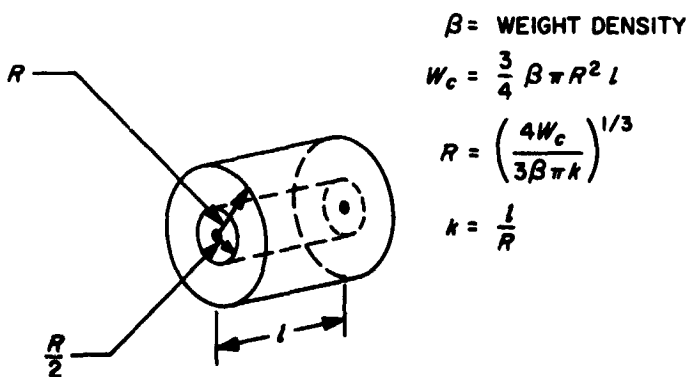


Fig. 12. Spacecraft whose configuration is a hollow cylinder

spacecraft. Likewise the minimum on-time for a jet was held at current levels, rather than being reduced to lower the cruise gas requirements in a large spacecraft.

The disturbance torques acting on the spacecraft have been lumped into two classes: In the first are those resulting from valve leakage. The second includes all other sources, such as unbalanced solar pressure and gravity gradients. The second must be estimated for any particular spacecraft configuration and mission.

2. Conclusions

The cruise gas consumption for a 200-day mission is displayed in Fig. 13. These results include triple redundancy but no tankage factor. The parameters not appearing on the figure are from *Mariner IV*. The results can be scaled directly for missions lasting other than 200 days. The curve for zero disturbance torque limit cycle gas consumption can be scaled inversely to the limit cycle deadband. Less gas would be required with the addition of solar panels and with the attitude control jets located at their extremities since both the constant disturbance torque and the zero disturbance torque gas consumptions are inversely proportional to the jet lever arm.

It is apparent that for the spacecraft geometry and other parameters assumed that cruise gas consumption

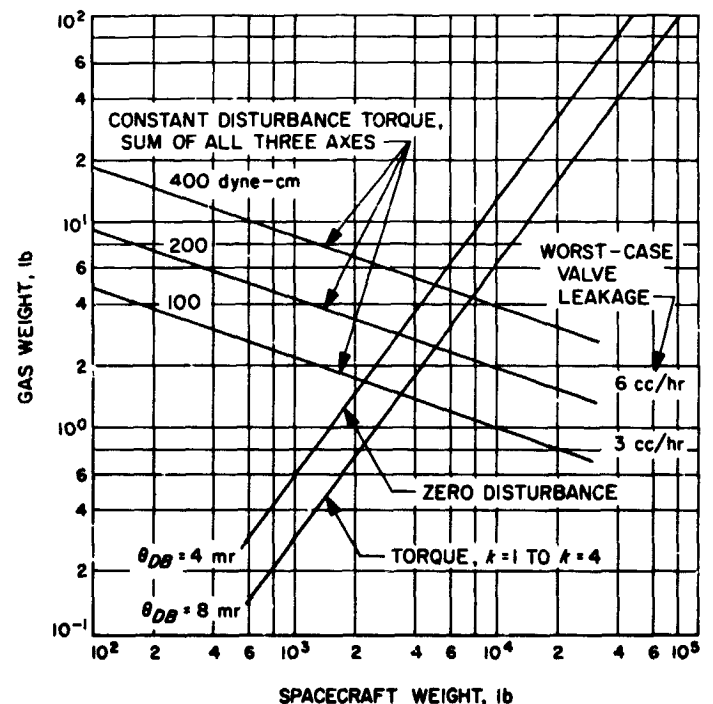


Fig. 13. Cruise gas consumption during 200-day mission

depends more on disturbance torque level than it does on the zero disturbance torque situation for spacecraft weighing less than about 2000 lb.

If the non-leakage disturbance torques are unspecified, it is reasonable in the light of the disturbance torques encountered on *Mariners II* and *IV* to allow 2.0 lb of gas for cruise purposes for spacecraft weighing less than 2000 lb. This is equivalent to about a 75 dyne-cm disturbance torque on a 500-lb craft without solar panels or about 225 dyne-cm on a 500-lb craft with them. On a 200-day mission, an additional 0.75 lb of gas will cover the worst-case valve leakage.

For missions where either the spacecraft is heavy or where the disturbance torques are expected to be small, the zero disturbance torque gas consumption will be the dominating factor and the gas consumption will vary according to spacecraft weight raised to the 4/3 power.

3. Gas Consumption

As the spacecraft studied is not equipped with solar vanes which passively control the spacecraft, the gas consumption will depend directly on the disturbing torque. For a constant disturbing torque, Fig. 14 shows the relationship between the magnitude of the torque and the weight of the gas consumed for a spacecraft which oscillates inside a symmetrical deadband.

To the left of τ_0 , the marginal torque in Fig. 14, the spacecraft oscillates and touches both sides of the dead-

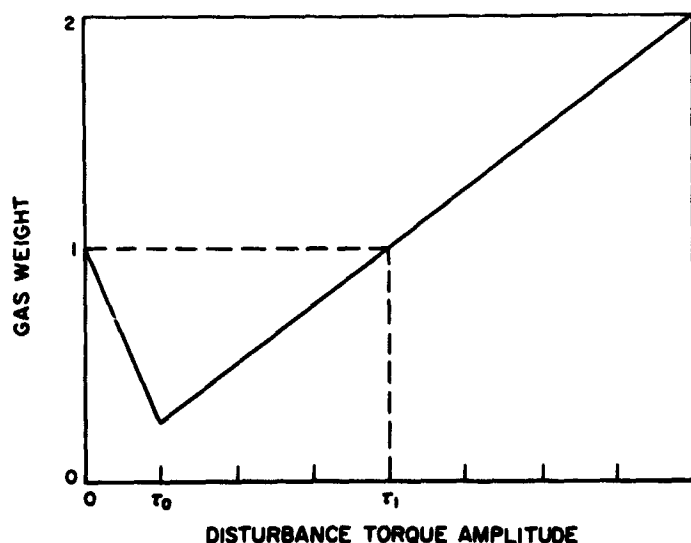


Fig. 14. Mass of consumed gas versus disturbance torques

band. To the right, the spacecraft touches only one side of the deadband.

If the maximum (worst-case) disturbance torque allowed is greater than τ_1 , then the gas consumption is determined totally by the disturbance torque and the mission duration. If the maximum disturbance torque is less than τ_1 , then the maximum gas consumption will occur with no disturbance torque present, and the gas consumption depends on a variety of vehicle parameters.

4. Disturbance Torques

A number of disturbances act on the spacecraft. They include externally caused torques such as those arising from solar pressure and gravity as well as internally generated ones from valve and piping gas leakage, as well as other sources.

Each disturbance torque can be related to a wide range of parameters. Here the parameters which depend on vehicle weight are called out explicitly while the others are held constant. So, for instance, the solar torques will be taken as proportional to vehicle area, or vehicle weight raised to the 2/3 power, neglecting the many other factors influencing solar torques. Valve leakage, which can produce a torque is taken as a worst-case constant, even though it varies from valve firing to valve firing. The same gas consumption will arise whether all twelve valves are leaking or whether only six are leaking and the other six must be fired to produce a restoring torque.

The gas weights required to counterbalance each of these is shown in Fig. 15. The constant disturbing torque

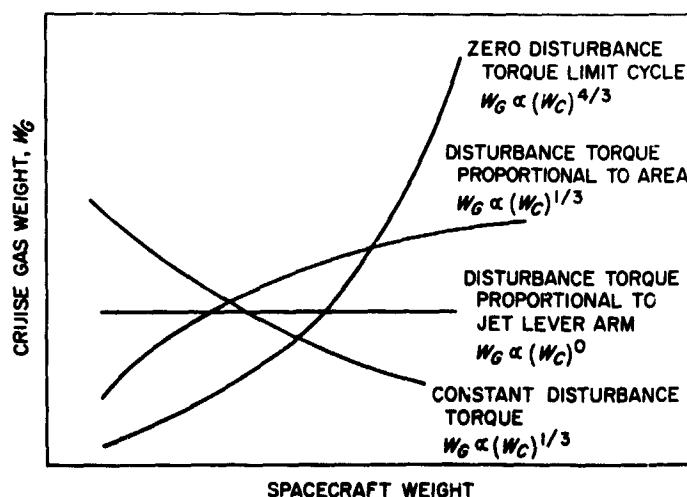


Fig. 15. Cruise gas consumption versus spacecraft weight and disturbance torque type

and the special case of zero disturbing torque gas consumptions are also included. In each case the counterbalancing torque is proportional to the lever arm, or spacecraft weight raised to the 1/3 power. Thus in the example of an area-dependent torque, the gas required increases only as the 1/3 power of the spacecraft weight although the disturbance increases by the 2/3 power.

From Fig. 15 it can be seen that there are two distinct regions: one where the disturbance torques control the gas consumption, and another where the zero disturbance torque gas consumption dominates. The weight of the gas required depends in the former region on the predominant disturbance and will vary with spacecraft weight according to the nature of the largest disturbance. In the former region, the worst worst-case valve leakage gas consumption adds directly to the other gas weights.

5. Mathematics of Gas Consumption

As has been shown in many other places (Refs. 3 and 4)¹ the weight of the gas consumed depends on several simple formulae,—making no allowances for either the triple gas redundancy or tankage weight factors. The formulae below give the gas consumption for each axis: The terminology is listed in Table 2.

a. Zero disturbance torque gas consumption:

$$W_g = \frac{I(\alpha\Delta t)^2 t_m}{4LI_{sp}\theta_{DB}}$$

b. Marginal disturbance torque gas consumption: Marginal torque is the smallest torque for which the attitude control system touches only one side of the deadband.

$$W_g = \frac{I(\alpha\Delta t)^2 t_m}{16LI_{sp}\theta_{DB}}$$

$$\tau_0 = \frac{(\alpha\Delta t)^2 I}{16\theta_{DB}}$$

c. Torque balance gas consumption:

$$W_g = \frac{t_m\tau_D}{LI_{sp}}$$

d. Largest disturbance torque with gas consumption equal to zero disturbance torque gas consumption:

$$\tau_1 = \frac{(\alpha\Delta t)^2 I}{4\theta_{DB}}$$

¹Also by H. K. Bouvier: *A Field Guide to Limit Cycles*, and *Mariner IV Disturbance Torques*, JPL-IOM 344-543.

Table 2. Nomenclature

W_g	gas weight
I	moment of inertia of spacecraft
L	gas jet lever arm
α	acceleration constant
Δt	minimum on time
t_m	mission time
I_{sp}	specific impulse
θ_{DB}	deadband
τ_0	marginal torque
τ_1	unbalance torque resulting in the same gas consumption as no disturbing torque
β	spacecraft weight density
k	spacecraft length to radius ratio
g	gravity
W_c	spacecraft weight
R	spacecraft radius
l	spacecraft length
τ_D	disturbance torque

6. General Spacecraft Configuration

The general spacecraft was chosen to be a hollow cylinder (Fig. 12) with a uniform mass density. The gas jets are placed so that the roll jets have a lever arm (L) equal to the radius (R), and the pitch and yaw jets so that they have a lever arm equal to the greater of the radius (R) or half the vehicle length (l). In the following functions, k is the ratio of spacecraft length to radius, and I is the inertia.

a. Pitch and yaw axis functions.

For all k

$$I = \frac{4W_c}{3g} \left(\frac{R^2}{4} + \frac{l^2}{12} \right) - \frac{W_c}{3g} \left(\frac{R^2}{16} + \frac{l^2}{12} \right)$$

or

$$I = \frac{W_c}{4g} \left(\frac{4W_c}{3\beta\pi k} \right)^{2/3} \left(\frac{5}{4} + \frac{k^2}{3} \right)$$

For $k \leq 2$, $L = R$, and

$$\frac{I}{L} = \frac{3\beta k\pi}{16g} \left(\frac{4W_c}{3\beta k\pi} \right)^{4/3} \left(\frac{5}{4} + \frac{k^2}{3} \right) = \frac{W_c R}{4g} \left(\frac{5}{4} + \frac{k^2}{3} \right)$$

For $k > 2$, $L = l/2$, and

$$\frac{I}{L} = \frac{3\beta\pi}{8g} \left(\frac{4W_c}{3\beta k\pi} \right)^{4/3} \left(\frac{5}{4} + \frac{k^2}{3} \right) = \frac{W_c R}{2kg} \left(\frac{5}{4} + \frac{k^2}{3} \right)$$

b. Roll axis function.

$$I = \frac{2W_c R^2}{3g} - \frac{W_c R^2}{24g} = \frac{5W_c R^2}{8g}$$

For any k , $L = R$, and

$$\frac{I}{L} = \frac{15\beta k\pi}{32g} \left(\frac{4W_c}{3\beta k\pi} \right)^{4/3} = \frac{5W_c R}{8g}$$

These factors, which enter directly into the zero disturbance torque gas consumption, are plotted in Fig. 16 for some typical parameters: $k = 1$ and $\beta = 15 \text{ lb/ft}^3$. On Fig. 16, the actual (I/L) values are plotted for *Mariner IV* and for a proposed Venus-Mercury 1973 craft. The differences are explained by the solar panels on these two craft. While the solar panels increase the inertia per unit spacecraft weight significantly, they increase the jet moment arm even more rapidly, giving rise to a net reduction in I/L .

7. Gas Weights and Results

Even with the concepts stated earlier, more assumptions are necessary before the gas requirements can be set down explicitly. Some of these concern the vehicle geometry, others concern the jet size and minimum on-time, while still others relate the disturbances and deadbands of the three axes.

The gas weight shown is for three-axes stabilization and includes the triple redundancy factor but does not allow for tankage. The gas weights should be multiplied by 2.6 to obtain gas weight plus variable tankage weight.

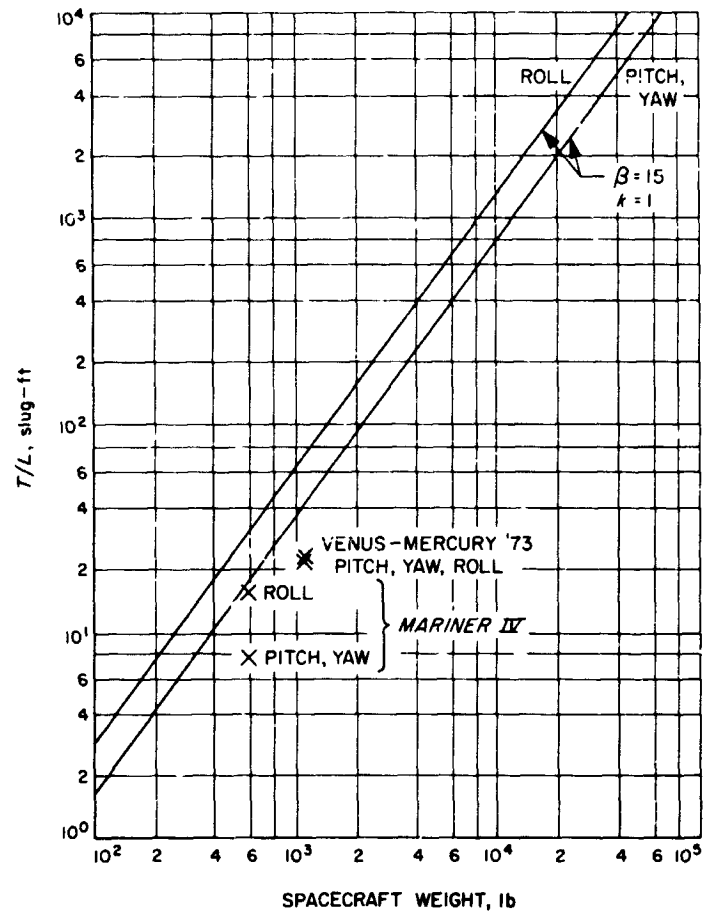


Fig. 16. Ratio of inertia to lever arm length versus spacecraft weight

The gas weight factor to account for half-system performance is 1.5 rather than 3 in the case of valve leakage.

In Fig. 13, gas consumption is shown as a function of spacecraft weight for $k = 1$ to $k = 4$, three levels of constant disturbance torque, two levels of gas leakage, and two deadband widths.

In Fig. 13 the following additional constants, taken from *Mariner IV* (see Ref. 4), are assumed:

$\theta_{DB} = \text{same for all three axes}$

$\alpha = 0.45 \text{ mr/sec}^2$

$\Delta t = 0.02 \text{ sec}$

$I_{sp} = 60 \text{ sec}$

$t_m = 200 \text{ days}$

$\beta = 15 \text{ lb/ft}^3$

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V. Guidance and Control Research

GUIDANCE AND CONTROL DIVISION

N67-29146

A. Temperature Dependence of the Velocity of Sound in Liquid Helium; I. Theory, W. M. Whitney

1. Introduction

For liquid helium, there exists a microscopic model that can be used as a basis for calculating the temperature dependence of the entropy, specific heat, and other equilibrium properties; this is the phonon-roton model of Landau (Ref. 1), which was described briefly in an earlier report (SPS 37-39, Vol. IV, pp. 43-48). The velocity of sound at sufficiently low frequencies is an equilibrium property, and it should therefore be possible to determine its temperature dependence directly from the model. Since there is known to be good agreement between the calculated and measured temperature dependences of the thermal functions upon which the velocity depends, such a calculation would be of interest only as an exercise if it were not for the fact that the model itself is not complete. Its microscopic basis has not been fully worked out; in particular, the density dependence of four parameters that characterize the energy spectrum of the phonons and the rotons cannot be given in terms of basic quantities

such as Planck's constant and the mass of the helium atom. Furthermore, experiments that have been done to date have not removed all doubt about what the density dependence of some of these parameters might be.

It will become clear from the present discussion that the temperature dependence of the sound velocity reflects certain relationships among the first and second derivatives of these parameters with respect to density. In this report, we will derive these relationships; in a later one, we will use them to analyze existing data (Ref. 2), including some measurements of the velocity at 10 MHz made recently in our laboratory. The calculations described below have been done in collaboration with C. E. Chase of the National Magnet Laboratory, M.I.T.

2. Hydrodynamic Considerations

It is known that liquid helium will sustain two kinds of wave motion: ordinary sound or pressure-density waves, arising from the elasticity of the fluid; and "second" sound, as it is called, which consists of the propagation of temperature and entropy fluctuations. The wave velocities for these two phenomena, at frequencies sufficiently low

that all dissipative effects can be ignored, are obtained from the hydrodynamic equations as roots of the following "compatibility" equation (Ref. 3):

$$(u^2 - V_1^2)(u^2 - V_2^2) = \frac{\gamma - 1}{\gamma} V_1^2 V_2^2, \quad (1)$$

where

$$V_1^2 = (\partial p / \partial \rho)_s, \quad V_2^2 = \frac{\rho_n}{\rho_n} \frac{T S^2}{C_v}.$$

In these expressions, T is the absolute (Kelvin) temperature, p is the pressure, S is the entropy per unit mass of fluid, and $\gamma = C_p/C_v$ is the ratio of the heat capacity per unit mass at constant pressure to that at constant volume. The normal fluid density ρ_n represents an effective mass associated with a heat flux, and can be calculated from the roton and phonon distribution functions. The superfluid density ρ_s is simply $\rho - \rho_n$.

In liquid helium, the difference $\gamma - 1$ does not exceed 2×10^{-3} at temperatures below 2°K , and is smaller than 10^{-4} over much of this region ($T < 1.3^\circ\text{K}$). For this reason, the right-hand side of Eq. (1) is set equal to zero in the usual treatments. Note that for $\gamma = 1$, the two wave modes appear uncoupled and propagate with the velocities V_1 and V_2 . In this approximation, the expressions for the velocity of "first" or ordinary sound are the same for liquid helium and for normal fluids. The fact that $\gamma \neq 1$ means that first and second sound are not entirely independent modes, and the coupling between them gives ordinary sound in liquid helium a slightly non-adiabatic character. The correction that arises in the expression for the velocity of ordinary sound when allowance is made for the coupling is small but not negligible. If we solve Eq. (1) for u_1 , treating $(\gamma - 1)$ as a small quantity and ignoring terms of higher order, we obtain

$$u_1 = u_T \left[1 + \frac{\gamma - 1}{2} (1 - V_2^2/u_T^2)^{-1} \right].$$

To put the equation in the form shown, we have made use of the thermodynamic identity $(\partial p / \partial \rho)_s = \gamma (\partial p / \partial \rho)_T$ and have set $u_T = (\partial p / \partial \rho)_T^{1/2}$. We have also made the approximation

$$\gamma^{1/2} = [1 + (\gamma - 1)]^{1/2} \simeq 1 + \frac{1}{2}(\gamma - 1).$$

Sufficient accuracy is retained if, in the correction term, we replace u_T by u_{10} , the sound velocity at $T = 0^\circ\text{K}$. We

have finally

$$u_1 = u_T + \frac{1}{2}(\gamma - 1) u_{10} (1 - V_2^2/u_{10}^2)^{-1}. \quad (2)$$

3. Thermodynamic Considerations

The pressure p can be written as a derivative of the Helmholtz free-energy function A (per unit mass of fluid):

$$p = \rho^2 (\partial A / \partial \rho)_T.$$

Since changes in the density are small over the temperature region of interest to us, we can expand p about the density ρ_0 at $T = 0$ to obtain

$$p = \rho_0^2 (\partial A / \partial \rho)_T + (\rho - \rho_0) \frac{\partial}{\partial \rho_0} \left(\rho_0^2 \frac{\partial A}{\partial \rho_0} \right)_T + \frac{1}{2} (\rho - \rho_0)^2 \frac{\partial^2}{\partial \rho_0^2} \left(\rho_0^2 \frac{\partial A}{\partial \rho_0} \right)_T + \dots \quad (3)$$

and

$$u_1^2 = (\partial p / \partial \rho)_T = \frac{\partial}{\partial \rho_0} \left(\rho_0^2 \frac{\partial A}{\partial \rho_0} \right)_T + (\rho - \rho_0) \frac{\partial^2}{\partial \rho_0^2} \left(\rho_0^2 \frac{\partial A}{\partial \rho_0} \right)_T + \dots \quad (4)$$

Following similar treatments of anharmonic effects in elastic solids (Refs. 4, 5), we write the Helmholtz function as the sum of a zero-point energy U which depends only upon density, and a thermal energy F which is a function of both temperature and density and represents the free energy of the thermal excitations (the phonons and the rotons):

$$A(\rho, T) = U(\rho) + F(\rho, T).$$

It is assumed that $F \ll U$. At $T = 0$, where $F = 0$, $U(\rho_0)$ represents the minimum energy of the liquid, so $dU/d\rho_0 = 0$; consequently for $T \neq 0$,

$$(\partial A / \partial \rho_0)_T = (\partial F / \partial \rho_0)_T.$$

The velocity of sound when the fluid is in its ground state depends only upon U :

$$u_{10}^2 = \frac{d}{d\rho_0} \left(\rho_0^2 \frac{dU}{d\rho_0} \right) = \rho_0^2 \frac{d^2 U}{d\rho_0^2}.$$

When the substitutions indicated above have been made and terms of second order in the small quantities

$(\rho - \rho_0)$ and F have been dropped, Eqs. (3) and (4) take the forms

$$p = (\rho - \rho_0) u_{10}^2 + \rho_0^2 (\partial F / \partial \rho_0)_T, \quad (5)$$

$$u_T^2 = u_{10}^2 + 2(\rho - \rho_0) u_{10} \frac{du_{10}}{d\rho_0} + \frac{\partial}{\partial \rho_0} \left(\rho_0^2 \frac{\partial F}{\partial \rho_0} \right)_T. \quad (6)$$

Eq. (5) can be regarded as the equation of state for the liquid. Since the measurements with which the theory will be compared have been made under the saturated vapor pressure, we substitute the vapor pressure $P(T)$ for p in (5) and solve for the density:

$$\rho_{sat} - \rho_0 = P(T) / u_{10}^2 - (\rho_0^2 / u_{10}^2) (\partial F / \partial \rho_0)_T.$$

After substituting this expression in (6) and taking the square root, remembering that the second and third terms on the right-hand side of that equation are small in comparison with the first, we obtain

$$u_T = u_{10} + \gamma_g \frac{P(T)}{\rho_0 u_{10}} + \frac{1}{2u_{10}} \times \left[\frac{\partial}{\partial \rho_0} \left(\rho_0^2 \frac{\partial F}{\partial \rho_0} \right)_T - 2\rho_0 \gamma_g \left(\frac{\partial F}{\partial \rho_0} \right)_T \right] \quad (7)$$

where $\gamma_g = (\rho_0 / u_{10}) du_{10} / d\rho_0$. Henceforth we drop the subscript zero from the density.

The contribution to u_T from the vapor pressure is of negligible importance (< 0.1 cm/sec) below 1°K, but it increases rapidly with temperature and at 2°K amounts to 25 cm/sec.

4. The Phonon and Roton Contributions to the Velocity

The hydrodynamic and thermodynamic constraints on the behavior of the velocity are embodied in Eqs. (2) and (7). To proceed further, we must introduce explicit expressions for the thermal energy F . It is at this point that the phonon-roton model is invoked.

It can be assumed that all of the thermal energy of liquid helium below approximately 1.8°K is carried by two types of energy quanta—the phonons and the rotons, each type making a separate contribution to the thermal energy F so that it is possible to write

$$F = F_{ph} + F_r.$$

To derive explicit formulas for F_{ph} and F_r from the model, one must know the relation between energy and momentum for each type of excitation. For the phonons, $\epsilon = cp$, where c is the phonon velocity. Inelastic neutron-scattering experiments (Ref. 6) show that, within experimental error, c is equal to the measured sound velocity at low frequency. In fact, this equality can be only approximate. The typical thermal phonon has a frequency $\nu = k_B T / h$ which amounts to ~ 20 GHz at 1°K, with a corresponding wavelength $\lambda \sim 120$ Å. (k_B = Boltzmann's constant, h = Planck's constant.) The propagation of a phonon at such a frequency, for which the period is much shorter than the mean time between collisions of the phonons with each other or with rotons, is quite a different physical process from the low-frequency one that we are considering here. In making our numerical comparisons it will be expedient, and approximately correct, to set $c = u_{10}$, but for the moment we shall carry the phonon velocity along as a separate physical constant for liquid helium.

For rotons, $\epsilon = \Delta + (p - p_0)^2 / 2\mu$. Values of the three parameters Δ , p_0 , and μ must be determined indirectly from measurements of the thermal properties of liquid helium, or directly from the dispersion curve $\epsilon(p)$ -vs- p , which can be traced out in experiments with thermal neutrons (Ref. 6). From the results of both types of measurements carried out at different pressures, the derivatives of these quantities with respect to density can be estimated, although as mentioned previously the results of different experiments do not always agree.

From the dispersion relations given above, it can be shown that

$$F_{ph} = - \frac{\pi^2}{90} \frac{k_B T}{\rho} \left(\frac{k_B T}{\hbar c} \right)^3,$$

$$F_r = - \frac{2p_0^2 \mu^{1/2}}{(2\pi)^{3/2} \hbar^3 \rho} (k_B T)^{3/2} e^{-\Delta/k_B T}.$$

The velocity u_T can be represented by the sum

$$u_T = u_{10} + \delta u_{vp} + \delta u_{ph} + \delta u_r,$$

where δu_{vp} denotes the vapor-pressure contribution. By taking the appropriate derivatives of F_{ph} and F_r , as required by Eq. (7), we obtain

$$\delta u_{ph} = G F_{ph} / u_{10}, \text{ where } G = 9\gamma_g^2 + \gamma_g - \frac{3}{2} \frac{\rho^2}{c} \left(\frac{\partial^2 c}{\partial \rho^2} \right)_T, \quad (8)$$

[we have assumed that $(\rho/c)(\partial c/\partial \rho)_T \simeq \gamma_g$], and

$$\delta u_r = \frac{F_r}{2u_{10}} [A + B(\Delta/k_B T) + C(\Delta/k_B T)^2], \quad (9)$$

where

$$A = 2\gamma_g \left[1 - \left(\frac{\rho}{w} \frac{\partial w}{\partial \rho} \right)_T \right] + \left(\frac{\rho^2}{w} \frac{\partial^2 w}{\partial \rho^2} \right)_T,$$

$$B = 2 \left(\frac{\rho}{\Delta} \frac{\partial \Delta}{\partial \rho} \right)_T \left[\gamma_g - \left(\frac{\rho}{w} \frac{\partial w}{\partial \rho} \right)_T \right] - \left(\frac{\rho^2}{\Delta} \frac{\partial^2 \Delta}{\partial \rho^2} \right)_T,$$

$$C = \left(\frac{\rho}{\Delta} \frac{\partial \Delta}{\partial \rho} \right)_T^2,$$

and

$$w = p_{0\mu}^{1/2}.$$

The fact that both the first and second derivatives of the roton and phonon parameters appear in the expressions for δu_{ph} and δu_r means that it will not be possible to make separate determinations of these quantities by fitting experimental data to the theory. However, the values of the first derivatives can be estimated from other sources, so from these and the results of the fit the second derivatives can be calculated.

It should be emphasized that the results that we have obtained will be valid only at frequencies sufficiently low that the thermodynamic constraints remain in force; in particular, the period of the sound wave must always be small compared with the time required for thermal equilibrium to be established in the presence of the wave. Since the characteristic relaxation times increase rapidly as the temperature falls, the range above zero frequency over which the theory will be applicable will grow smaller and smaller as the temperature decreases. Generally speaking, measurements carried out at frequencies above 1 MHz cannot be compared with the theory at temperatures much below 1°K without making allowance for the frequency dependence of the velocity.

5. Behavior of the Velocity in the Phonon Region

Below 0.6°K, the rotons play a very minor role in sound propagation, and the temperature dependence of the velocity at zero frequency can be ascertained by considering the phonons alone. It is not difficult to show that

the predicted low-temperature form of the equation for u_1 is

$$u_1 = u_{10} + u_{10} \frac{\rho_{nph}}{\rho} \left\{ \frac{1}{4} (3\gamma_g + 1) - \frac{3}{2} \left[\gamma_g^2 - \frac{1}{4} \frac{\rho^2}{c} \left(\frac{\partial^2 c}{\partial \rho^2} \right)_T \right] - \frac{1}{4} \gamma_g (3\gamma_g + 1) \right\}, \quad (10)$$

where

$$\rho_{nph}/\rho = \frac{2\pi^2}{45} \frac{k_B T}{\rho c^2} \left(\frac{k_B T}{\hbar c} \right)^3.$$

The first term inside the braces in Eq. (10) reflects the coupling between first and second sound and the factor γ that relates the isothermal and adiabatic compressibilities; the second term arises from the compressibility of the phonon gas; and the third term represents the change in velocity attributable to the shift in the zero-point energy $U(\rho)$ as the density changes. Corresponding expressions for the velocity at low frequencies in the phonon region have been given recently by Khalatnikov and Chernikova (Ref. 7) and by Disatnik (Ref. 8), whose approaches were quite different from that given here. They obtain an expression equivalent to Eq. (10) but with the last term left out. In both papers, no allowance has been made for the shift in zero-point energy. The numerical consequences of this omission are not negligible because the term that has been left out is comparable in magnitude (and identical in sign) with the sum of the terms left in. The treatments of Mott and Jones and of Landau and Lifshitz (Refs. 4 and 5) make it clear that the shift in zero-point energy cannot be ignored, so the expressions given by Khalatnikov and Chernikova and by Disatnik must be considered to be in error.

B. Boundary Trapping in Unipolar Space-Charge-Limited Current, D. Stoffa, V. Rodriguez and M.A. Nicolet¹

1. Introduction

Within the limits of the approximations of Mott and Gurney, the flow of unipolar space-charge-limited current (sclc) at steady state follows the general law

$$I = \epsilon \mu V^2 g \quad (1)$$

¹At the California Institute of Technology, performing work supported by the Jet Propulsion Laboratory.

where V is the applied voltage, ϵ and μ are the dielectric constant and the mobility of the medium through which the current I passes, and δ is a factor which depends only on the geometrical configuration of the emitter and collector electrodes (Ref. 9). Values of δ are reported in the literature for the five distinct cases of the planar, cylindrical and spherical geometry (Ref. 10):

Planar

$$\delta = 9a/8W^2 \quad (1a)$$

Cylindrical

$$\delta = 2\pi l r_c^2 [(k^2 - 1)^{1/2} - \arccos(k^{-1})]^{-2}, \quad 1 \leq k \quad (1b)$$

$$\delta = 2\pi l r_c^2 [\operatorname{arcosh}(k^{-1}) - (1 - k^2)^{1/2}]^{-2}, \quad k \leq 1 \quad (1c)$$

Spherical

$$\delta = 6\pi r_c^2 \left[\int_1^k (z^3 - 1)^{1/2} z^{-2} dz \right]^{-2}, \quad 1 \leq k \quad (1d)$$

$$\delta = 6\pi r_c^2 \left[\int_k^1 (1 - z^3)^{1/2} z^{-2} dz \right]^{-2}, \quad k \leq 1 \quad (1e)$$

where, in the planar geometry, W is the width between emitter and collector boundaries, and a is the area under consideration. In the cylindrical geometry, r_c is the radius of the emitter, kr_c is the radius of the collector, and l is the length of the cylinder under consideration; r_c and k have the same meaning in the spherical geometry.²

Rose (Ref. 11) and Lampert (Refs. 12, 13) have shown that if the medium contains a uniform distribution of shallow traps—that is, traps whose population is in a Boltzmann equilibrium with the free charge carriers surrounding it—sc is suppressed by a constant trapping factor $F \ll 1$. The modified characteristic

$$I = \epsilon \mu V^2 \delta F \quad (2)$$

is valid in a range $V \ll V_{TFL}$, a condition which reflects the assumption made in Eq. (2) that the space charge of the trapped charge carriers dominates and that the mobile charge carriers contribute negligibly to the divergence of the field.

²A confusion between $k = r_c/r_e$ and the Boltzmann constant is avoided if it is noticed that the latter appears only as a product kT , while the former never does.

It was shown recently (Ref. 10) that the value of the trapping factor F does not depend only on the energy distribution of the shallow traps, but that their spacial distribution also affects the value of F . In particular, shallow traps outside of the immediate neighborhood of the emitter are of little significance, except possibly for large peaks in the trap distribution. Since traps are most likely to occur in connection with imperfections, and since boundaries represent the most prominent imperfection of a lattice, trapping at the emitter boundary stands out as a particularly significant phenomenon. The purpose of this note is to discuss the properties and the significance of this effect.

2. Model for Boundary Trapping

Trapping at the emitter boundary will be described here by the most simple model, assuming that only one type of trap is present at the boundary in a surface concentration Σ and that the trap possesses one discrete trapping energy E_t in the forbidden band (see Fig. 1). A generalization including several discrete energy levels and/or trapping bands can be made along the lines developed in the analyses of bulk trapping and will not be considered here. It will be further assumed that $E_b - E_t$ is always at least several kT , so that Boltzmann statistics apply to the mobile charge carriers. No restriction will be imposed on the relative locations of E_t and E_i ,

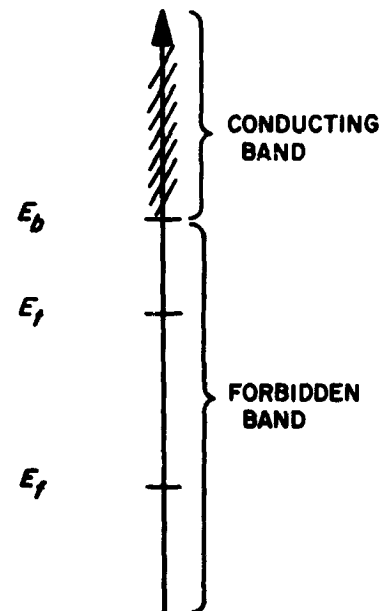


Fig. 1. The position of the energy level E_t of the trap is assumed to be known and fixed with respect to the band edge E_b ; E_f shall not approach E_b by less than at least several kT

so that the population of the traps will have to be described by Fermi statistics. The trapped charge σ per unit area of the emitter boundary then is (q = value of electronic charge)

$$\sigma/q\Sigma = [(\exp(E_t - E_f)/kT) + 1]^{-1} \quad (3)$$

while the mobile charge carrier concentration ρ at the emitter is

$$\rho/qN = \exp[-(E_b - E_f)/kT] \quad (4)$$

where N is the effective density of states of the conducting band. Introducing $\gamma = \exp[(E_b - E_f)/kT]$ for E_f and $\beta = \exp[(E_b - E_t)/kT]$ as a parameter one obtains

$$q\Sigma/\sigma = \gamma\beta^{-1} + 1 \quad (3a)$$

and

$$qN/\rho = \gamma. \quad (4a)$$

The trapped charge will generate an electric field of a value $E = \sigma/\epsilon$, and that same field will be responsible for the flow of current at the emitter boundary; hence, assuming that the field is constant on the emitter boundary,

$$I = a\mu\rho E = a\mu\rho\sigma/\epsilon \quad (5)$$

where a is the total area of the emitter. Eqs. (3) to (5) specify ρ , σ and E_f or γ uniquely. The solution in terms of γ is

$$\gamma = (\beta/2) [-1 + (1 + 4b/\beta)^{1/2}] \quad (6)$$

which specifies the location of the Fermi level for any given trap depth and current via $\beta = \exp[(E_b - E_t)/kT]$ and

$$b = a\mu q^2 N \Sigma / I \epsilon. \quad (7)$$

3. SCLC Dominated by Boundary Trapping ($V < V_{TFL}$ and $F < 1$)

In the limit of a "shallow" trap, that is, when the Fermi level lies below E_t by at least several kT , the population of the traps is governed by Boltzmann statistics as well and Eq. (6) simplifies to

$$\gamma = (\beta b)^{1/2} \quad (8)$$

which states that the mobile charge carrier concentration at the emitter varies as

$$\rho = (I\epsilon N/a\mu\beta\Sigma)^{1/2} \quad (9)$$

while the ratio ρ/σ is constant

$$\rho = \sigma N/\beta\Sigma. \quad (10)$$

Since the current at the emitter boundary is $I = a\mu\rho E$ and ρ is given by Eq. (9), one obtains

$$I = (\epsilon\mu a N/\beta\Sigma) E^2. \quad (11)$$

The voltage V , on the other hand, is determined solely by the field E since the space charge of the mobile charge carriers is negligible. One thus finds by direct integration

$$\text{Planar} \quad V = E \cdot W \quad (12a)$$

$$\text{Cylindrical} \quad V = E \cdot r_e \ln(k), \quad 1 \leq k \quad (12b)$$

$$V = E \cdot r_e \ln(k^{-1}), \quad k \leq 1 \quad (12c)$$

$$\text{Spherical} \quad V = E \cdot r_e (1 - k^{-1}), \quad 1 \leq k \quad (12d)$$

$$V = E \cdot r_e (k^{-1} - 1), \quad k \leq 1. \quad (12e)$$

All of these expressions are of the form

$$V = E\eta \quad (12)$$

where η depends on the geometry of the sample. The V - I characteristic therefore becomes

$$I = \epsilon\mu V^2 \cdot a\eta^{-2} N/\beta\Sigma \quad (13)$$

and in agreement with Eq. (2).

In the limit where trapping dominates, the effect of boundary trapping at the emitter thus consists in introducing a factor $a\eta^{-2}N/\beta\Sigma$ into Eq. (1). Since $a\eta^{-2}$ depends on the geometry, this indicates that sclc will vary differently with the size of a structure depending on whether

trapping takes place uniformly in the volume or on the emitter boundary only, thus allowing us to distinguish between the two cases experimentally.

In the planar case, for example, shallow traps with a discrete energy level E_t and distributed uniformly throughout space at a density n_t will suppress sclc by a factor (Ref. 11):

$$F = N/\beta n_t \equiv \theta \quad (14)$$

for $V \ll V_{TFL}$, so that

$$I = \epsilon \mu V^2 \cdot (9a/8W^3) \cdot \theta \quad (15)$$

while by Eq. (13) the same traps, when present at the emitter boundary only, will yield

$$I = \epsilon \mu V^2 \cdot (a/W^2) \cdot (N/\beta \Sigma). \quad (16)$$

This means that in this case F has a value of $8WN/9\beta\Sigma$, which may be written as

$$F = (8/9) \theta_{eff} \quad (17)$$

where

$$\theta_{eff} = WN/\beta \Sigma \quad (18)$$

has a form identical to that occurring in Eq. (14). The main difference between Eqs. (15) and (16) is in the thickness dependence of the current. For volume trapping F is constant and $I \simeq W^{-3}$, while for boundary trapping at the emitter F is proportional to W and $I \simeq W^{-2}$.

A comparison of the same two cases in cylindrical geometry readily shows that for volume trapping

$$I = \epsilon \mu V^2 \cdot 2\pi l r_e^2 \times [(k^2 - 1)^{1/2} - \arccos(k^{-1})]^{-2} \cdot \theta, \quad 1 \leq k \quad (19)$$

while for boundary trapping at the emitter

$$I = \epsilon \mu V^2 \cdot 2\pi l r_e^{-1} \ln^{-2}(k) \cdot N/\beta \Sigma, \quad 1 \leq k. \quad (20)$$

In this case also the dependence on the geometrical scaling factor r_e is reduced by one order for boundary

trapping. For the last expression F now has the value

$$F = \left(\frac{[(k^2 - 1)^{1/2} - \arccos(k^{-1})]}{\ln(k)} \right)^2 \cdot \theta_{eff} \quad (21)$$

where

$$\theta_{eff} = r_e N/\beta \Sigma. \quad (22)$$

Analogous hold for $k \leq 1$.

A similar comparison in spherical geometry leads to a remarkable fact. For volume trapping

$$I = \epsilon \mu V^2 \cdot 6\pi r_e^{-1} \left[\int_1^k (z^3 - 1)^{1/2} z^{-2} dz \right]^{-2} \cdot \theta, \quad 1 \leq k \quad (23)$$

but for boundary trapping at the emitter

$$I = \epsilon \mu V^2 \cdot 4\pi (1 - k^{-1})^{-2} \cdot N/\beta \Sigma, \quad 1 \leq k. \quad (24)$$

Again, the dependence on r_e has been reduced by one order. The current I is now *independent* of the scaling factor r_e . Since the variation with k disappears also for $k \gg 1$, *unipolar sclc dominated by boundary trapping is essentially independent on sample dimension in spherical geometry and $r_c \gg r_e$* . This property, combined with the experimental significance of the point contact, may offer new possibilities in the study of sclc or the evaluation of material parameters such as μ , N or Σ because it does not require accurate control or even the knowledge of sample dimensions. It should be noted, however, that this result holds only for $1 < k$. For $k < 1$ Eq. (24) contains an additional factor k^2 which reintroduces a geometrical factor into the equation.

The absence of r_e in Eq. (24) is due to a combination of mutually independent effects. The following arguments will help to reveal their origin. At the emitter, the current density is given by $j = \mu \rho E$. The model is such that with the knowledge of E the applied voltage is also determined by direct integration over the same field E ; hence, $V = E r_e (1 - k^{-1})$. This expression is independent of k for $k \gg 1$ because the field decreases so rapidly for $r_c \gg r_e$ in the spherical geometry that the overall potential is determined essentially by the field distribution in the vicinity of the emitter only. This explains the independence of the current on the geometrical factor k and establishes that $E \simeq V/r_e$. But E also specifies the surface density of the trapped charge $\sigma = \epsilon E$ and through it the

free charge carrier concentration $\rho = \sigma(N/\beta\Sigma)$, so that the current density j is proportional to E^2 . The current is thus $I = 4\pi r_c^2 \cdot j \simeq 4\pi r_c^2 \cdot V^2/r_c^2$. For fixed applied voltage V , a change in emitter radius thus generates a variation in $E \cdot \rho$ which exactly cancels the effect of the changing emitter area.

The results derived in this section are summarized in Table 1. The voltage V_{TFL} which sets an upper limit for the validity of these results is also given. It is found by assuming in Eq. (12) that all traps are filled, that is, by setting $E = q\Sigma/\epsilon$.

4. Complete V-I Characteristic

The preceding considerations are limited to the range $V \ll V_{TFL}$ by the assumption that Boltzmann statistics governs the population of the traps. For a general discussion, Fermi statistics has to be used and Eq. (8) must be replaced by the more general relation (6). The ratio of ρ/σ is no longer constant and the analysis becomes more tedious. The space charge of the mobile carriers is no longer negligible either. The solution must now be obtained by a formal integration of the set of equations

$$\begin{aligned}\nabla \cdot \epsilon \mathbf{E} &= \rho \\ \nabla \cdot \mathbf{j} &= 0 \\ \mathbf{j} &= \mu \rho \mathbf{E}\end{aligned}\quad (25)$$

which may be combined into a single differential equation

$$\nabla \cdot (\mathbf{E} \nabla \cdot \mathbf{E}) = 0. \quad (26)$$

The boundary conditions are as discussed in Sect. 2. The analysis can be carried out explicitly for both the planar and the cylindrical geometry, while numerical integrations are required in the spherical geometry. Fig. 2 shows the result for the planar case. Voltage is normalized with respect to $V_{TFL} = q\Sigma W/\epsilon$ and current is given in units of the current $I_{TFL} = e\mu V_{TFL}^2 (9a/8W^3)$ which would flow in the absence of traps at a bias V_{TFL} . As expected, the characteristic follows a square law for $V \ll V_{TFL}$ and F has the constant value $(8/9)(WN/\beta\Sigma)$ given in Table 1.

The main feature of Fig. 2 is the rapid increase in current in the vicinity of V_{TFL} . This reflects the fact that the traps are being all filled at this point ($E_f \simeq E_t$). Beyond this, scic is rapidly approaching its trap-free limit $\bar{I} \simeq \bar{V}^2$. The overall characteristic is in all respects similar to that

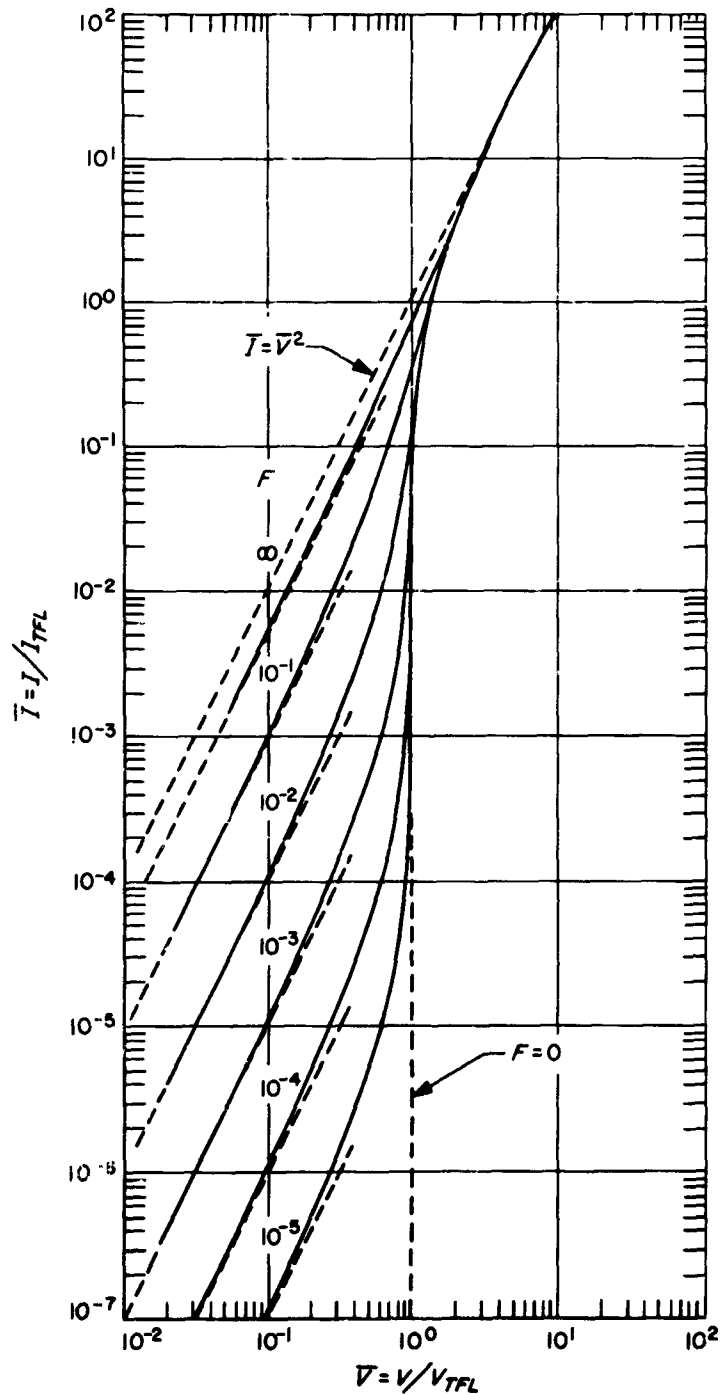


Fig. 2. Complete V-I characteristic of unipolar scic in the presence of traps at the emitter boundary and for a planar configuration ($V_{TFL} = q\Sigma W/\epsilon$; $I_{TFL} = e\mu V_{TFL}^2 (9a/8W^3)$; $F = 8WN/9\beta\Sigma$)

predicted by volume trapping (Refs. 11, 12), but in this latter case $F = 0$ is independent of the base width W .

5. Discussion and Conclusion

The effect of volume trapping on the V-I characteristic of scic at steady state is indistinguishable from that of

Table 1. The V - I characteristic of unipolar scs dominated by boundary trapping at the emitter ($V < V_{TFL}$ and $F < 1$) for the five distinct cases of the planar, cylindrical and spherical geometry

Geometry	$0 \leq k = \frac{r_c}{r_e}$	I	or	$I = \epsilon_\mu V^2 \delta F (F < 1)$	V_{TFL}
Planar		$\epsilon_\mu V^2 \cdot \frac{a}{W} \cdot \frac{N}{\beta \Sigma}$		$F = \frac{8}{9}$	$\frac{q \Sigma}{\epsilon} \cdot W$
Cylindrical	$1 \leq k$	$\epsilon_\mu V^2 \cdot \frac{2\pi l}{r_e \ln^2(k)} \cdot \frac{N}{\beta \Sigma}$		$F = \left[\frac{(k^2 - 1)^{1/2} - \arccos(k^{-1})}{\ln(k)} \right]^2 \cdot \frac{r_e N}{\beta \Sigma}$	$\frac{q \Sigma}{\epsilon} \cdot r_e \ln(k)$
	$k \leq 1$	$\epsilon_\mu V^2 \cdot \frac{2\pi l}{r_e \ln^2(k^{-1})} \cdot \frac{N}{\beta \Sigma}$		$F = \left[\frac{\operatorname{arccosh}(k^{-1}) - (1 - k^2)^{1/2}}{\ln(k^{-1})} \right]^2 \cdot \frac{r_e N}{\beta \Sigma}$	$\frac{q \Sigma}{\epsilon} \cdot r_e \ln(k^{-1})$
Spherical	$1 \leq k$	$\epsilon_\mu V^2 \cdot \frac{4\pi}{(1 - k^{-1})^2} \cdot \frac{N}{\beta \Sigma}$		$F = \frac{4}{6} \left[\int_1^k \frac{(z^3 - 1)^{1/2} z^{-2} dz}{1 - k^{-1}} \right]^2$	$\frac{q \Sigma}{\epsilon} \cdot r_e (1 - k^{-1})$
	$k \leq 1$	$\epsilon_\mu V^2 \cdot \frac{4\pi}{(k^{-1} - 1)^2} \cdot \frac{N}{\beta \Sigma}$		$F = \frac{4}{6} \left[\int_k^1 \frac{(1 - z^3)^{1/2} z^{-2} dz}{k^{-1} - 1} \right]^2$	$\frac{q \Sigma}{\epsilon} \cdot r_e (k^{-1} - 1)$

boundary trapping at the emitter. This fact corroborates the conclusion reached in Ref. 10 whereby a measurement of the V - I characteristic of scic in a device is insufficient to specify the nature of the traps, if trapping is observed. Additional information is therefore essential if interpretations are to have more than a merely suggestive character. Such information can be obtained from measurements of the V - I characteristics on a set of comparable devices of different sizes and geometries. The present note demonstrates that the two limiting cases of uniform trapping throughout the base without boundary trapping and boundary trapping at the emitter without volume trapping can readily be distinguished in this manner. Mixtures of both types of trapping in the same device will of course lead to intermediary results. To clarify more complicated systems such as these then also demands more sophisticated experimental studies.

Only few investigators have given this problem the attention it deserves. There are, correspondingly, only a few experiments reported in the literature in which samples of various geometries but otherwise identical features have been produced and studied. Where trapping is observed in this fashion, the evidence points towards volume trapping.

An independent check on the nature of the trapping is offered by V_{TFL} if this quantity is experimentally accessible. Volume and boundary trapping each predict a specific dependence of V_{TFL} on the geometrical scaling factor W or r_c . It is invariably assumed in the literature that volume trapping is present when a threshold V_{TFL} is reported. Experimental observations of V_{TFL} are not numerous, and its thickness dependence is studied rarely. The evidence, again, seems to favor volume trapping.

In summary it can be stated that most of the observations reported in the literature and interpreted in terms of trapping in the bulk can readily be re-interpreted in terms of boundary trapping at the emitter or, more generally, in terms of any arbitrary distribution of traps in space. Insight into the nature of the traps can be gained from changes observed in V_{TFL} and in the characteristic of scic below V_{TFL} as the size and geometry of the sample is varied. Such measurements have been performed very infrequently. The two idealized models of pure volume trapping and pure boundary trapping thus offer equally simple alternatives of interpretation. In view of some remarkable consequences predicted for boundary trapping on point contacts, an experimental clarification of the actual location of traps and their distribution in space is most desirable.

C. Applications of Superconductivity in Spacecraft, P. V. Mason

1. Introduction

The properties of superconductors which appear to be of engineering interest include: (1) zero dc resistance, (2) flux exclusion, (3) switching to the normal state induced by temperature, by applied magnetic field, and by self current, (4) quantization of flux, and (5) tunneling currents between superconductors, both single particle and Josephson.

In judging applications of superconductivity, we have used the following criteria: First, the time scale for final application can be long range—say up to 10 yr; we assume that larger weight and power limits and much more complex missions will prevail. Second, the size and weight of refrigeration will be relatively large, certainly not less than 100 lb and several hundred kilowatts for systems that must be operated continuously; for short-term operation such as a planetary encounter this can be considerably reduced.

Thus the payoff must be large; the application must either be a function that can be done in no other way, or a large number of tasks must be performed more efficiently than can be done otherwise. If the application is a short-term one, with very low heat loads, this requirement may be relaxed. The applications may be divided into the following categories:

a. Active and signal-handling devices. These include (1) switching devices such as cryotrons and related devices, (2) linear amplifiers, and (3) parametric amplifiers.

b. Memories. Superconductors may serve in (1) random access memories, and (2) associative memories.

c. High-field solenoids. These may be used in (1) spacecraft shielding, and in (2) fields for masers and other devices.

d. High-Q circuits and transmission lines. Superconductivity may be applied to (1) selective circuits, and (2) data transmission and compression devices.

e. Power generation and handling. These include (1) generators, (2) converters, and (3) energy storage devices.

f. Sensors. Improved performance may be obtained with (1) magnetometers, (2) infrared detectors, (3) gyroscopes and accelerometers, (4) voltage to frequency conversion and vice-versa, and (5) alpha particle detectors.

g. Propulsion.

In this summary, we will discuss only items a, b, and c, above. The remainder will appear in later issues.

2. Active Devices

A number of successful devices have been built around the superconducting-to-normal transition induced by the magnetic field of a current-carrying wire or film. Such systems are usually in thin-film form in order to reduce inductance and increase normal resistance, since the major factor determining speed is the ratio of the inductance to normal resistance.

As usually seems to be the case in integrated circuit devices, which these closely resemble in physical form, switching devices have been most successful, while linear amplifiers have followed relatively slowly. Packing densities of 10^4 devices/in.² of substrate and perhaps 10^5 to 2×10^5 /in.³ of refrigerated space are now possible; perhaps 10^6 /in.² and 10^7 to 10^8 /in.³ is possible in the future.

Speeds are now in the sub-microsecond range, and increases to the 10-nanosec range seem possible. Impedances are typically in the milliohm range, so voltages are characteristically of the order of microvolts to millivolts, and currents of the order of tens of milliamperes.

In order to raise the voltage level, various linear amplifiers have been developed. The principal problem is that of obtaining high gain and short time constant simultaneously or, to put it another way, a good gain bandwidth product. Principal advantage is high sensitivity. A recently developed device has a sensitivity of 2×10^{-13} volts and a bandwidth of 750 kHz.

In another direction, a family of parametric microwave amplifiers based on the nonlinear properties of thin films has been developed. Typical figures are 27 db gain at 2 GHz and 6 db at 54 GHz. However, power outputs are in the microwatt range.

A promising development is in the area of a family of devices—diodes and triodes—based on the Josephson effect. Very preliminary measurements have shown switching speeds in the sub-nanosecond range.

The advantages of cryotron devices lie in their high density, moderate speed, low heat dissipation and (presumably) high reliability. Their disadvantages, aside from the obvious ones connected with refrigeration, lie in the

decrease in speed with number of cascaded elements, low impedances and voltage levels, and difficulties of interconnection between substrate planes. Over-all, it does not seem that the advantages are enough to justify the cost for spacecraft applications. However, if some other device, for example, a cryoelectric memory or magnetometer were required, then the use of cryotrons may be justified.

The Josephson devices, if they fulfill their promise, may be useful as the need for sophisticated high-speed computing increases.

In addition, active devices will be necessary as components of the cryoelectric memory described below; hence a study of them will be needed if cryoelectric memories are to be developed.

3. Cryoelectric Memories

One of the most promising areas of application of superconductors is the large-scale random-access and associative memories. Cryoelectric memories are based on the following effects; first, that current circulating in a superconducting ring will flow indefinitely, and second, that it can be destroyed by making part of the ring normal by generating magnetic field by passing current through a nearby wire. Currents can be excited in either direction by suitable timing of an applied current and a switching current. Readout is made by noting the voltage generated in a sense line when the current is destroyed.

Thus, all elements of a memory rather similar to the conventional magnetic core memory are available. As in the cryotron, best performance is obtained when the device is formed of thin films. Hence batch fabrication is both easy and desirable. Densities for simple random access cells are similar to those for cryotrons. A 10^8 bit random access memory in 1 ft³ of refrigerated volume is now being built, and 10^9 bit appears well within reach.

Advantages of cryoelectric memories are: high storage density, zero power dissipation while holding a bit, batch fabrication and high speed. Individual cells can be switched in 5 to 25 nanosec, although cycle times are about a microsecond because of delays in the cryotron selection trees which select an individual bit. Disadvantages are: tolerances and yields in fabrication, difficulties in low impedance interconnections between substrate planes, and low voltage outputs—typically 20 μ v. The latter is alleviated by the fact that the signal-to-noise ratio is good, so that transformers or active amplifiers can be used to raise the level.

Another attractive possibility lies in the flexibility which the fabrication process allows for incorporating logic into the individual memory cells. At the cost of increased area per cell, a wide variety of functions can be performed. The basic function is a comparison of the cell contents with an external signal, yielding an agree/disagree or a less-than/equal/greater-than/output. One form this takes is an associative or content-addressed memory. A word held in an external register can be compared with part or all of all words in memory and a positive output obtained if one of the words is the same. Then the matched word can be read out. There have also been proposals to perform arithmetic operations in the memory.

Over-all, the cryoelectric memory seems to be the most likely candidate for spacecraft applications such as data compression, where very large amounts of data must be handled and stored.

4. High-Field Magnets

Certain alloy superconductors can carry currents of up to 10^6 amp/cm² at zero field, and 3×10^5 amp/cm² at 100 kilogauss with no voltage drop. Thus, high-field large-volume magnets can be made that require no steady-state energy input. Solenoids as large as 1 ft bore by 10 ft long with a field of 40 kilogauss are under construction for use on plasma power-generation experiments. A Helmholtz pair 14 ft in diameter with a 2-ft separation, and a field of 30 kg, is being designed for a hydrogen bubble-chamber application at Brookhaven National Laboratory.

The economics strongly favors superconductors in such applications because the refrigeration is much cheaper than the large amounts of power required for conventional coils.

The most direct application so far suggested for high field coils in spacecraft is a scheme to shield manned spacecraft from high energy particles. The spacecraft is charged to a high positive voltage to repel positively charged particles and the magnetic field deflects the negative particles. The field required is about 2500 gauss over the entire protected volume. Such a system would be prohibitively heavy if done with normal conductors, but would weigh in the vicinity of 1000 to 2000 lb if done with superconductors.

Other possible applications include providing fields for devices which already require cooling such as masers. Also if plasma devices are used to provide power, high magnetic fields could be efficiently generated by superconducting magnets.

D. Enhanced Electron Emission in Plasma Mode,

K. Shimada and S. Luebbbers

1. Introduction

An experimental study of a thermionic-energy converter which utilizes an applied axial magnetic field has been initiated. The objective of this study is to investigate the possibility of increasing the output voltage of a thermionic converter by increasing the ionization efficiency. Such an increase would reduce the plasma-sustaining voltage. A test converter having an interelectrode gap of 28 mils was designed and fabricated by the Thermo-Electron Engineering Company (TEECO), Boston, Massachusetts. This diode differs from conventional thermionic converters which have been tested at JPL with regard to: (1) the interelectrode gap, which is nearly 10 times larger, and (2) the emitter side walls, which are kept at the same temperature as the planar part of the emitter.

The experimental diode is operated at considerably lower cesium pressures and emitter temperatures than conventional thermionic converters. Such a mode of operation is chosen to minimize internal losses due to electron scattering by cesium atoms. During operation of the experimental converter, with no applied magnetic field, the observed currents were two or three orders of magnitude greater than normally anticipated. In addition, the diode would go into a self-sustained discharge which is indicative of positive-ion bombardment of the emitter. Heat balance experiments were performed to allow separation of electron and ion-current effects and to give a better understanding of the phenomena involved.

To give assurance of credibility to the observed high currents, a conventional converter was tested. Once again, anomalously high currents resulted under similar temperature conditions. The conventional converter has a nominal interelectrode spacing of 0.002 in., and minimizes emitter-side-wall emission. As would be expected from similarity considerations, the conventional converter (closely-spaced) required a higher cesium pressure to maintain a constant pressure-spacing product.

2. Converter Characteristics

Before describing our experiments, we briefly describe the characteristics of conventional thermionic-energy converters.

The charge transport in thermionic converters may be understood by considering two planar electrodes separated from each other by an interelectrode gap d . The

emitter electrode is operated at elevated temperatures to utilize thermionic emission, and the collector is operated at a lower temperature to provide "contact potential" between the electrodes. The cesium vapor present in the interelectrode gap serves to modify the electrode work functions (Ref. 14); when ionized, the cesium reduces the effect of negative space charge within the interelectrode gap. Fig. 3 is a schematic representation of a thermionic converter. The interelectrode gap is grossly exaggerated for clarity and is nominally 0.002 to 0.030 in.

When the converter is operated at high emitter temperatures ($T_E \simeq 2000^\circ\text{K}$) and at large cesium pressures ($pd > 10$ mil-torr), volt-ampere curves such as those shown in Fig. 4 are obtained. However, if the emitter temperature and pd are reduced, another volt-ampere curve, which is also shown in Fig. 4, results. In the latter curve, two fundamental modes, the unignited mode (ABC) and the ignited mode (DEF), are indicated. In contrast to this case, at high emitter temperatures no clear change in mode is evident, and the entire region may be considered as the ignited mode.

An essential difference between these modes lies in the fact that volume ionization of cesium exists in the ignited mode, whereas surface ionization is the dominant process of ion generation in the unignited mode. Corresponding motive diagrams, which are the energy diagrams for electrons, are shown in Fig. 5. The region AB (in

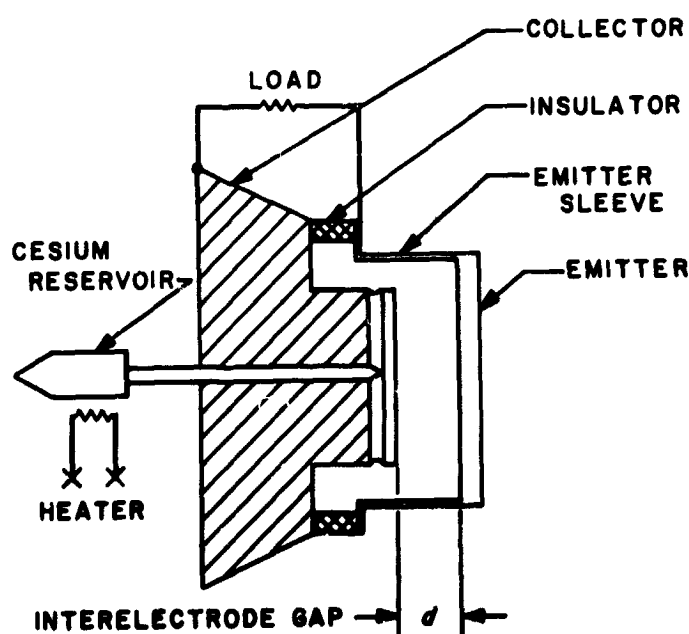


Fig. 3. Schematic representation of a planar thermionic converter

Fig. 4) is the Boltzmann region where the current depends exponentially on the applied voltage, since the motive diagram is strongly electron-retarding. The region BC is a quasi-saturation region where the current is much less than the temperature-saturated emission current from the emitter. The saturation current is reduced by an exponential factor $\exp(-e\Delta V/kT_E)$, where ΔV is the height of the electron potential in excess of the emitter work function Φ_E . In the region BC, ΔV remains relatively unchanged as the output voltage is varied. As the voltage across the diode is further reduced, the motive diagram becomes more electron-accelerating, and inelastic collisions between thermionically emitted electrons and cesium neutrals cause cesium ionization. The generation of cesium ions results in a reduction of the aforementioned ΔV , and high currents are obtained. Operation in the ignited mode is accompanied by a plasma-sustaining voltage drop, V_d , of the order of 0.5 v. At a point E, which is a knee of the volt-ampere curve, the electron-retarding sheath near

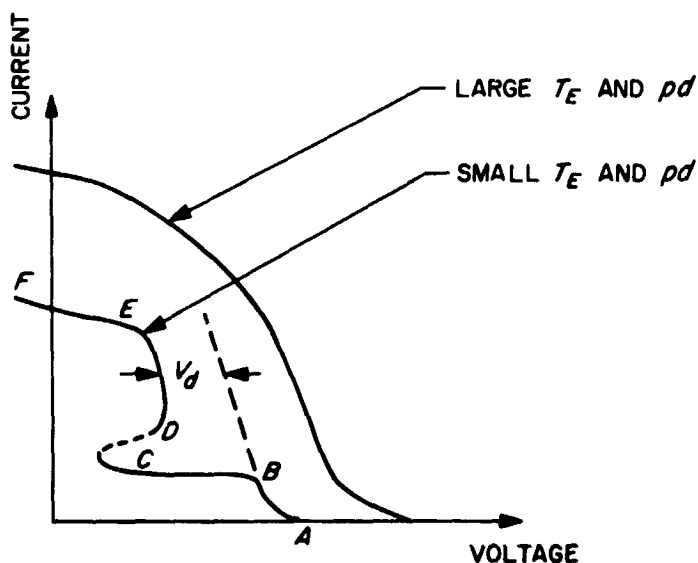


Fig. 4. Converter volt-ampere characteristics illustrating the different modes of operation

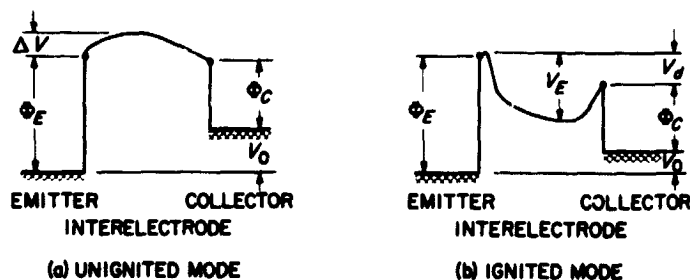


Fig. 5. Motive diagrams illustrating electron potential energy within a thermionic converter

the emitter vanishes. Therefore, the current again saturates near a value equal to the temperature-saturated emission current but is reduced due to electron scattering in the interelectrode space. In the region EF , electron scattering and Schottky enhancement modify the current.

The cesium diode tested in our laboratory operates with a low emitter temperature and in a regime where the pressure-spacing product (pd) is less than unity. These conditions produce relatively small currents in the power-producing quadrant; however, as voltage is applied to the diode an anomalous ignition is observed and the current "soars" to a value two to three orders of magnitude higher than anticipated from normal saturation current. To distinguish this high-current operation from the normal ignited mode, we shall refer to it as the plasma mode. Observed characteristics in the plasma mode are discussed in the subsequent sections.

3. Experiment

External emitter-heating power was supplied by electron bombardment from a Pierce gun (SPS 37-41, Vol. IV, p. 46), designed and fabricated by JPL. Utilization of an electron gun, displaced from the emitter by 10 cm or more, permits calculation of heat input power to the emitter without inclusion of radiation-heating by the gun filament. The bombardment voltage and current were both accurately measured on a digital voltmeter. The emitter temperature was observed in a "hohlraum" using a Mikro-Werk pyrometer. All other temperature monitoring was provided by uninterrupted Chromel-Alumel thermocouples.

The tests performed on both the experimental and conventional converter included: volt-ampere performance characteristics, electrode work-function measurements, and emitter heat balance. The latter experiments provide the necessary information for calculation of ion current and emitter work function in the plasma mode. The heat balance results strongly suggest we look for an unconventional explanation of the work-function adjustment in the presence of a highly ionized cesium plasma or electron emission resulting from some secondary mechanism. The remainder of this paper will be confined primarily to heat balance experiments and discussion of the observed plasma mode.

4. Results

The semi-log representation of the volt-ampere curve shown in Fig. 6 was obtained from the experimental converter. The applied voltage is shown on the abscissa and

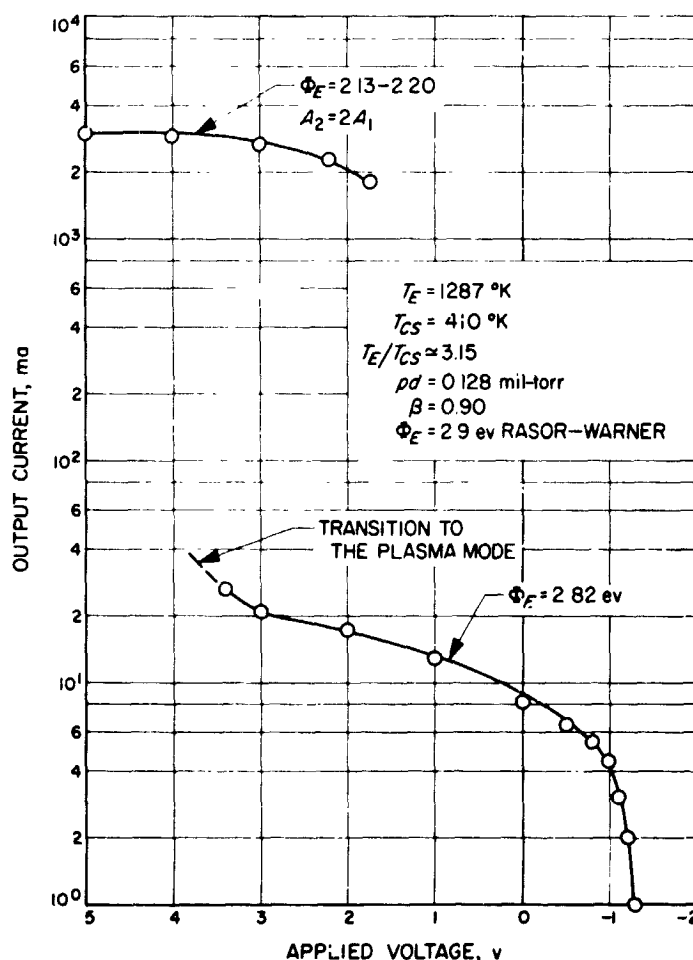


Fig. 6. Volt-ampere characteristics at $T_E = 1287^\circ\text{K}$

spans the range of +5 to -2 v. The power generation quadrant (negative voltage) is seen to contain the linear Boltzmann behavior and a gently sloping saturation due to Schottky effects. As the voltage across the diode approaches the ionization potential of cesium (3.89 v), the current continues the slow rise and suddenly makes the transition into the plasma mode. The lower portion of the curve is the anticipated behavior discussed in Sect. 2, "Converter Characteristics." In this lower region, the converter is temperature-saturated as long as the diode operates in an ion-rich mode. If the current at zero volts is considered to be the saturated current value, the emitter work function calculated from the Richardson equation is 2.82 eV. This is in reasonable agreement with the theoretical value of 2.9 eV determined from a Φ_E -versus- T_E/T_{CS} plot for an emitter with a bare-work function of 4.5 eV. On the other hand, the saturation current value observed in the plasma mode would require an emitter work function of 2.15 eV for thermionic saturation. This is in strong disagreement with known behavior of refractory emitters in contact with cesium vapor. It

should be noted that the curve in Fig. 6 is a semi-log plot and the discontinuity in current spans two decades.

Volt-ampere curves of the converter operating in the plasma mode for various cesium reservoir temperatures are shown in Fig. 7. Power input measurements at each data point in the figure permit the distinction to be made between electron and ion currents. This distinction is apparent from a plot of the additional heating power required to maintain a constant emitter temperature for varying applied voltages. For voltages greater than 1 v, the current approaches saturation. This means that the electron current as well as the ion current reaches saturation. Consequently, the electron cooling of the emitter remains constant as the applied voltage is increased, whereas the energy returned to the emitter from the plasma increases as the applied voltage is increased. Ions impinging on the emitter not only transfer their kinetic energy but also give energies to the emitter as they recombine (Ref. 15). The energy returned from the plasma by excited cesium atoms and by photons is considered to be negligible compared with the energy returned by ions.

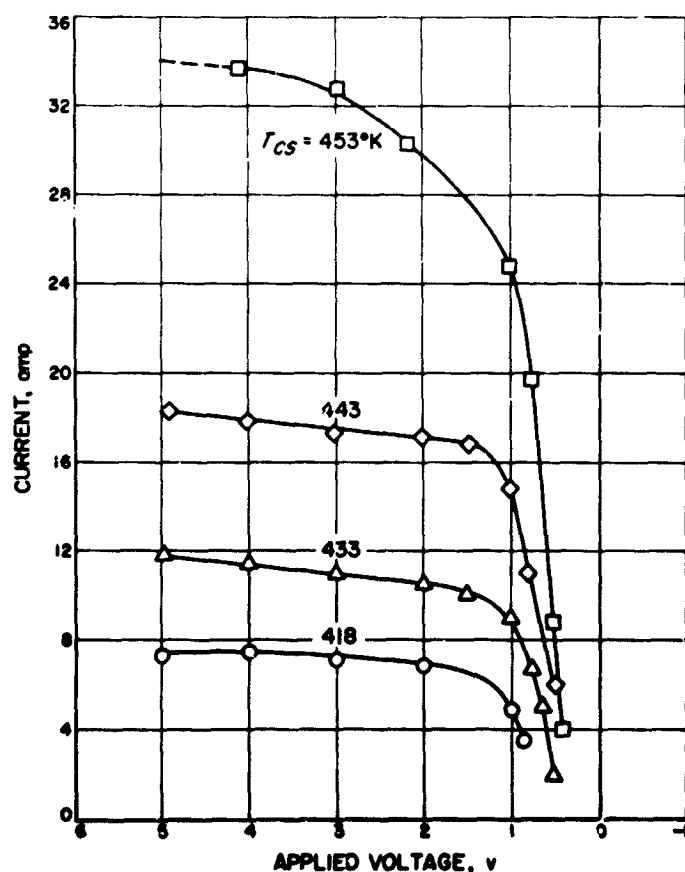


Fig. 7. Volt-ampere characteristics in plasma mode

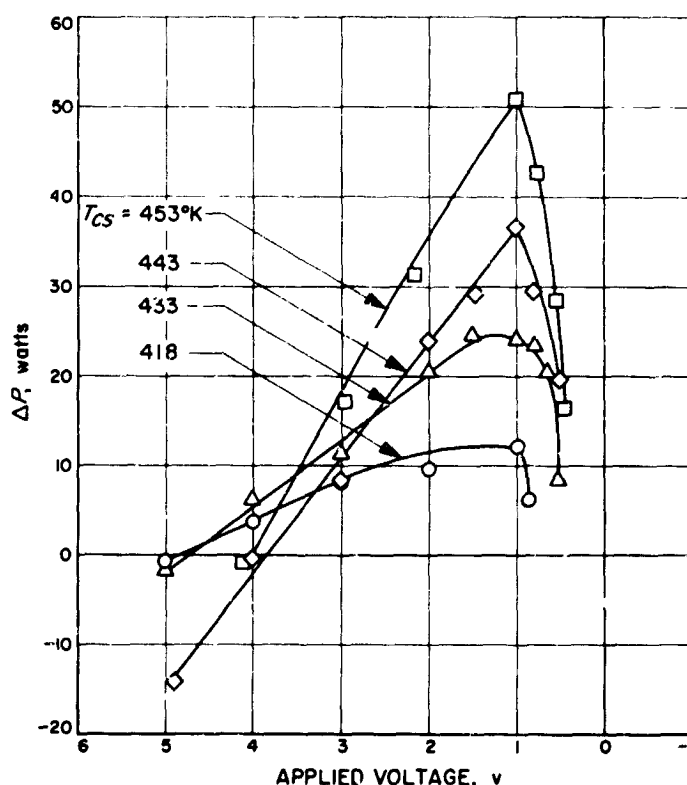


Fig. 8. ΔP versus applied voltage $T_e = 1190^\circ K$

If the emitter sheath voltage V_s , which has a polarity to accelerate ions flowing out of the plasma to the emitter, varies linearly with the applied voltage, we would expect a corresponding linear decrease in the input power ΔP as the applied voltage is increased. This behavior is exhibited in Fig. 8 for applied voltages in excess of 2.0 v. The slope of the linear region yields the value of the saturated ion current. The results for the four curves shown in Fig. 8 are tabulated in Table 2. In all four curves we see that the ion current is comparable with the electron current.

Table 2. Saturated ion and electron currents

$T_{cs}, ^\circ K$	i_e, amp	i_i, amp	i_{ion}, amp
418	7.5	3.0	4.5
433	12.0	4.6	7.4
443	18.0	5.3	12.7
453	34.0	16.0	18.0

Subtraction of the i_i current from the net current yields the forward electron current. Using this reduced current to determine ϕ_s from the Richardson equation, we calculate the values of ϕ_s and tabulate them in Table 3. Also shown are the conventional work function values

Table 3. Comparison of emitter work function values

T_{cs} , °K	Φ_E { Calculate from Richardson Eq. (experimental)	Φ_E { From Φ_E versus T/T_{cs} plot
418	1.94	2.55
433	1.90	2.45
443	1.87	2.35
453	1.77	2.27

anticipated from a Φ_E versus T/T_{cs} plot. We see marked disagreement between the calculated and anticipated values.

5. Conclusions

If the electron currents tabulated in Table 2 originate from thermionic emission, the emitter work function values calculated from the Richardson equation (column 2 of Table 3) must be correct. An obvious difference between the plasma mode and the ignited mode is the fact that the ion current arriving at the emitter in the plasma mode is many times larger than ordinarily encountered. In other words, the theory for the work function of cesiated surfaces, which is based upon the arrival rate of cesium neutrals and not of cesium ions, may have to be modified to be applicable to the plasma mode. In fact, the degree of ionization appears to be near 100% in the plasma mode, while it is a small fraction in conventional

thermionic energy converters operating in the ignited mode.

The presence of a large ion arrival rate at the emitter admits the possibility of other secondary electron-emission processes. The higher ion concentration at the emitter will produce large changes in potential (see Fig. 5b) adjacent to the emitter and, hence, high-field or Schottky enhancement seems possible. A calculation of the field strength at the emitter required to produce the enhancement present in the plasma mode quickly discounts both of these possibilities. Schottky enhancement could account for an increase in current by a factor of 3, while field emission effects are negligible. Neither of these secondary mechanisms explains the observed increase in current. Auger emission is also being considered as a possible secondary mechanism; however, at this time, no concrete conclusion may be drawn.

The dependence of the emitter work function upon the ion arrival rate and the possibility of Auger emission are both under investigation. These investigations are to be conducted in a guard-ringed test vehicle to minimize extraneous emission from undefined electrode surfaces. In addition, a glass tube is being fabricated which will permit visual observation of the discharge.

It is hoped that a suitable explanation of the plasma mode will result from these tests.

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VI. Materials

ENGINEERING MECHANICS DIVISION

N67-29147

A. Carbon and Graphite, D. B. Fischbach

1. Tensile Deformation Dimension Changes

Although brittle at ordinary temperatures, carbons and graphites exhibit considerable plasticity at high temperatures. An investigation is being made of the mechanisms of plastic deformation which are responsible for this high-temperature plasticity. Currently, attention is being directed to the deformation behavior of pyrolytic and vitreous carbons. Recent precise data on the gage-section dimension changes which accompany high-temperature tensile deformation parallel to the substrate in pyrolytic carbon have provided some significant clues to the nature of the active deformation processes.

It had been found that the tensile deformation of as-deposited pyrolytic carbon stressed parallel to the substrate occurs in two stages¹ (SPS 37-33, Vol. IV, pp. 75-81). Stage I is characterized by straightening of the initially wrinkled layer planes, and is accompanied by crystallite growth and the development of layer-plane stacking order (graphitization). At the end of stage I, after about 10% elongation, the sample gage section is well graphitized and highly oriented with the layer planes aligned closely parallel to the stress axis. Stage II deformation occurs in this highly oriented graphite. Earlier results

showed that deformation occurs at approximately constant volume to at least 20% elongation. During stage I the width of the gage section (parallel to substrate) increased slightly, while the thickness (perpendicularly to the substrate) decreased rapidly, consistent with layer straightening. During stage II, the width and thickness appeared to decrease at approximately the same rate as a function of elongation¹ (Ref. 1).

More precise measurements have now confirmed these early results and revealed some significant details which were not apparent earlier. The better quality of the new data resulted from the use of ink-line fiducial marks and other improved techniques reported earlier (SPS 37-43, Vol. IV, pp. 121-125). The reduction in cross-sectional area as a function of uniform gage elongation is shown in Fig. 1. The solid-line curve was calculated on the assumption of constant volume. The data for pyrolytic carbon (circles) show that during stage I the volume of this material decreases (densification). During stage II the volume remains constant, about 2% less than the initial volume. This observed volume decrease is accounted for

¹Fischbach, D. B., and Kotlensky, W. V., "On the Mechanism of High Temperature Plastic Deformation in Pyrolytic Carbons," *Electrochemical Technology*, to be published.

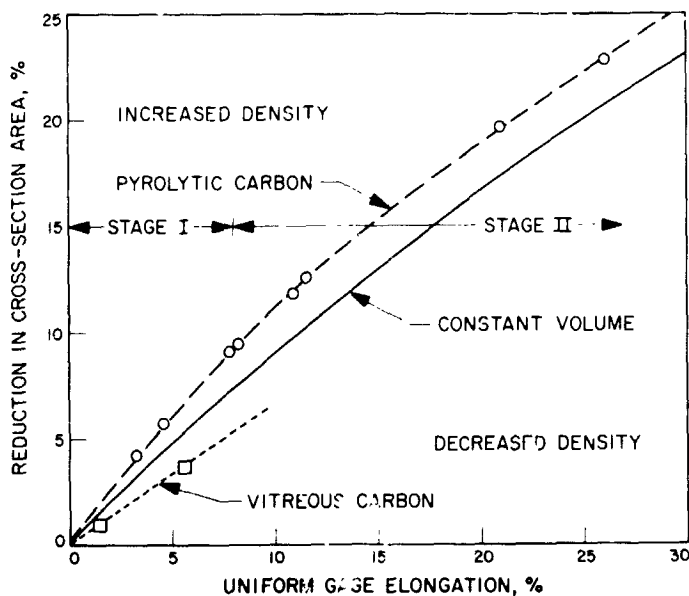


Fig. 1. Reduction in area as a function of tensile elongation for pyrolytic and vitreous carbons at 2500–2900°C

by the 2% decrease in interlayer spacing, measured by x-ray diffraction, which accompanies graphitization. Initial results on vitreous carbon (squares) show that some volume increase accompanies tensile deformation in this material.

The changes in gage-section width and thickness with elongation are shown in Fig. 2. Stage I is completed at 8% elongation with a width increase of about 3.2% and a thickness decrease of 12% in this particular carbon. One-sixth of the thickness decrease (2%) may be attributed to graphitization, while the rest of the change in thickness and all of the width increase must be attributed to layer straightening. The solid lines in the stage II region were calculated on the assumption of isotropic (width and thickness decrease at same rate) constant volume deformation beginning at 8.0% elongation. Within the precision of the data, the results for pyrolytic carbon agree very well with the expected behavior of an isotropic material. The results on vitreous carbon show that it also deforms isotropically, but not at constant volume, consistent with the reduction in area data. This behavior is expected for vitreous carbon, which has a macroscopically isotropic porous structure. Isotropic deformation in pyrolytic carbon is quite unexpected because the structure is highly anisotropic. The changes in gage-section length and width occur parallel to the layer planes; changes in thickness occur perpendicular to the layer planes. Most properties of graphite differ greatly parallel and perpendicular to the layer planes.

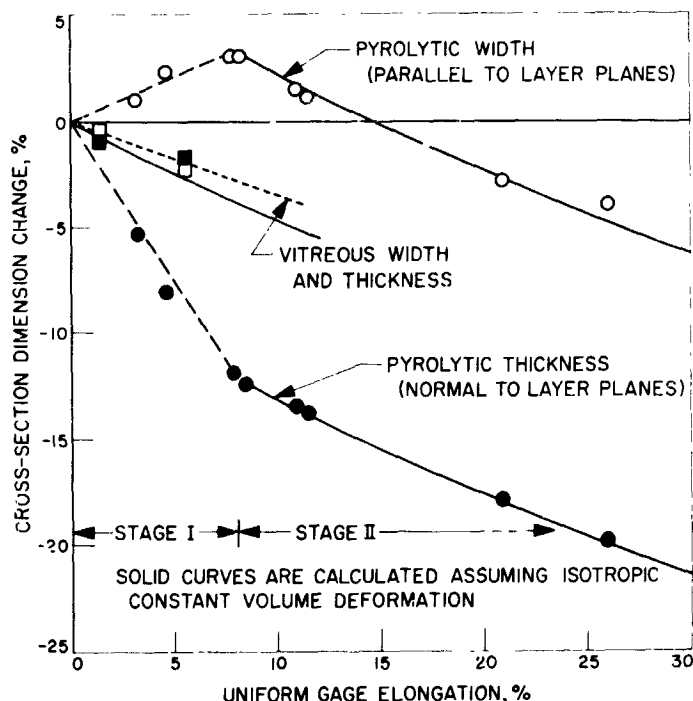


Fig. 2. Change in cross-section dimensions as a function of tensile elongation for pyrolytic and vitreous carbons at 2500–2900°C

The observed isotropy of dimension changes in pyrolytic carbon stage II deformation has significant implications for the deformation mechanisms and associated processes such as diffusion. If deformation occurs primarily by a dislocation mechanism, then slip must occur with equal ease on prism- and pyramid-type planes. If Nabarro-Herring diffusion mass transport is the dominant deformation mechanism, as suggested by current studies on the stress dependence of the creep rate (SPS 37-43, Vol. IV, pp. 121–125) then diffusion transport of atoms from the layer plane face and edge sides to the edge ends of the crystallites must occur at the same rate. From this it may be inferred that there is little difference in diffusion rate parallel and perpendicular to the layer planes. More information on deformation and diffusion mechanisms in graphite is required to determine the true significance of these results.

2. Some Bending and Kinking Deformation Effects

It is well known that graphite will deform by kinking and twinning under compression parallel to the layer planes. In a twin, the layer planes bend sharply through a crystallographically prescribed angle along a crystallographically prescribed direction. A kink, as used here, is similar but the angle and line of bend are not crystallographically determined. More gradual bending is also

observed. Kink bands and bending are easily visible in polarized light micrographs of sections normal to the layer planes because of the sharp change in layer-plane orientation in the kink or bend. In the course of an investigation of the mechanisms of tensile deformation in pyrolytic carbon parallel to the substrate, several interesting varieties of kinking and bending deformation have been observed. All are incidental to the gage-section tensile deformation.

Frequently, the layer-plane faces of highly elongated ($\geq 10\%$) tensile specimens have bent-up ridges along the edges, especially in the throat regions. Fig. 3a shows a polarized light micrograph of a transverse cross section

of the gage region of a deformed sample. (The location and orientation in the specimens of the micrographs in Figs. 3-6 are shown in Fig. 7.) The tensile stress was normal to the plane of the figure. The layer plane orientation is parallel to the cleavage and delamination cracks (the sample cleaved into two pieces during preparation). Throughout most of the cross section the layer planes are horizontal, but in wedge-shaped regions in the corners the planes are bent 28-47 deg away from the horizontal (the lower left corner is dark and cannot be distinguished). This peculiar phenomenon has been traced to adherent deposits of evaporated carbon from the graphite heater which condense on the specimen edges, especially near the grips (Fig. 3b). Deposits are not usually observed on the specimen faces; evidently the evaporated carbon does not adhere to layer-plane surfaces. X-ray diffraction showed that these deposits were well graphitized but randomly oriented. The thermal expansion coefficient of the deposited carbon would be expected to be $\leq 4 \times 10^{-6}/^{\circ}\text{C}$; the expansion coefficient of the highly oriented pyrolytic gage section, perpendicular to the layer planes, is about $28 \times 10^{-6}/^{\circ}\text{C}$. Therefore, on cooling from the 2600-2900°C test temperature a considerable stress is developed perpendicular to the layer planes on the specimen edges. This stress is relieved by kinking and bending as shown schematically in Fig. 3c (lines represent layer planes; shaded area is carbon deposit). Carbon deposition can be eliminated or greatly reduced by using shields of flexible pyrolytic foil held in close contact with the specimen edge surfaces by carbon yarn ties. When this is done, kink ridge formation is correspondingly eliminated or reduced.

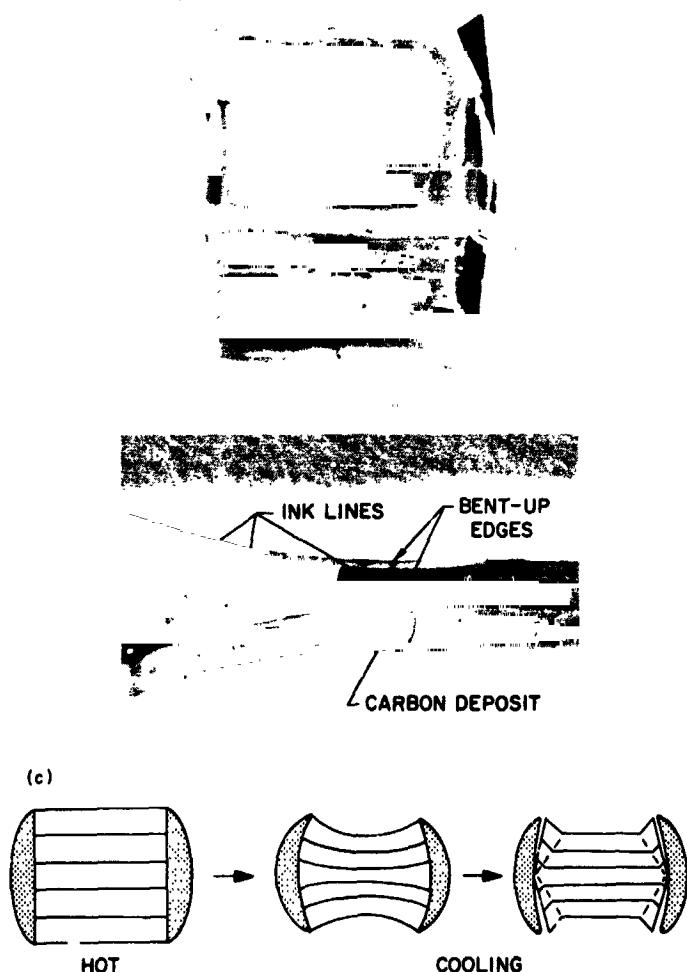


Fig. 3. Bent-up edges on tensile-deformed pyrolytic carbon: (a) transverse cross section of gage region showing bent edges (40X); (b) evaporated carbon deposit on sample sides in throat region and bent-up edges (approximately 4X); (c) schematic of bent edge formation

In specimens tensile-deformed in the as-deposited condition, there is a rather sharp gradient in microstructure and deformation-induced dimension changes in the throat regions at each end of the gage section. In the gage section the material is highly graphitized and very well oriented with very straight layer planes. In the butt and grip portions, the turbostratic, wrinkled sheet structure of the as-deposited carbon is largely retained. Narrow kink bands inclined 45-60 deg to the stress direction are often observed in longitudinal sections (normal to substrate and parallel to stress) of the throat region. An unusually good example is shown in Fig. 4a. More often, a number of short kink bands terminating at the specimen surface or on delamination cracks are seen (Fig. 4b). Kink bands have not been observed in transverse sections of the throat region, or in either transverse or longitudinal sections of the uniform gage region. Evidently, the kinks in the throat help to accommodate the structure and thickness gradient which occurs there. They may



Fig. 4(a). Kink bands in throat region of tensile-deformed pyrolytic carbon; long kink in structure and thickness gradient (40 X)

(b)



Fig. 4(b). Kink bands in throat region of tensile-deformed pyrolytic carbon; smaller kinks terminating at external or crack surfaces in another specimen (50 X)



Fig. 5. Structure at grip interface (at right) of tensile-deformed annealed pyrolytic carbon (40 X)

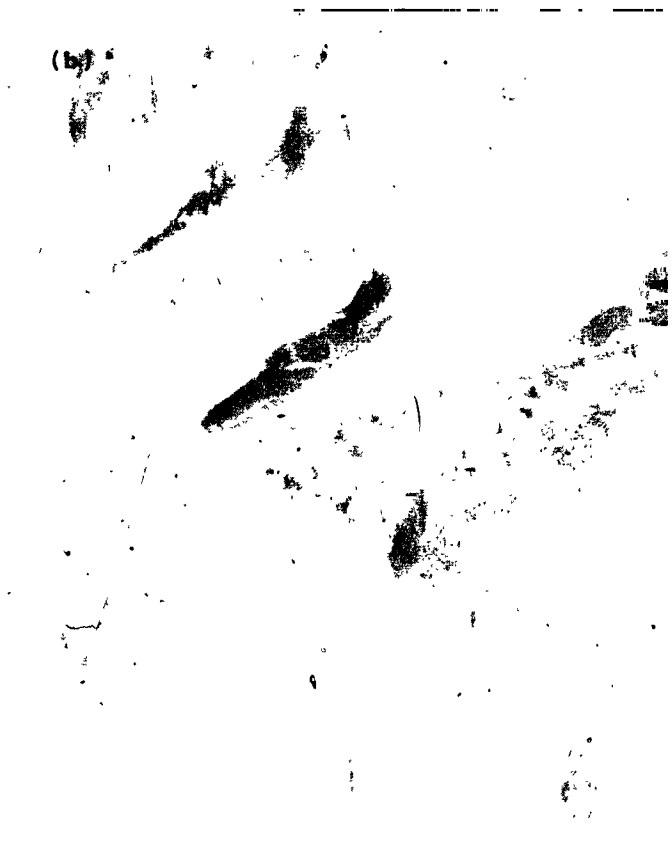


Fig. 6. Grip section structure in annealed pyrolytic carbon specimen tensile-deformed with ends clamped normal to substrate plane: (a) transverse section normal to grip interface, adjacent to grip at right, near centerline of sample at left (40 $\times$); (b) longitudinal (parallel to tensile axis) section of grip portion of specimen (40 $\times$)

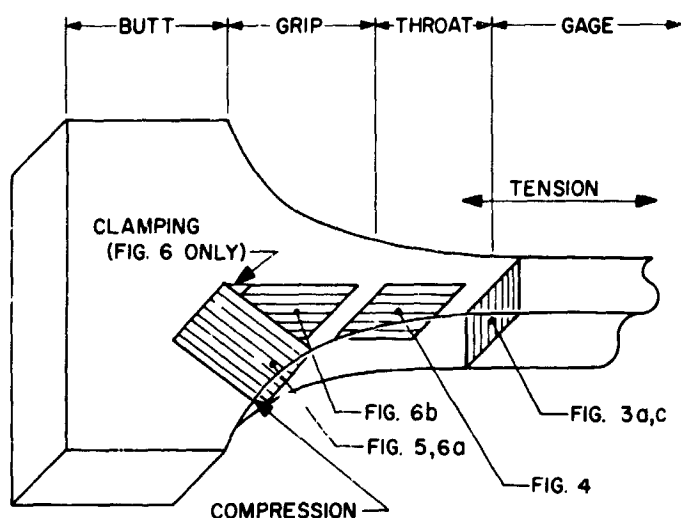


Fig. 7. Location, and orientation in specimens of microstructures in Figs. 3-6.

also be associated with bending produced by the slight bow in the substrate "plane" which is characteristic of pyrolytic carbon deposits.

When pyrolytic carbon which has been annealed at 2900°C for 1 hr under 100-psi stress normal to the substrate is tested in tension, most of the recorded elongation results from massive kinking under the compressive stress parallel to the substrate which is developed in the grip region (SPS 37-43, Vol. IV, pp. 121-125). A polarized light micrograph of a cross section of the grip region normal to the radiused grip surface is shown in Fig. 5. Material has been extruded laterally and large diagonal kink bands developed adjacent to the grip surface (at right). However, a short distance from the grip surface the microstructure is the same as in the as-annealed sample (left edge of Fig. 5). If the specimen is tightly clamped laterally (on and perpendicular to the substrate faces), using shims in the grips (SPS 37-43, Vol. IV, pp. 121-125), extrusion is inhibited. A pattern of "flow lines" parallel to the radiused grips on the sample grip region faces shows that deformation effects extend across the whole width of the sample. A cross section normal to the grip surface (Fig. 6a) shows large kink bands near the grip surface (at right) and an unusual chevron-like microstructural pattern which extends to the centerline of the specimen (at left). This latter pattern suggests a polygonized structure with straight orientation-change boundaries approximately perpendicular to the layer planes. There is little delamination cracking. When viewed in a direction approximately normal to the compression direction (longitudinal section) near the center line of the specimen grip region, the microstructure has quite a

different appearance, as shown in Fig. 6b. The contrast here evidently results mostly from varying inclination of the layers to the plane of the section. These microstructures resulting from parallel compression of annealed pyrolytic carbon are unusual and do not appear to have been observed before.

These examples illustrate the common occurrence and wide variety of layer-plane kinking and bending in the deformation behavior of graphite. More detailed microstructural investigation will be needed before this type of deformation and its relationship to other deformation mechanisms is fully understood.

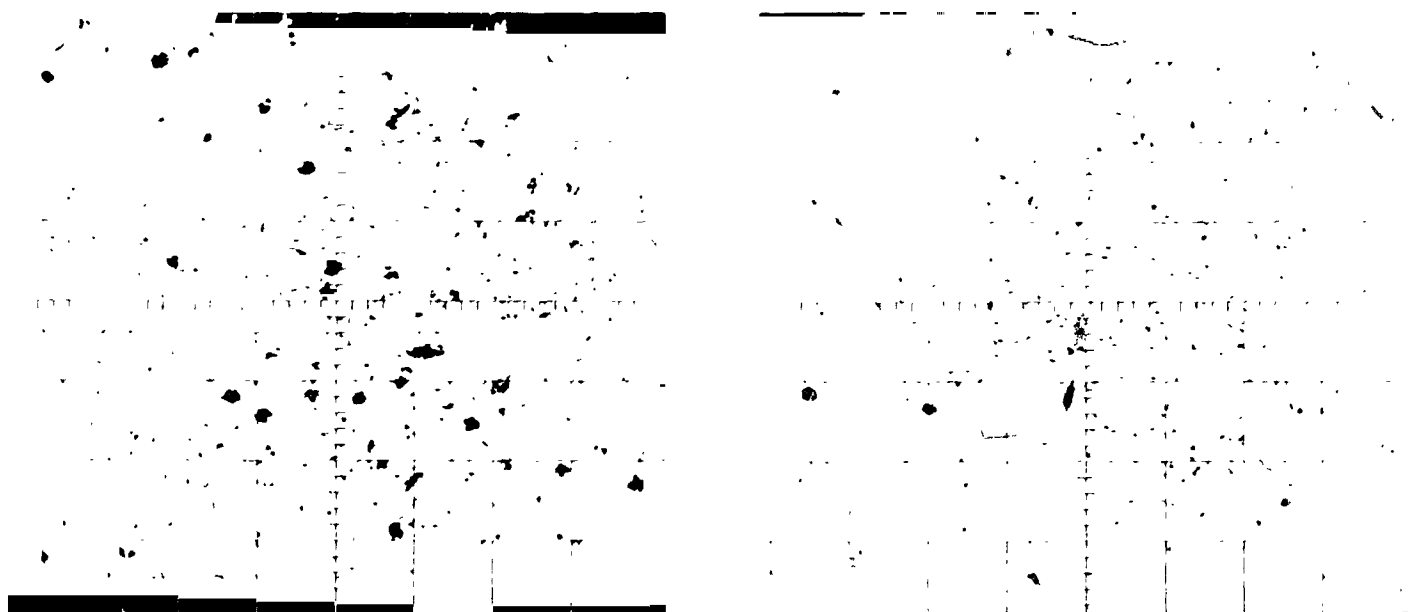
B. Pure Oxide Ceramic Research, M. H. Leipold

The problem of nonhomogeneous distribution of impurities in ceramic oxides was reported previously (SPS 37-34, Vol. IV, pp. 85-87 and Ref. 2) to be a severe problem and likely to have a significant influence on the behavior of polycrystalline oxides. The previous results did not clarify the mechanism of segregation, and further work has been completed to investigate the driving force for this impurity segregation.

The materials, techniques, and equipment are all identical to those indicated in the reports cited above. The only difference introduced was use of varied specific cooling rates. Two specimens were cut from the same pieces of both Fisher M300 MgO and JPL MgO. They were held in air at a maximum of 2,000°C for ½ hr. The slowly-cooled specimens were cooled at a rate of 50°C/hr to a maximum temperature of 1200°C. At this temperature diffusion rates are too slow to permit additional atomic redistribution in these materials. The quenched specimens were dropped from the furnace chamber at 2,000°C approximately 3 ft into a beaker of distilled water. The cooling rate was such that interior color was no longer visible after approximately 3 sec. Random walk calculations indicate that movement of calcium is less than 0.1 μ for this cooling rate.

Fig. 8 shows the results for calcium (Ref. 3) distribution in relatively large areas (0.16 mm²) of two Fisher M300 MgO specimens that are identical except that they were cooled at different rates. The large amount of segregated calcium in the slow-cooled specimen shows that the movement to the grain boundary occurred primarily upon cooling, although small amounts of segregation did exist in the quenched specimen. Similar results for silicon (not shown) were obtained. Distribution of iron and aluminum was sought but not found. The results for the

CALCIUM DISTRIBUTION



OPTICAL MICROGRAPHS

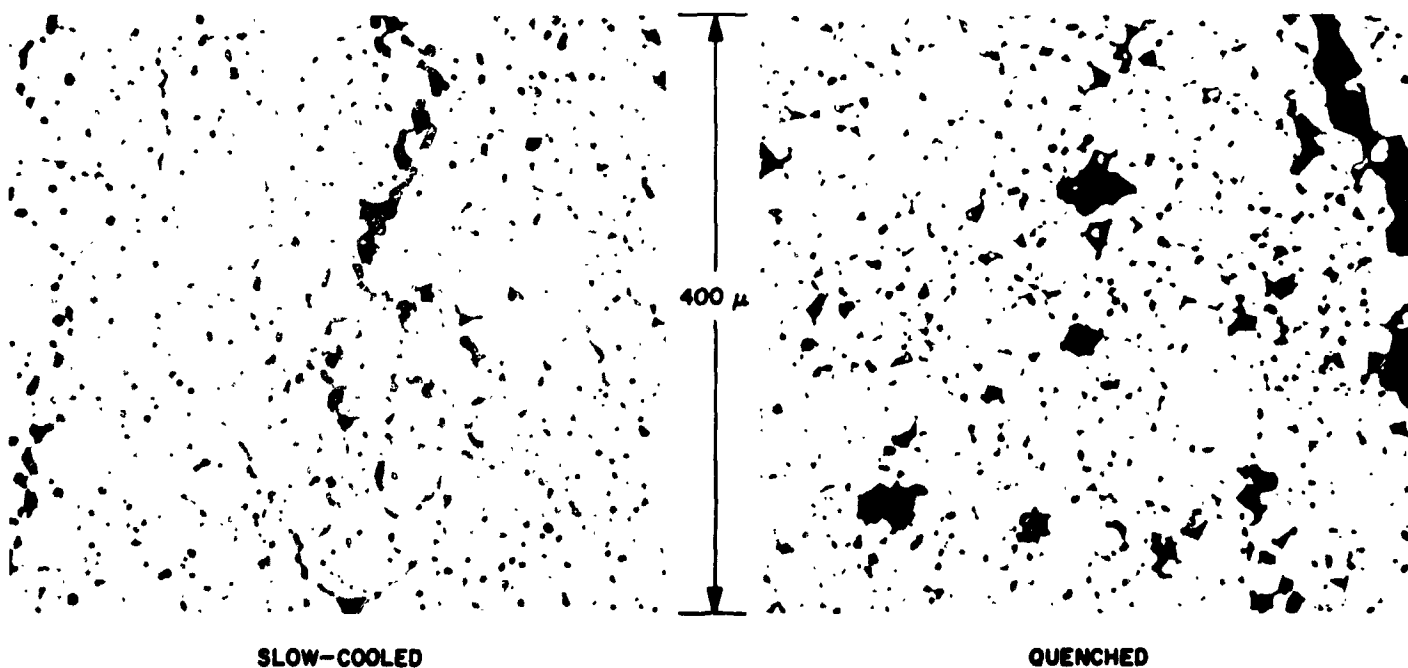


Fig. 8. Calcium distribution in Fisher M300 MgO after heating at 2000°C for 1/2 hr then cooling

higher purity JPL materials were similar, but were less obvious because of lower total concentration.

Fig. 9 shows calcium and silicon distributions at higher magnification in the slowly-cooled Fisher material. These results are similar to the previously reported work, although less severe because of the lower re-heat temperature (2000 vs 2200°C). Note that the segregated impurities are generally evident as a second phase and largely at triple points.

Fig. 10 shows one of the infrequent segregated regions in the quenched specimen. Note that here the impurities

are located along the grain boundaries, and were not optically discernible as second phase. The existence of these impurities in solution in the quenched specimen could be a result of true quenching or merely lower segregated quantities. Consequently, effort was made to find locations having the same concentration of impurity existing as second phase in the slowly-cooled specimens, and in solution in the quenched specimens. This effort was not successful, so true quenching of impurities into solution cannot be claimed.

The difference in behavior between quenched and slow-cooled specimens helps considerably to understand

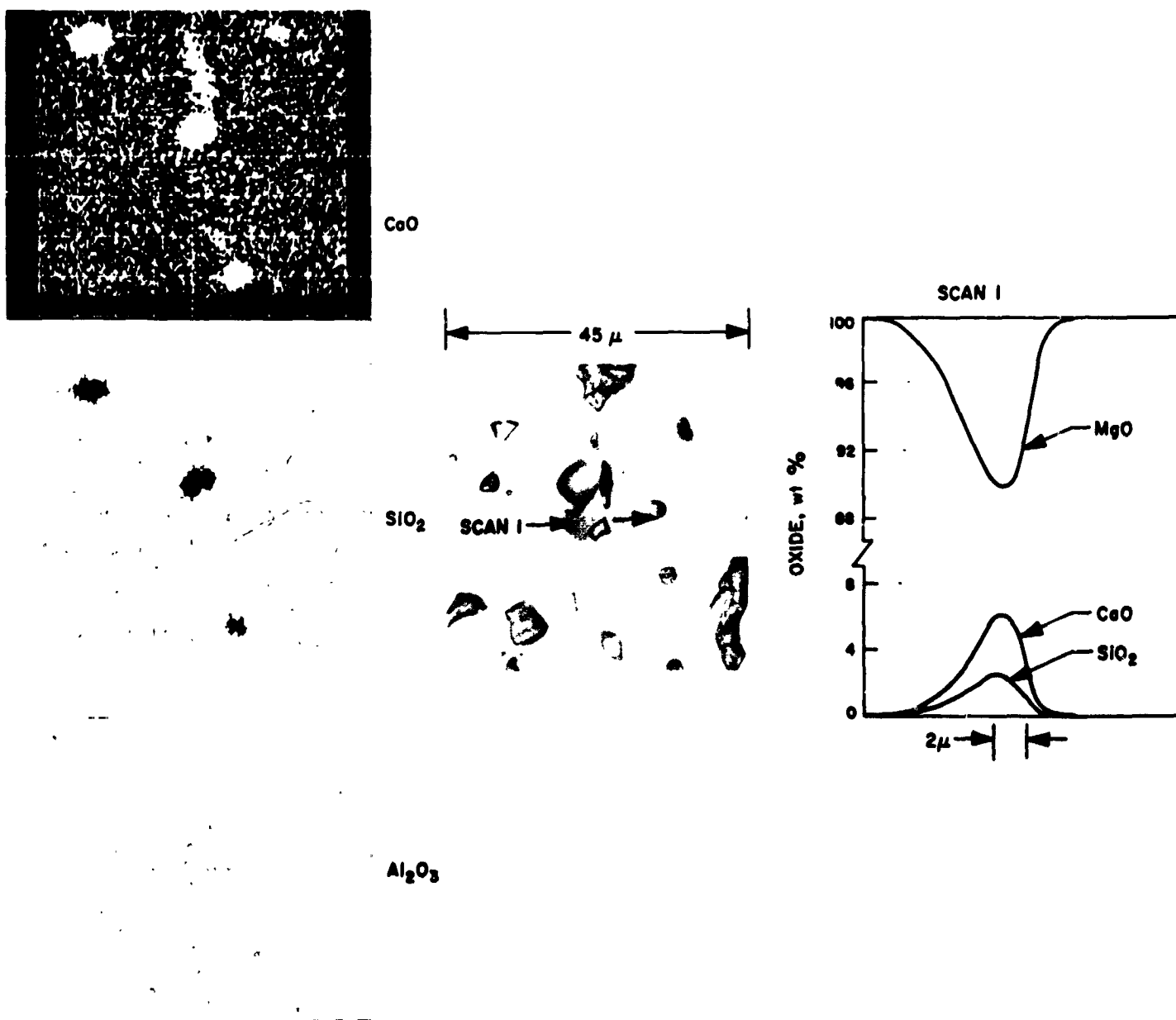


Fig. 9. Impurity distribution in Fisher M300 MgO cooled at 50°C/hr

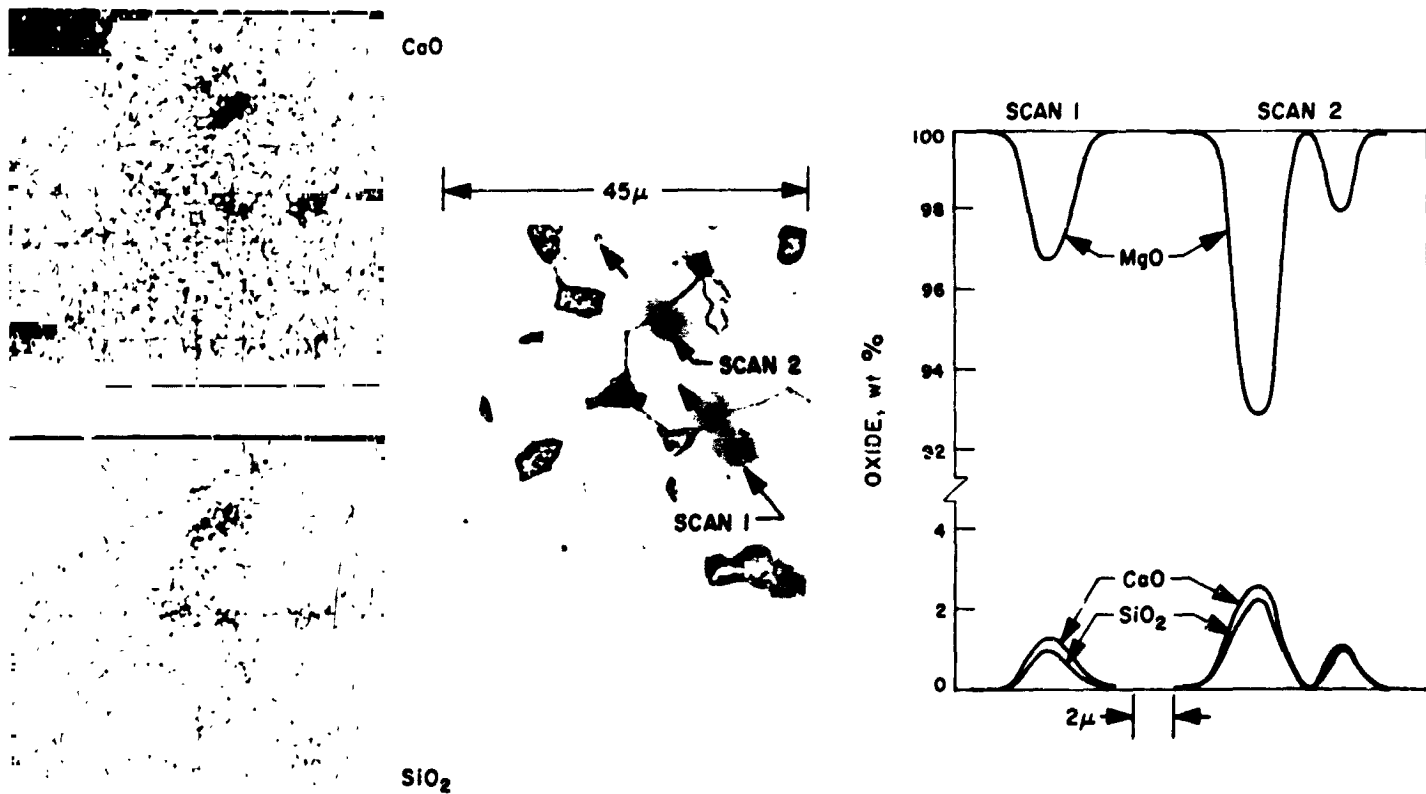


Fig. 10. Impurity distribution in quenched Fisher M300 MgO

the nature of the segregation. The fact that it occurs during the cooling cycle invalidates an equilibrium segregation mechanism as the primary factor. Although differences in energy may still exist for an impurity at a normal lattice site when compared to a grain boundary location, this cannot be the primary factor. Rather, a mechanism of the type proposed by Westbrook², et al (Refs. 4 and 5) is supported. They propose that as the number of equilibrium thermal vacancies in the lattice become excessive during cooling, these excess vacancies migrate to the grain boundaries carrying impurities with them. The vacancies are absorbed into the grain boundary leaving the impurity atoms effectively trapped.

C. Heat Shields for Out-of-Orbit Entry into Mars, R. G. Nagler

Under the auspices of an informal Intercenter *Voyager* Heat Shield Coordination Group with representatives from Ames Research Center, Langley Research Center,

²Westbrook, J. H., "Impurity Effects at Grain Boundaries in Ceramics," *Proceedings of the Third Joint Meeting of the British and Dutch Ceramic Societies*, Bristol, July 1965 (to be published).

and JPL, a preliminary investigation of the available materials for entry into the Martian atmosphere from orbit has been begun. The first phase of this investigation, out-of-orbit ablation screening, is now complete. This phase consisted of a series of 3 test conditions (Table 1) simulating the expected Martian entry environment from out of orbit. The tests were done by Mr. R. Pope of Ames Research Center on materials furnished by JPL.

The test data on these 23 materials are shown in Table 2. The back-surface temperature and the mass of material lost were thought to be sufficient test information for this early screening activity. There were some side

Table 1. Test conditions for out-of-orbit ablation screening activity

Test condition	Cold wall heating rate, Btu/hr ² sec	Stream enthalpy, Btu/lb	Pressure, atm	Total heating, Btu/hr ²
1	15	1900	0.01	1500
2	30	3500	0.01	1500
3	50	6000	0.01	1500

Table 2. Preliminary results of ablation screening at Ames Research Center
for Voyager-Mars out-of-orbit missions

Material	Density ρ , lb/ft ³	Weight ρx , lb/ft ²	Temperature rise at back surface, °K			Total mass loss, %		
			q = 15"	q = 30"	q = 50"	q = 15"	q = 30"	q = 50"
Armstrong cork 2755	31.2	2.0	41	32	31	11.33	12.35	11.72
	31.2	1.0	56	46	37	20.3	22.9	21.4
Armstrong cork 2755 (sterilized)	26.1	2.0	44	36	23	11.39	12.94	12.47
GE 1004AP	33.0	2.0	92	75	64	4.85	5.45	4.80
GE 1004AP (sterilized)	30.5	2.0	59	61	52	3.98	5.01	4.69
GE 1004AP (low density)	22.2	2.0	57	54	44	3.68	4.77	5.48
	22.2	1.0	137	135	111	4.6	8.6	9.8
Hughes phenolic nylon	36.0	2.0	80	56	42	9.33	10.70	10.93
Avco 5026-39 (with HCG)	33.3	2.0	77	56	49	9.92	10.50	9.84
Avco Mod 5 (purple blend)	42.4	2.0	81	72	62	6.78	7.67	6.67
	42.4	1.0	141	118	103	11.4	15.3	14.3
McDonnell B45 RF (without HC)	40.3	2.0	98	70	62	6.16	7.38	6.23
McDonnell B45 RF (with HC)	37.2	2.0	77	71	63	7.23	7.14	5.84
High density teflon	137.0	2.0	240	176	141	10.65	20.58	18.46
Avco 5026-99	24.3	2.0	51	40	29	8.1	8.3	7.5
	24.3	1.0	86	71	55	13.5	17.6	16.8
Avco foamed teflon	46.8	2.0	68	50	40	22.5	23.9	20.8
Avco foamed teflon	33.7	2.0	49	37	30	25.2	24.8	22.1
Boeing polyborazole	34.9	2.0	52	41	42	11.1	12.3	21.1
Avco 5026-39/p8	33.3	2.0	65	56	48	9.4	9.5	10.0
Avco Mod 7	42.4	2.0	87	69	64	7.6	8.1	7.8
Avco Mod 20	42.4	2.0	75	62	58	8.6	9.1	8.5
McDonnell B 46RF	36.5	2.0	83	64	52	5.9	6.0	6.3
McDonnell B-47RF	25.2	2.0	53	40	35	9.4	12.3	9.4
Avco high cork	33.3	2.0	47	32	28	9.1	10.4	10.0
Lockheed quartz fiber	15.5	1.0	79	58	52	0.6	0.6	1.1
Martin SLA-561	12.5	1.0	62	51	42	11.3	12.0	11.1

* q = cold-wall heating rate, Btu/ft² sec

heating effects due to the small size of the models (1.25-in. diameter), but these negative effects were similar for all tests and should not affect the relative performance of the various materials significantly.

Several fairly obvious conclusions can perhaps be drawn from Table 2. First, low density is desirable. The lower the density, the lower the thermal conductance, and, therefore, the better the insulation capacity. Second, the use of cork as a filler appears to provide the lowest composite conductance. Cork is the only material available which exists in the form of hollow polygons on the

micro level. Third, sterilization is apparently somewhat beneficial since it drives out low-molecular-weight interstitially-absorbed fluids, lessening the composite conductance by more than can be explained by weight loss alone. Fourth, foamed plastics without fillers may be an area with significant future application since foamed teflon, which compares favorably with the more exotic composites, has one of the higher conductances of any pure plastic.

A plot of the temperature rise at the back surface versus density is shown in Fig. 11. For the out-of-orbit

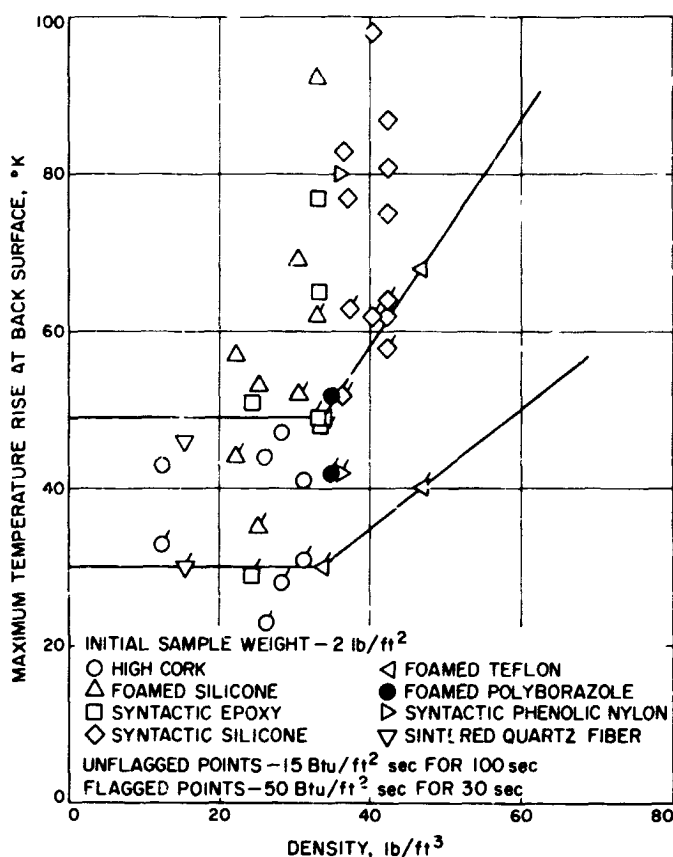


Fig. 11. The effect of density on the insulation capability of various candidate material systems for out-of-orbit entry into Mars

entry use, where the heating loads are small and where the insulation capability is the single most important concept, this figure is somewhat meaningful. The lowest value, regardless of density, represents, of course, the most efficient insulator. On the other hand, if more than one density of the same material is available, the two or

more points show a trend which indicates the improvement in insulation capability which might be possible if homogeneous samples could be achieved in even lower densities. Every material has a minimum density into which it may be fabricated in a homogeneous form. This minimum is based on the limitations in the fabrication techniques used to form it into a sample.

An example is shown with the teflon lines drawn on Fig. 11. For its density, foamed teflon appears to be the most efficient material in the figure. The lowest density, though, is presently the lowest available using the given processing method. Hence, the flat extension of the line across the figure to the zero density line results. Any material which is below this line for either heating rate, or which can be hypothesized to be below this line in those lower density versions which could be realistically assumed to be fabricable, is worth investigating further for the out-of-orbit entry mode. Those materials in the figure which meet this criteria are primarily materials high in cork (greater than 50%). A syntactic epoxy foam material, one variety of foamed silicone elastomer, foamed polyborazole, and perhaps the sintered quartz fiber are also somewhat competitive and warrant some further consideration as candidate materials for the out-of-orbit mode of entry into Mars.

Other activities, presently in process, as part of the cooperative efforts of the Intercenter *Voyager* Coordination Group, are sterilization and transit environment screening, an investigation of ablation processes in the direct hyperbolic entry mode into Mars, and the relative RF transparency before and after ablation of the better heat shield materials from the above tests. The results of these activities will be reported as they become available.

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VII. Applied Mechanics

ENGINEERING MECHANICS DIVISION

N67-29140

A. An Experimental Comparison of Three Typical Mars Entry-Body Shapes on the Basis of Total Equilibrium Shock-Layer Radiation, H. J. Stumpf

The AVCO Advanced Research and Development Division, Wilmington, Massachusetts has completed an experimental program for measuring and comparing the total (over the whole body) equilibrium shock-layer radiative heat transfer for three entry-body shapes in a simulated Mars atmosphere. The work was performed under JPL Contract 951331. A description of the test program, entry-body shapes, and experimental facilities is given in SPS 37-40, Vol. IV, pp. 28-31.

The approximate comparison of the equilibrium shock layer radiation was made for the three potential Mars entry body shapes at 0-, 45-, and 90-deg angles of attack by conducting radiation measurements in a shock tube and shock shape measurements in a shock tunnel. Using these data, radiation results were extrapolated to a trajectory condition for the full-size vehicles using a simplified analytical model described in Ref. 1. The extrapolation method was tested by comparing zero-angle-of-attack extrapolated results of the shock tube to the radiation measurements obtained in the ballistic range experiments (which simulated the true entry conditions except for size) and to theoretical predictions. The agreement was found to be satisfactory for the 60-deg blunt

cone and *Apollo* shapes; but due to a lack of range data and because the simplified theory may not be valid for the tension shell, the results of the comparison made for that shape are questionable.

The results of the relative, full scale, total radiative heat loads W to the vehicle surface during Mars entry are summarized in Table 1.

Local radiation distribution measurements were performed in the shock tube for a 60-deg blunt cone, using a model instrumented with fiber optics. The results of this test confirmed previous calculations which indicated that the radiation at locations on the body other than at the stagnation point may be higher than at the stagnation point.

Table 1. Relative, full scale, total radiative heat loads to vehicle surface

Angle of attack, deg	$\frac{W_{\text{Apollo}}}{W_{\text{cone}}}$	$\frac{W_{\text{tension shell}}}{W_{\text{cone}}}$
0	2.9	16.0
45	1.7	1.5
90	3.0	0.7

A more stringent test of the usefulness of the extrapolation method can be made when the results of theoretical calculations are available (SPS 37-40, Vol. IV, pp. 31-32).

B. Analysis of the Crushing of a Dovetail Phenolic Honeycomb Spherical Impact Limiter, A. C. Knoell

1. Background

In the development of unmanned space vehicles considerable emphasis has been placed on the development of energy-dissipating materials and devices capable of protecting scientific payloads during lunar or planetary landings. Conditions of omnidirectional impact have generally required that the payload configuration be spherical. This requirement implies that the energy-dissipating medium possesses spherical curvature capability, thereby giving rise to an impact limiter configuration consisting of a spherical payload encased in an energy-dissipating shell.

A new type of nonhexagonal cell phenolic honeycomb possessing good spherical curvature capability and energy-dissipating characteristics is currently under development (Ref. 2). This honeycomb, known as Dovetail, is shown schematically in Fig. 1. It has a constant cell foil thickness and consists of phenolic resin built up on a glass cloth core. The Dovetail is classified according to cell size (honeycomb "land" dimension L_D), configuration (wing angle ϕ), and bulk density (cell foil thickness t).

2. Object and Scope

The purpose of this analysis is to develop a theoretical method for predicting the static and dynamic response characteristics of a Dovetail phenolic honeycomb spherical impact limiter during vertical crushing against a flat unyielding surface. The static response includes the de-

velopment of the crushing force as a function of the Dovetail material properties and limiter geometry, and the expression of this force in terms of the crushing depth. The dynamic response includes the prediction of the maximum impact velocity and deceleration experienced by a spherical Dovetail limiter during crushing. Included in the analysis are the effects on response of thickness efficiency (a parameter describing the limit of Dovetail crushability before "bottoming out" occurs) and stress waves generated by high velocity impact of the limiter. The so-called cannonball effect, i.e., the penetration of the payload sphere into the Dovetail honeycomb during impact, is considered by determining the deceleration limit at which this phenomenon occurs. Finally, it is intended that the theoretical results obtained from the static response analysis be compared to experimental results as they become available. Comments will also be presented concerning the physical limitations that must be placed on the analytical limiter model.

3. Assumptions

In developing the analytical model for a Dovetail impact limiter, it is assumed that the payload is a rigid body and that the cells of the Dovetail honeycomb are geometrically identical and uniformly distributed throughout the impact limiter. This latter assumption implies that spherical geometric symmetry exists, and as a consequence there is no variation in circumferential bulk density of the Dovetail. For small Dovetail cell sizes this assumption is valid since the honeycomb tends to approach a continuous medium. Also, since the Dovetail honeycomb is composed of small cell sizes, it is assumed that continuous functional theory applies. Finally, it is assumed that the Dovetail honeycomb is infinitely rigid in shear and that the only failure mode is crushing of the honeycomb.

4. Analysis Extension

Although not presented here, the analysis has been extended to include both concave and convex surface geometry. The case of rock penetration of the limiter is currently under study. In the future, however, the analysis will be extended to include the effects on response of:

- (1) Nonvertical impact
- (2) Finite limiter shear stiffness
- (3) Nonrigid soil
- (4) Nonrigid payload
- (5) Nonuniform cell geometry and distribution

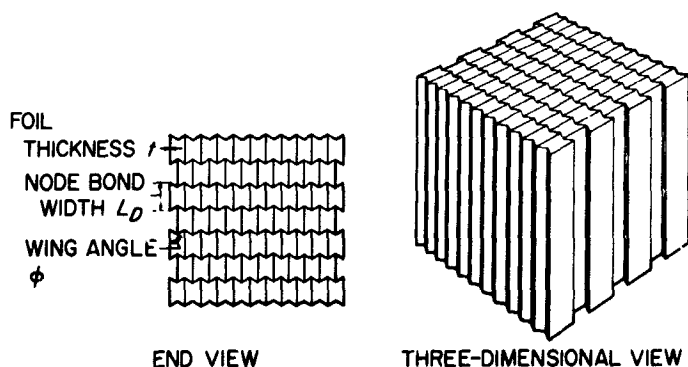


Fig. 1. Sketch of Dovetail phenolic honeycomb specimen

5. Analysis Synopsis

The analysis begins with the development of the force at the crushing boundary as a function of angular displacement θ , and is derived from the relation that

$$F_{(\theta)} = \int_{V_{(\theta)}} \sigma_r dA \quad (1)$$

The crushing stress σ_r can be expressed as

$$\sigma_r = \sigma_{r(\rho, a)} \xi_{(\phi)} \quad (2)$$

where

$\sigma_{r(\rho, a)}$ = axial crushing stress as a function of Dovetail radial density ρ and cell area a .

$\xi_{(\phi)}$ = reduction parameter to account for decrease in axial crushing stress due to angle of cell loading ϕ .

The terms on the right-hand side of Eq. (2) are derived from experimental data and expressed as algebraic polynomial functions. The Dovetail density, cell area (which combines the effects of cell configuration and size on response), and differential surface area dA are developed as functions of θ and ϕ . Then, by appropriate substitution in Eq. (1), the force at the crushing boundary is developed as a function of θ .

The analysis is continued by deriving the equation of motion which relates the force at the crushing boundary to the deceleration experienced by the impact limiter. To properly account for the stopped mass effect on the deceleration process, the derivation proceeds utilizing principles of fluid mechanics. Specifically, Newton's second law is applied to a system of particles contained within a control volume that is accelerating and has mass crossing its boundaries. The resulting equation, neglecting stress wave effects on response, is

$$F = m_{uc} \ddot{x} \quad (3)$$

where m_{uc} = uncrushed limiter mass, and $\ddot{x}$ = limiter deceleration.

Solving Eq. (3) for the maximum impact velocity that can be attained by the limiter during crushing results in

$$V_o^2 = 2R_L \int_0^{\theta_{max}} \left(\frac{F}{m_{uc}} \right) \sin \theta d\theta \quad (4)$$

where R_L = total limiter radius.

Substituting Eq. (1) in Eq. (4) yields

$$V_o^2 = 2R_L \int_0^{\theta_{max}} \left[\frac{\int_{V_{(\theta)}} \sigma_r dA}{m_{uc}} \right] \sin \theta d\theta \quad (5)$$

In Eq. (5) the upper limit of integration θ_{max} is derived from thickness efficiency considerations. The approach taken is to determine that geometric Dovetail cell volume which is just sufficient to house the volume of crushed particles of the centrally-loaded cell. Further loading of the limiter to reduce that geometric cell volume will cause "bottoming out" to occur. The size of the resulting cell volume is therefore a direct measure of thickness efficiency and gives rise to an analytic expression for θ_{max} .

By introducing the concept of a payload fraction, i.e., the ratio of the payload weight to the total limiter weight, in Eq. (5), it is shown that the maximum impact velocity is independent of the limiter mass. It is also shown that the deceleration, as given by the bracketed term of the integrand of Eq. (5), varies as the inverse of the payload radius. In addition to Eq. (5), other design relations pertaining to levels of constant deceleration and limiter size are developed which are also independent of limiter mass. These relations significantly reduce the number of cases to be analyzed for design purposes.

The effect of stress waves on limiter crushing response is developed in a qualitative sense by conservatively determining the number of stress wave reversal cycles occurring in the centrally-loaded Dovetail cell during the total stopping event. It is assumed that if the number of stress wave reversals exceeds ten the solution as given by Eq. (5) is satisfactory since, in effect, it represents the average integrated dynamic response of the limiter.

Finally, an expression is developed for the payload deceleration at which "cannonballing" will occur. Both tensile and compressive forces acting on the payload during deceleration are considered in the development of this expression. In a design sense the results of this relationship are compared to the maximum limiter deceleration to insure that the cannonballing does not occur.

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VIII. Electronics Parts Engineering

ENGINEERING MECHANICS DIVISION

N67-29149

A. Effects of Nuclear Radiation on Electronic Parts, E. T. Klippenstein

Deep space missions possibly requiring nuclear devices for electrical power and propulsion systems have been considered at JPL. In exploring the use of nuclear devices, a matter of concern is the shielding necessary for protection of the electronics.

A two-phase program was initiated in 1962 to study the effects of nuclear radiation as part of the environment of space. The first phase consisted of reviewing the literature available and formulating an experimental program designed to provide the required information. The second phase consisted of performing the test program and reporting the analysis of the results. The total program was completed in 1966. This article will describe the program briefly, and present a summary of observations based on an engineering analysis of the test results.

The total program consisted of exposing electronic parts in seven combinations of environmental conditions (Table 1) with and without a nominal electrical stress. The radiation exposure time was 10,000 hr. The parts included in the program represented a cross section of parts typically found in spacecraft fabricated during

Table 1. Environmental conditions

Group	Temperature, °C	Pressure, torr	Power	Radiation	Time, hr
I	100	Ambient atmosphere	On	None	10,000
II	100	10^{-5}	On	None	10,000
III	100	10^{-5}	On	a	10,000
IV	100	10^{-5}	On	b	10,000
V	50	10^{-5}	On	a	10,000
VI	100	10^{-5}	Off	a	10,000
VII	100	Ambient atmosphere	Off	None	10,000

a 10^6 ncm⁻² hr⁻¹, 10^5 rad gamma hr⁻¹ exposure for 10,000 hr.
b 10^{11} ncm⁻² hr⁻¹, 10^5 rad (carbon) gamma hr⁻¹ exposure for 100 hr followed by 10,000 hr without radiation.

1962-1965. Included were capacitors, diodes, transistors, resistors, relays, switches and other parts representing 33 different part types, or a total of 3,418 individual piece parts (Table 2).

The total radiation exposure was approximately 10^{11} ncm⁻² ($E > 0.1$ mev) concurrent with 10^7 rad (carbon)

Table 2. Test specimens

Part	Type	Number
Capacitors	Metallized paper (P323ZN105K Aerovox)	120
	Metallized paper-mylar (118P10592S2 Sprague)	100
	Mylar (683G10592W2 Good-All)	140
	Tantalum foil (15K106AA6 G.E.)	120
	Tantalum wet slug (HP56C50D1 Fansteel)	100
Diodes	Silicon, switching (FD1184 Fairchild)	140
	Silicon, switching (FD643 Fairchild)	100
	Silicon, switching (1N916 Texas Instruments)	100
	Silicon controlled switch (3N58 General Electric)	140
	Power rectifier (1N2063 International Rectifier)	24
	Zener (1N822 Hoffman)	140
	Zener (PS4653 Pacific Semiconductors)	100
Resistors	Carbon film (CG-104 Texas Instruments)	120
	Metal-oxide-film (C-07-104 Corning Glass)	140
	Carbon composition (CB-1045 Allen Bradley)	120
	Potentiometer, conductive plastic (78PSH-128-16 NEI)	140
Transistors	Silicon NPN planar (2N911 Fairchild)	100
	Silicon NPN planar, epitaxial (2N914 Fairchild)	140
	Silicon NPN planar (2N915 Fairchild)	140
	Silicon PNP medium power (2N1132 Texas Instruments)	140
	Silicon NPN planar, epitaxial (2N2297 Fairchild)	100
	Silicon NPN planar (2N930 Fairchild)	100
	Silicon NPN power (2N1050 Texas Instruments)	140
	Silicon PNP planar epitaxial (2N2412 Texas Instruments)	100
	Silicon PNP alloy (2N861 Philco)	140
Relays	DPDT, dual coil (32RJD90GD-GSP Sigma)	100
Switches	SPDT (1HM1 Minneapolis-Honeywell)	100
Transformers	Interstage (SP-13 [TF5RX13ZZ] Triad)	100
Connectors	DEM-9P-NM-10 (Cinch)	30
	DEM-9S-NM-10 (Cinch)	30
	PT06A-8-4P (Bendix)	30
	PT00A-8-4S (Bendix)	30
Miscellaneous	Photomultiplier tube (7817, CBS)	24
	Fiber optic disc (Mosaic Fabrications, Inc.)	10
	Cadmium sulfide cells (CL-605, Clairex)	20

gamma.¹ The study was performed at the pool-type research reactor at the Battelle Memorial Institute, Columbus, Ohio.

Based on an analysis of the experimental results, some general observations regarding the nuclear radiation conditions of this test program are summarized as follows:

- (1) Transistors in general experienced permanent radiation damage. The effects observed were current gain decrease and saturation voltage increase with only a small percentage of recovery after radiation turn off. The amount of damage was appreciable in the medium power and power transistors. A 75% degradation of current gain was observed. However, for the fast switching types of transistors the amount of degradation was less severe and probably not significant.
- (2) Capacitors, diodes, resistors, relays, switches, transformers and connectors were apparently not affected by the amount of radiation exposure of this program.
- (3) The 10^{11} ncm⁻² environment was applied to several groups of parts over 10,000 hr, while another group was subjected to this level of radiation over a 100-hr period followed by the 10,000-hr life test. The data showed that the radiation effect was essentially the same whether applied in a 100-hr period or a 10,000-hr period. For the group which was exposed to 100-hr radiation followed by the 10,000-hr life test, there appeared to be a small annealing effect.

In addition to the nuclear radiation testing, a small number of parts having light sensitivity measurements were tested for shorter periods of time in only a gamma radiation environment of 10^4 rad/hr. Based on the analysis of the data, the following was observed regarding the gamma radiation environment:

- (1) The cadmium sulfide cells decreased in cell resistance to about 50% at dim illumination and 10% at bright illumination. They were considered functional, however, at the end of the test.
- (2) Dark current increased in the photomultiplier tubes. (Testing was discontinued because dark current was nearly equal to light current after 100 hr of test.)
- (3) Transmittance decreased in the fiber optic discs to less than 1.0% of the initial value for the shorter

¹ncm⁻² = neutrons cm⁻².

wave lengths. (Testing was discontinued after 200 hr because of early degradation.)

This test program also provided information about reliability and the effects of vacuum, temperature, and

power. Very briefly stated, the reliability of parts varied and the results in this area were comparable with other JPL qualification tests. There were no apparent effects caused by vacuum. Temperature and power, however, caused significant degradation in some part types.

IX. Lunar Spacecraft Development

ENGINEERING MECHANICS DIVISION

N67-29150

A. 22-inch Air Gun for High-Impact Testing, S. G. Taylor

This article describes the major aspects of the establishment of a high-impact test facility utilizing a 22-in. air gun. Pursuant to the requirements of the High-Impact Survival Program, an existing 22-in. air gun is being converted to accelerate a mass of up to 350 lb to a muzzle velocity of 320 ft/sec. With the proper selection of target material, this provides the desired impact of 10,000 g, 1-msec duration, rectangular wave. The conversion is nearly complete. Two major problems remain to be solved. These will be described in detail.

1. The Air Gun

Fig. 1 shows the air gun located in a subsurface, concrete channel. It shoots a sabot into a target at one end of the channel. Between the two center flanges is a seal plate which separates the accumulator from the barrel. The sabot is held against the seal plate by a tension-release link. Accumulator pressure stresses the link to approximately 80% of its yield strength. The gun is fired by remotely triggering a ram-set. This drives a projectile into the weakest part of the link, causing failure. Only minor modifications were made to the gun itself.

2. Recoil Constraint

A welded steel buttress (Fig. 2) is used to constrain the gun during recoil. This is bolted to the existing deck rails. Its design was based on the maximum allowable floor loading and limits the accumulator pressure to 130 psig. Two steel cables hold the gun carriage firmly against the recoil constraint and a third, at the muzzle, prevents pitching.

3. Target

End grain redwood (6 in. thick) with a nominal density of 25 lb/ft³ and a crush strength of approximately 4000 psi is the impact material used to provide the desired impact profile. Fig. 3 shows the target system, which consists of the matrix of redwood blocks, the peripheral constraints, 8 in. of steel plate backing, and approximately 50 railroad ties for load distribution. The target system is held against the end wall of the channel which is 5 ft thick. A 10,000-g impact crushes the redwood to a depth of about 2 in.

4. Sabot

The existing sabot is both too expensive and too heavy for practical testing. Some improvements have resulted in

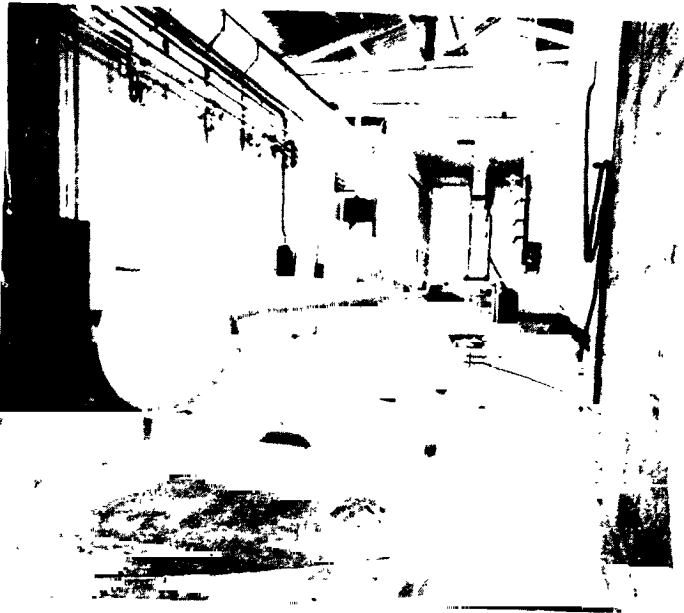


Fig. 1. Air gun in channel

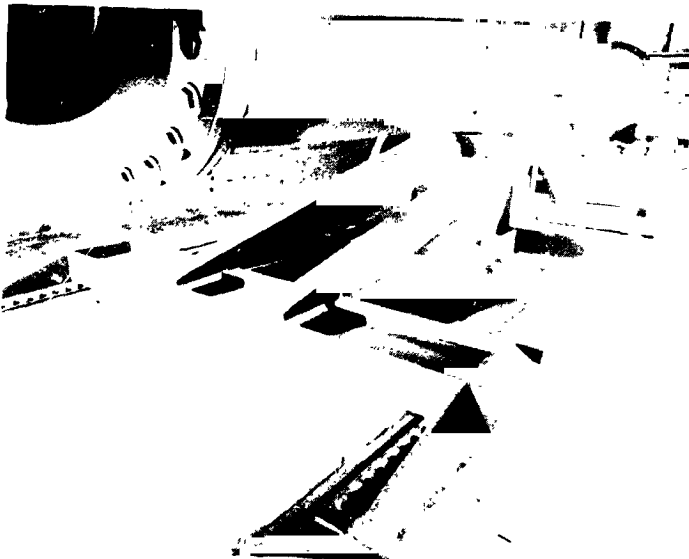


Fig. 2. Welded steel buttress

cutting the mortality rate from one per shot to one per two or three shots; however, a failure usually requires scrapping the entire sabot. Fig. 4 shows the existing sabot with the front (impact) plate removed, and Fig. 5 shows a model of what is expected to be an improved design. An attempt is being made to design a sabot that will be lighter in weight and strong enough to allow re-use of at least the more expensive parts. The parts will be bolted together so that damaged parts can be replaced. A scaled model of this design (Fig. 5) is undergoing impact tests on a 2.75-in. air-gun test facility for design optimization.



Fig. 3. Target system



Fig. 4. Sabot with impact plate removed



Fig. 5. Model of improved sabot design

Its full-scale weight would be approximately 160 lb. This compares favorably with the present sabot weight of 260 lb.

5. Instrumentation

Muzzle velocity is measured with magnetic pick-ups and shear pin pick-ups mounted in the gun barrel. These signals are fed into electronic counters which display the time the sabot took to travel between pick-ups. Impact acceleration is measured with a piezoelectric accelerometer mounted on the inside face of the impact plate of the sabot. Its signal is fed into an oscilloscope equipped with a camera for photographing the impact wave form. All data is simultaneously recorded on magnetic tape. The most critical problem is getting the acceleration signal from the sabot to the readout instrumentation. Three different cable pay-out systems have been tried without success. A Micro-dot coaxial cable supported by fiberglass tape and an AWC #20, two-conductor, shielded, jacketed

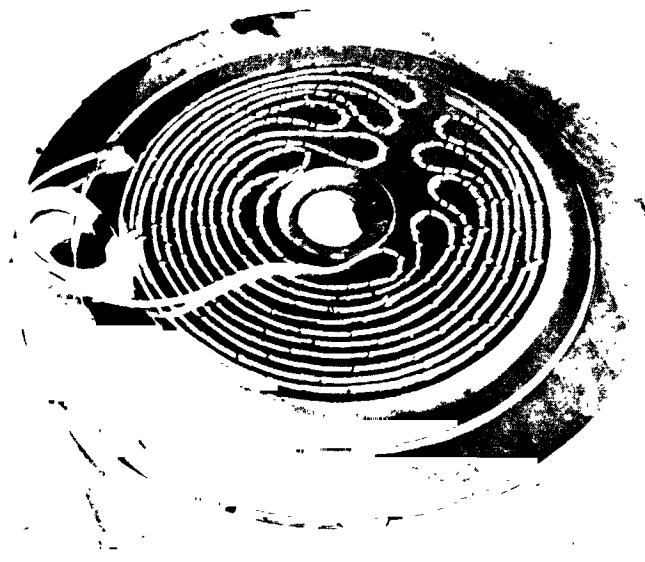


Fig. 6. Cable cast in silicone rubber

cable both failed in tension. An Amphenol coaxial cable failed in shear due to twisting, but satisfactorily resisted tensile loading and abrasion. Fig. 6 shows this same cable cast in silicone rubber in a non-twist configuration. This system will be used on the next test.

6. Status

Three test shots have been made to date. Several more will have to be made before the remaining problems are solved and an adequate system calibration is obtained. There remain two major problems: sabot design and the pay-out cable system. Each of these shows promise of solution in the immediate future. The tests thereafter will be for the purpose of obtaining calibrations of acceleration versus pressure and velocity versus pressure. Measurements of the crush strength of the redwood and the depth of penetration of the sabot have given a rough indication that the calculated accumulator pressure based on an adiabatic expansion process has been close. These indications, however, are not adequate for actual test purposes. Several measured test points will have to be obtained before the facility is operational.

X. Solid Propellant Engineering

PROPULSION DIVISION

N67-29151

A. Pressure and Density Gradients in Propellant Gases, O. K. Heiney

The problem of determining the fraction of propellant energy required to accelerate propellant gases in the ballistic system of burning propellants utilized for acceleration of a piston has been studied intermittently since the time of La Grange.

The problem is of interest not only from an energy fraction standpoint, but also because its solution directly gives a value for the pressure profile in the gas and, therefore, a value for the pressure drop from static gas to gas at the base of the piston, moving at a velocity equal to that of the piston. The analysis is performed for essentially two cases, (1) when the velocity of sound in the combustion products is a function of the adiabatic flame temperature T_0 ; and (2) when the velocity of sound is a function of $7/10$ of T_0 , with the realistic results somewhere between these boundaries. The symbols used in this analysis are defined in Table 1.

1. Density Distribution Analysis

La Grange, as covered in Ref. 1, takes the continuity equation and applies it to the system illustrated in Fig. 1,

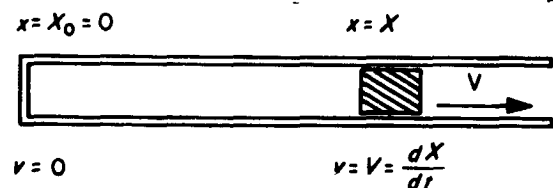


Fig. 1. System and boundary conditions

as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v) = 0 \quad (1)$$

Then, La Grange assumes the constant density

$$\frac{\partial \rho}{\partial x} = 0$$

This assumption changes Eq. (1) to

$$\frac{1}{\rho} \frac{\partial \rho}{\partial t} = - \frac{\partial v}{\partial x} = C \quad (2)$$

Table 1. Definition of symbols

A	bore area
C_w	initial propellant charge
E_G	kinetic energy in accelerating gas
F_P	impetus of propellant
g	acceleration due to gravity
m_p	mass of propelled device
P_0	static pressure
P_s	pressure at base of shot
P_{av}	average plenum pressure
R	gas constant
T	gas temperature
T_0	isochoric flame temperature
V	piston velocity
v	gas velocity
V_c	plenum volume
X_0	initial piston reference
X	distance from piston base to X_0
x	chamber reference
γ	gas specific heat ratio
ρ	gas density
δ	density distribution factor

In turn, Eq. (2) is integrated with the boundary conditions shown in Fig. 1, giving

$$\int_0^x dv = C \int_0^x dx \quad v = Cx \quad (3)$$

The boundary conditions also give

$$C = \frac{1}{X} \frac{dX}{dt}$$

then

$$v = \frac{x}{X} \frac{dX}{dt} \quad (4)$$

In Eq. (3), it is shown that the constant density assumption of La Grange will directly result in a linear gas velocity profile. Constant density is then a sufficient condition for velocity linearity, although not a necessary condition.

The energy in the gas using the above approach is then:

$$\begin{aligned} E_G &= \frac{1}{2} \int_0^x A \rho v^2 dx = \frac{A \rho}{2} V^2 \int_0^x \frac{x^2}{X^2} dx \\ &= \left(\frac{1}{2} \right) \left(\frac{1}{3} \right) A \rho X V^2 \end{aligned}$$

but

$$A \rho x = \frac{C_w}{g}$$

therefore

$$E_G = \left(\frac{1}{2} \right) \left(\frac{1}{3} \right) \frac{C_w V^2}{g} = \frac{1}{2} \frac{C_w V^2}{\delta g} \quad (5)$$

where

$$\delta = 3 \text{ for the La Grange solution.}$$

The factor δ was introduced by Hirschfelder (Ref. 2) in his extension of the work of Kent (Ref. 3), who allowed the gas density to vary, and solved for δ as a function of the charge-to-projectile mass ratio.

Solutions of the form

$$\delta = f(C_w/m_p g)$$

are useful for the approximate closed-form ballistics of Hirschfelder, but are less useful for incremental computer solvable interior ballistic systems such as that discussed in a previous JPL report (SPS 37-44, Vol. IV, pp. 167-170). The results obtained by La Grange with $\delta = 3$ essentially state that the energy in the accelerating gas is equivalent to $\frac{1}{3}$ of the gas mass traveling at the shot velocity. This is a good approximation at velocities where the gas density is almost constant. This approximation is inaccurate, however, at velocities where the density is no longer uniform.

For incremental computations, δ is most effectively formulated as a function of shot velocity. To accomplish this, however, it is first necessary to describe the density variation behind the projectile as a function of shot velocity. It is then assumed that a linear velocity gradient exists behind the shot and that isentropic flow relations may be used during an interval to describe the flow behind the shot. The flow, during the short interval

considered, must be considered as quasi-isentropic. According to Ref. 4, the density ratio then becomes:

$$\rho/\rho_0 = \left(1 + \frac{\gamma - 1}{2} \frac{v^2}{gRT}\right)^{-1/\gamma-1} \quad (6)$$

The question then arises as to what temperature should be used to define the speed of sound in the environment of the burning gases. An upper bound for the tempera-

ture would be the isochoric flame temperature of the gases, T_0 , while a reasonable lower bound would be $7/10 T_0$. The analysis is then carried out for these two temperature extremes. For lower velocity ($< 3,000$ ft/sec) and during propellant burn, the values near T_0 are probably more accurate. At higher velocities, and subsequent to propellant burnout, the temperature tends to fall from the T_0 value. The results of Eq. (6) are illustrated in Fig. 2 for $\gamma = 1.222$ and $R T_0 = 375,000$ ft-lb/lb.

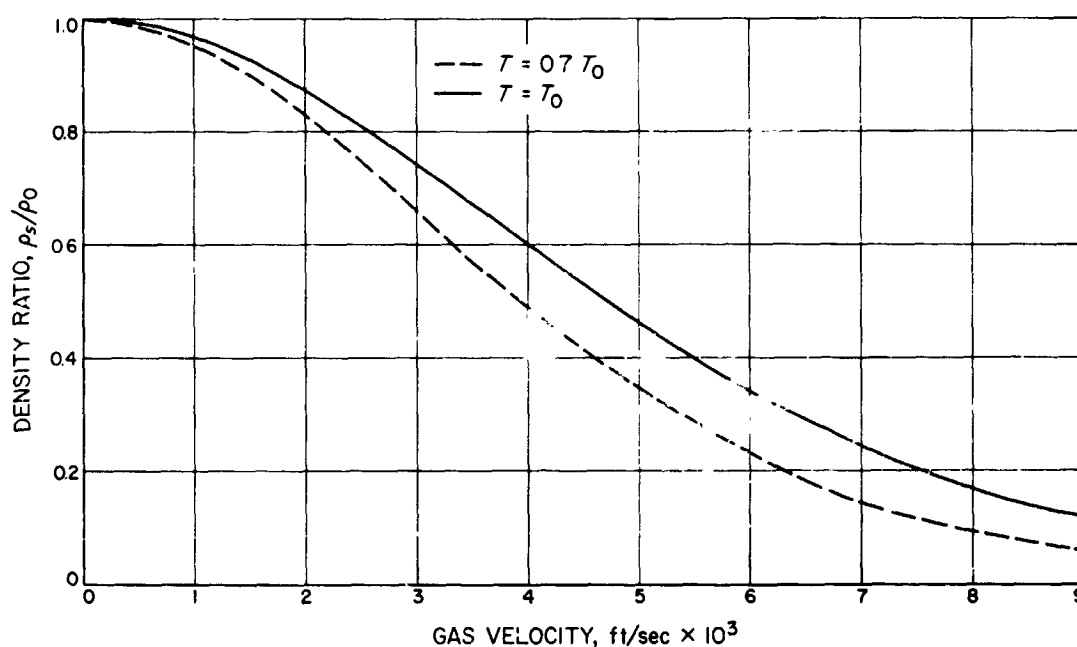


Fig. 2. Density ratio as a function of shot velocity

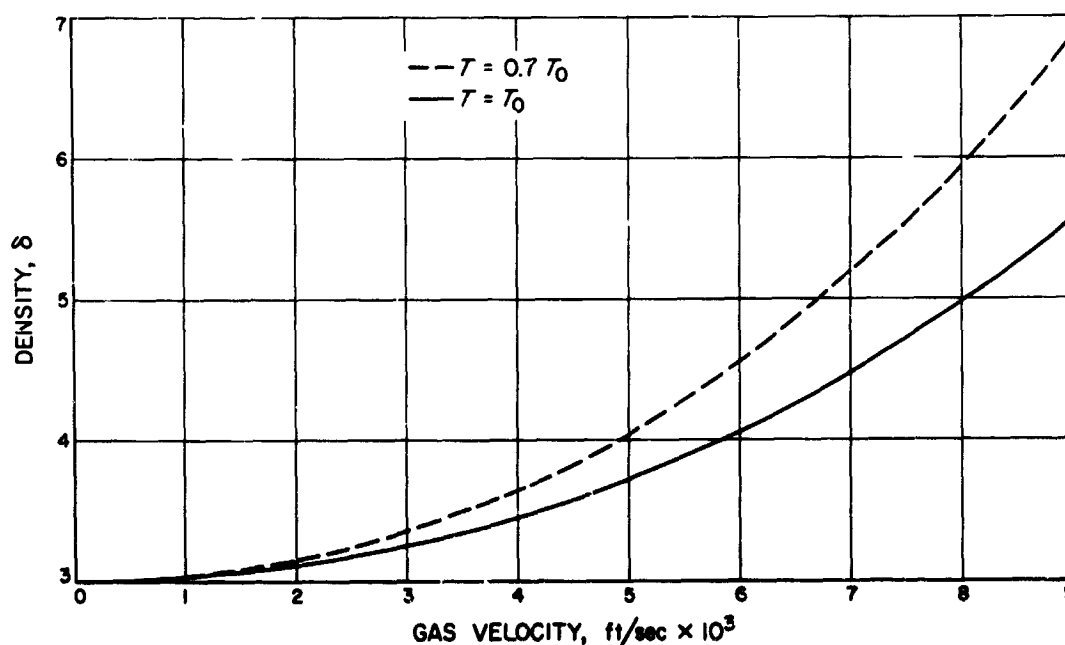


Fig. 3. Value of δ as a function of shot velocity

The energy increment in a small control volume of gas will then be

$$E_t = \frac{1}{2} \rho_N v_N^2 A \Delta x_N$$

and

$$E_G = \frac{A \rho_0}{2} \sum_N \frac{\rho_N}{\rho_0} v_N^2 \Delta x_N = \frac{1}{2} \frac{C_H}{g \delta} V^2 \quad (7)$$

The value of δ is determined by performing the numerical summation of Eq. (7) on a digital computer. Fig. 3 illustrates the results of this procedure. As anticipated, the value of δ approaches 3.0 at low velocities.

2. Pressure Gradient Analysis

The determination of the pressure drop from P_0 or P_{ar} to the value at the base of the projectile, P_s , is central to any interior ballistic solution because the motion of the projectile is a function of P_s while the system energy equations give a value for P_{ar} .

A value for the ratio of P_s/P_0 may be obtained by the quasi-isentropic assumption previously used for densities.

$$P/P_0 = \left(1 + \frac{\gamma - 1}{2} \frac{v^2}{gRT} \right)^{-\gamma/(\gamma-1)} \quad (8)$$

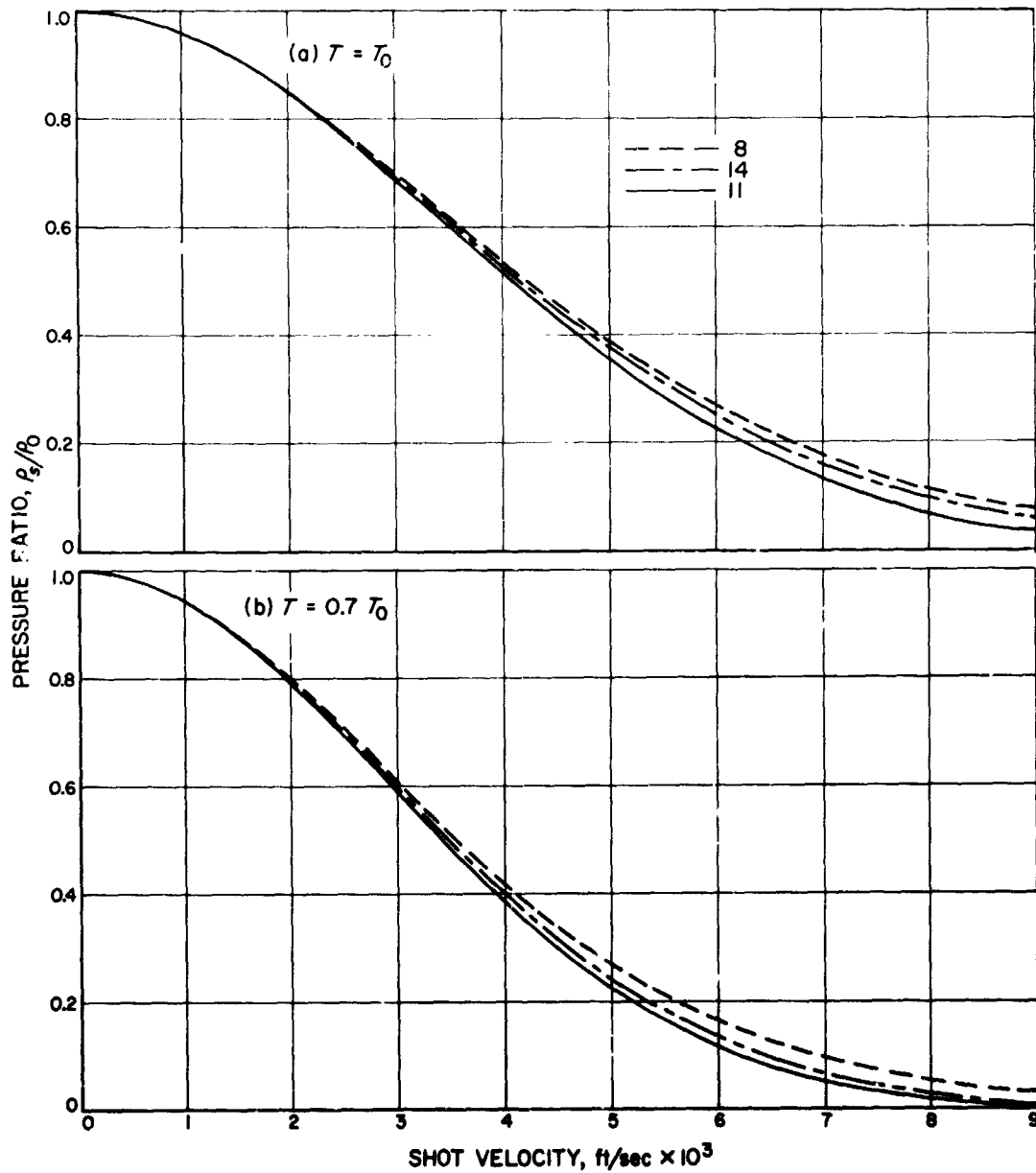


Fig. 4. Shot pressure to static pressure ratio as a function of shot velocity

The results of this relation are illustrated in Fig. 4 for the temperatures $T = T_0$ and $T = 0.7 T_0$, respectively.

Another approach to the pressure gradient solution would be to assume a constant temperature process. In this case Euler's equation and the equation of state give

$$P = \rho RT \quad (9)$$

$$v dv = - \frac{dP}{\rho} \quad (10)$$

The combination of Eqs. (9) and (10) and integration give

$$\begin{aligned} \int_0^v v dv &= - \int_{P_0}^{P_s} \frac{RT}{P} dP \\ \frac{V^2}{2} &= - RT \ln(P_s - P_0) \\ \frac{P_s}{P_0} &= e^{-V^2/2RT} = e^{-V^2/2gF_P(T/T_0)} \end{aligned} \quad (11)$$

The results of Eq. (11) are also shown in Fig. 4. The constant temperature equation always predicts a lower shot pressure than the quasi-isentropic relation. The required relation is that for P_s/P_{ar} ; Eqs. (8) and (11) give expressions for P_s/P_0 . The problem being that for the interior ballistic formulism utilized, a term is already included for the energy in the accelerating gas, to give a value for P_{ar} . A heuristically reasonable approach would be to assume that a relation of the form:

$$\frac{P_s}{P_{ar}} = \left(1 + \frac{\gamma - 1}{2\phi} \frac{v^2}{gRT}\right)^{-\gamma\gamma-1} \quad (12)$$

with

$$\phi > 1$$

would be satisfactory.

To investigate this possibility, a value must be developed for P_{ar} which only accounts for the gas kinetic energy. This is accomplished as follows:

$$\begin{aligned} P_0 V_c - \frac{1}{2\delta} m V^2 &= \rho_0 V_c \\ 1 - \frac{V^2}{2\delta gF_P(T/T_0)} &= P_{ar}/P_0 \end{aligned} \quad (13)$$

The results of Section 1, giving $\delta = f(V)$, may then be employed with $\phi = 1.5$. The combined results of Eqs. (12) and (13) give a value for P_s/P_0 (Fig. 4). For the plotted

values shown in Fig. 4 a ϕ of 1.5 was empirically chosen. This value of ϕ gives consistent results for both temperature extremes.

$$\frac{P_s}{P_0} = \frac{P_s}{P_{ar}} \frac{P_{ar}}{P_0} \quad (14)$$

3. Conclusion

The energy required to accelerate the combustion gases has a unique status in ballistic theory because, at low velocities, this energy may be considered second order and, at higher velocities, it assumes primacy. This energy is, in fact, the causal factor of the upper velocity limit of standard ballistic devices. The results expressed in Fig. 3 provide the δ values required for adequately averaging the gas acceleration energy in a given interior ballistic formulism, while the relations plotted in Fig. 4 give the bounds for determining the pressure gradient in a given device. Experimental ballistic firings will be conducted to empirically narrow these bounding relations. Recourse to empirical techniques is a requisite because the movement of a piston by burning propellant is neither adiabatic, isothermal, isentropic, or anything else.

B. Application Technology Satellite Motor Development, R. G. Anderson, R. A. Grippi

1. Historical Background

In January 1963 the Jet Propulsion Laboratory initiated a development program to provide a solid propellant apogee motor for a second-generation *Syncom* satellite. This program, under the management of the Goddard Space Flight Center, was designated *Advanced Syncom*. It was to result in a spin-stabilized, active repeater communications satellite weighing approximately 750 lb, operating at synchronous altitude (22,300 mi), which would handle voice communications, teletype, and monochrome and color television signals.

In January 1964 this program was redirected to include a number of experimental instruments in addition to the original communication instruments. This expanded program, the Application Technology Satellite (ATS), will result in a general-purpose satellite capable of operation at synchronous altitude with experimental instruments in meteorology, communications, radiation, navigation, gravity gradient stabilization, and various engineering experiments. To place these satellites in synchronous orbit, JPL will provide a solid propellant rocket motor

for the final required velocity increment at the apogee of the elliptical transfer orbit. This rocket motor is designated the JPL SR-28-1 (steel chamber) or JPL SR-28-3 titanium chamber. At present, only the JPL SR-28-3 unit will be for flight use.

Previous progress reports on the development of this motor were published in SPS 37-34 to 37-43, Vol. IV.

2. Program Status Summary

The motor development program calls for static firing of 4 heavywall motors and 26 flightweight motors, including two with flight-design titanium chambers, prior to conducting an 8-motor qualification program. To date, 4 heavywall motors plus 24 flightweight motors have been static-fired, 4 of which were under simulated high-altitude conditions at Arnold Engineering Development Center (AEDC), Tullahoma, Tennessee. All of the flightweight motors tested to date were with Type 410 chromium steel chambers, with the exception of Dev. G-8T, G-9T, E-3T, and Q-9T, which used titanium chambers. During the past reporting period, two additional development units were tested. The testing of the first storage unit (Dev. F-1) was performed, after 16 months of storage, in January 1967. On March 10, 1967 a live unit (Dev. Q-9T) was tested as the fourth and final test in a program to evaluate an omnidirectional antenna design.

The ATS apogee motor qualification phase was conducted at AEDC in July and August of 1966. The results are reported in a previous JPL report (SPS 37-41, Vol. IV, pp. 91-96).

The first of five Applications Technology Satellites (ATS-B) was successfully launched in December 1966. The satellite is presently on station and functioning as planned.

3. ATS Storage Units

The three ATS apogee motors, cast in September 1965, have been stored at 80°F or ambient temperature following completion of post-loading requirements in November 1965. The units were processed and loaded to flight-standard procedures. All hardware components are of flight design with the exception of the chamber and the number of layers of chamber insulation. In place of the 6AL-4V titanium chamber, a Type 410 chromium steel unit was used since the titanium chambers were not available at the time the motors were loaded. The insulation configuration lacks one layer of 0.030-in. material at

each end of the motor chamber. This difference in insulation configuration is a result of changes initiated in the flight design after the first titanium motor was fired in November 1965. These deviations from the flight configuration should not affect the storability of the motor.

These units are being removed at periodic intervals and subjected to various inspections. These inspections include weight determinations, visual examination of all components, dimensional checks, and hardware alignment. The last periodic inspection of the storage units was performed during November 1966 (SPS 37-43, Vol. IV).

The first storage unit (Dev. F-1) was statically tested at the JPL-ETS facility on January 31, 1967, approximately 16 months after being cast. Prior to static firing, the unit was subjected to a series of environmental tests including booster vibration, booster acceleration, and temperature cycle. These environments simulate, but do not duplicate, the conditions an apogee unit will encounter during the boost phase into orbit. Visual and radiographic inspection of the unit tested revealed no anomalies. Table 2 summarizes the weight and measurement data for Dev. F-1. This table includes data taken after the unit was cast in September 1965, and data recorded 16 months later just prior to static testing. All pre-fire data indicated that the apogee unit could be stored for at

**Table 2. Weight and measurement data summary for
ATS storage unit Dev. F-1**

Parameter	Post-loading (Sept. 1965)	Prefire (Jan. 1967)
Chamber assembly weight, lb (chamber, insulation, propellant, and handling ring)	868.3	868.7
Nozzle assembly weight, lb	39.01	38.89
Hardware thrust alignment, in./in.	0.00007	0.00014
Propellant alignment Port diameter, TIR, in. <u>inches from nozzle base</u>		
4	0.015	0.015
14	0.017	0.016
24	0.014	0.011
Port diameter, in. <u>inches from nozzle base</u>		
4	10.147	10.191
14	10.143	10.168
24	10.139	10.140

least 16 months, after which it could survive a series of simulated launch environments.

The igniters for the storage units were originally assembled in February 1965. Prior to the static test of F-1, an igniter was disassembled and inspected. This visual inspection showed that the igniter hardware was not affected by long-term ambient storage.

Dev. F-1 was tested on January 31, 1967, at a grain temperature of 10°F while mounted in the vertical spin stand operating at 100 rpm. The unit appeared to operate normally during its 45-sec burning time. A post-fire inspection of the spent motor hardware indicated nominal operations. The data presented in Table 3 represent some

of the motor's performance parameters and other pertinent data related to this unit.

A complete analysis of all data related to this first storage unit appeared nominal with the exception of the characteristic velocity value (W^*). The measured value of W^* was 4903 ft/sec, approximately 1% below the average values obtained during testing of the development and qualification units. All data measurements related to W^* have been checked several times and appear accurate. A review of past ATS motor firings indicates that some decrease in motor performance may occur after the unit has been stored for 10 to 12 mo. A detailed review of all data related to this storage problem is being conducted and will be reported in a future SPS.

**Table 3. Static test data summary for
ATS storage unit Dev. F-1**

Test conditions	
Type of test	atmosphere/spin
Test location	JPL-ETS, Bldg. E-60
Date	Jan. 31, 1967
Run No.	E-781
Grain temperature, °F	10
Propellant weight, lb	764.7
Physical hardware (S/N)	
Chamber	P-32
Nozzle	F-26
Igniter	SYC-257
Squib	SDI-56
Pressure data	
Characteristic velocity, W^* , ft/sec	4903.2
Chamber pressure integral, psia-sec	8857.7
Igniter peak pressure, psia at msec	"
Chamber ignition peak pressure, psia at msec	229 at 30
Chamber starting pressure, psia at sec	103 at 0.19
Chamber run peak pressure, psia at sec	243 at 34
Time	
Ignition delay, msec	10
Run time, sec	44.96
Nozzle dimensions	
Throat diameter, in.	
initial	4.083
final	4.106
average	4.095
Throat erosion (area), %	1.13
* Amplifier malfunction, data not available.	

4. ATS Omnidirectional Antenna Test No. 2

The second omnidirectional antenna test was conducted at the L-ETS facility on February 17, 1967. These antenna tests are conducted jointly by JPL and Hughes Aircraft.

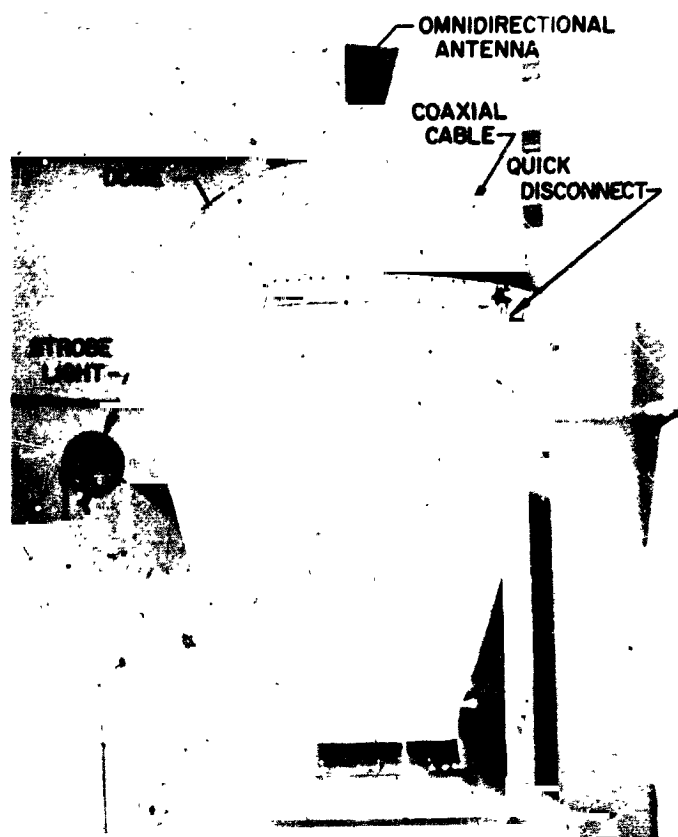


**Fig. 5. Omnidirectional antenna-dome combination
on apogee meter**

The antenna for Test No. 2 was structurally modified based on the results of Test No. 1 (SPS 37-43, Vol. IV). The major design changes were an increase in the thickness of the fiberglass dome and in the method of attaching the antenna to the hemispherical dome. In the present configuration, the antenna is attached to a fiberglass cone which, in turn, is bonded to the main spherical dome.

The purpose of this test was to evaluate the release mechanism of the redesigned antenna-dome combination. The test consisted of releasing the antenna coaxial cable disconnects and confirmation of the antenna's free-flight path. These design changes were evaluated to ensure the structural integrity of the unit during its release and free-flight period. Fig. 5 shows the antenna-dome combination.

Since the first test confirmed that the ignition impulse was sufficient to release the antenna, the same apogee motor configuration was used for Test No. 2. During this test, the following JPL motor hardware was used:

- (1) Motor Chamber P-6 (inert grain).
- (2) Nozzle F-18.
- (3) Igniter Basket, SYC-284.
- (4) Squib, PC-37, S/N 37897.

The test setup consisted of the equipment shown in Fig. 6. The motor was mounted on the vertical spin test stand at the north test bay of Building E-60, at the JPL Edwards Test Station. Throughout the test, the motor and test article were spun at 100 rpm. Four 16 mm

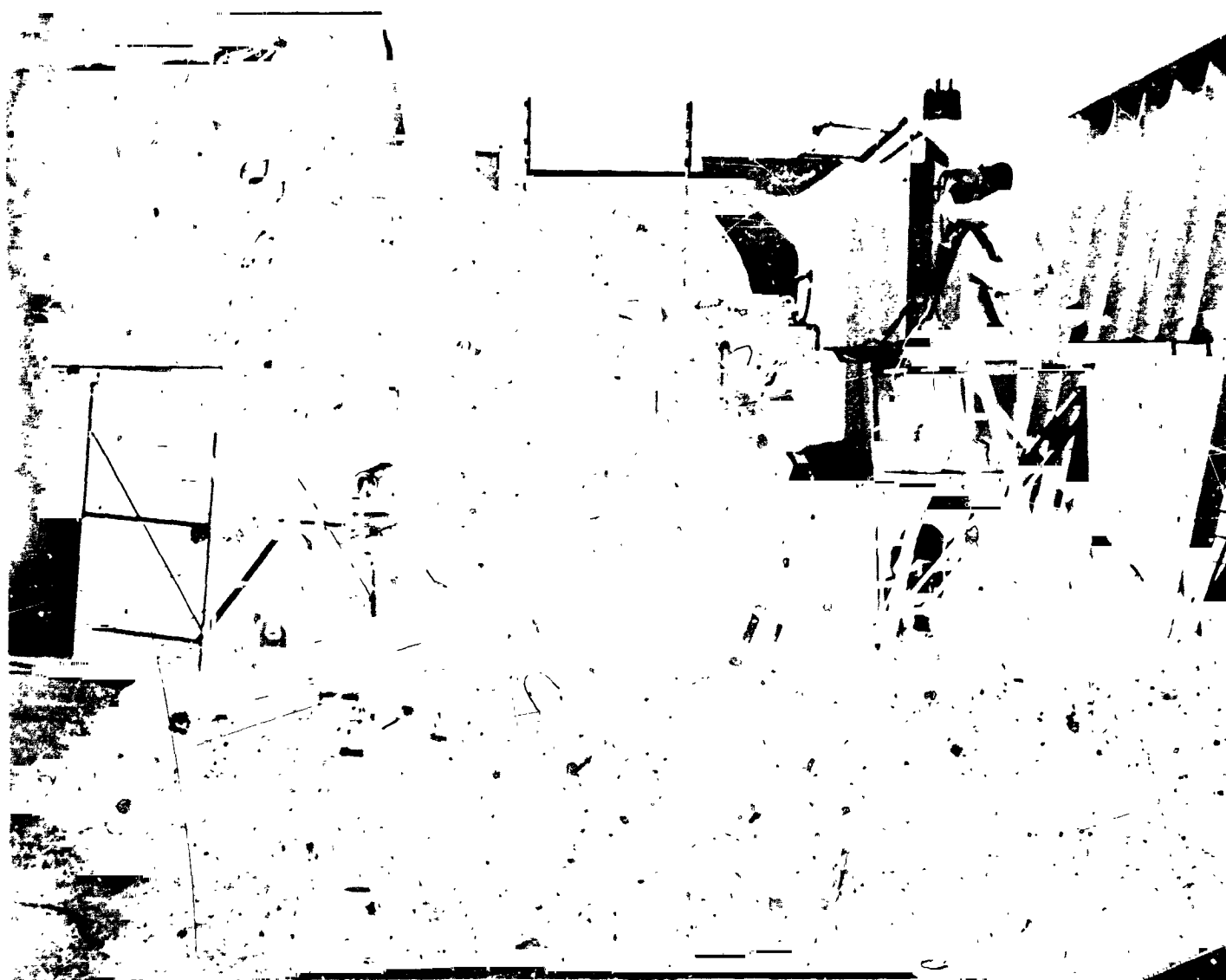


Fig. 6. Pre-test installation of test article and high-speed cameras

Fastex cameras, operating at 2,000 frames/sec, were used to photograph the release and initial free flight of the antenna-dome combination. A 16 mm Milliken camera, operating at 500 frames/sec, was used for overall test coverage. Chamber pressure was recorded by two pressure transducers while another transducer recorded igniter basket pressure. The output of all transducers was recorded on an oscillograph (paper speed of 40 in./sec). A special circuit was created to indicate the release of the coaxial cables. The signal indication, which occurs after the antenna has traveled 0.25 in., was also recorded on the oscillograph.

The Fastex camera does not have the capability of incorporating a timing mark on the film; therefore, a strobe light was placed within the range of sight of all Fastex cameras. The strobe light was triggered at the same time current was supplied to the squib, thus producing on the film a positive indication of ignition sequence start. Fig. 7 shows the electrical circuits for the high-speed cameras, igniter ignition, and strobe light. As shown in the figure, the firing of the squib and strobe light and the starting of each camera were controlled by a Fastex goose box. The test was initiated by actuating the start switch on each goose box.

Following a dry run, during which the five cameras were tested, the test was performed. All data were adequately recorded, except for ignition current. The oscillograph revealed that it was still possible to correlate the start of the ignition sequence without an ignition current trace. At the time current was supplied to the squib circuit, a slight noise signal was generated in all pressure transducer circuits. This noise signal is discernible on all transducer traces recorded on the oscillograph (Figs. 8 and 9). This noise signal corresponds to zero time (t_0) or the first indication of squib current.

The results of the recorded pressures (Test No. 2) are given in Fig. 8. A comparison of the oscillograph records from Test No. 2 with those of Test No. 1 (Fig. 9) shows excellent agreement in both pressure levels and times.

The impulse from the igniter resulted in a nominal release of the antenna-dome combination with visual estimates giving a straight-up free-flight height of approximately 150 ft. The antenna was structurally intact throughout the trajectory, except for one coaxial cable which separated from the dome during its flight. Upon impact, the dome hit the ground at the antenna position, causing the fiberglass cone to separate from the dome,

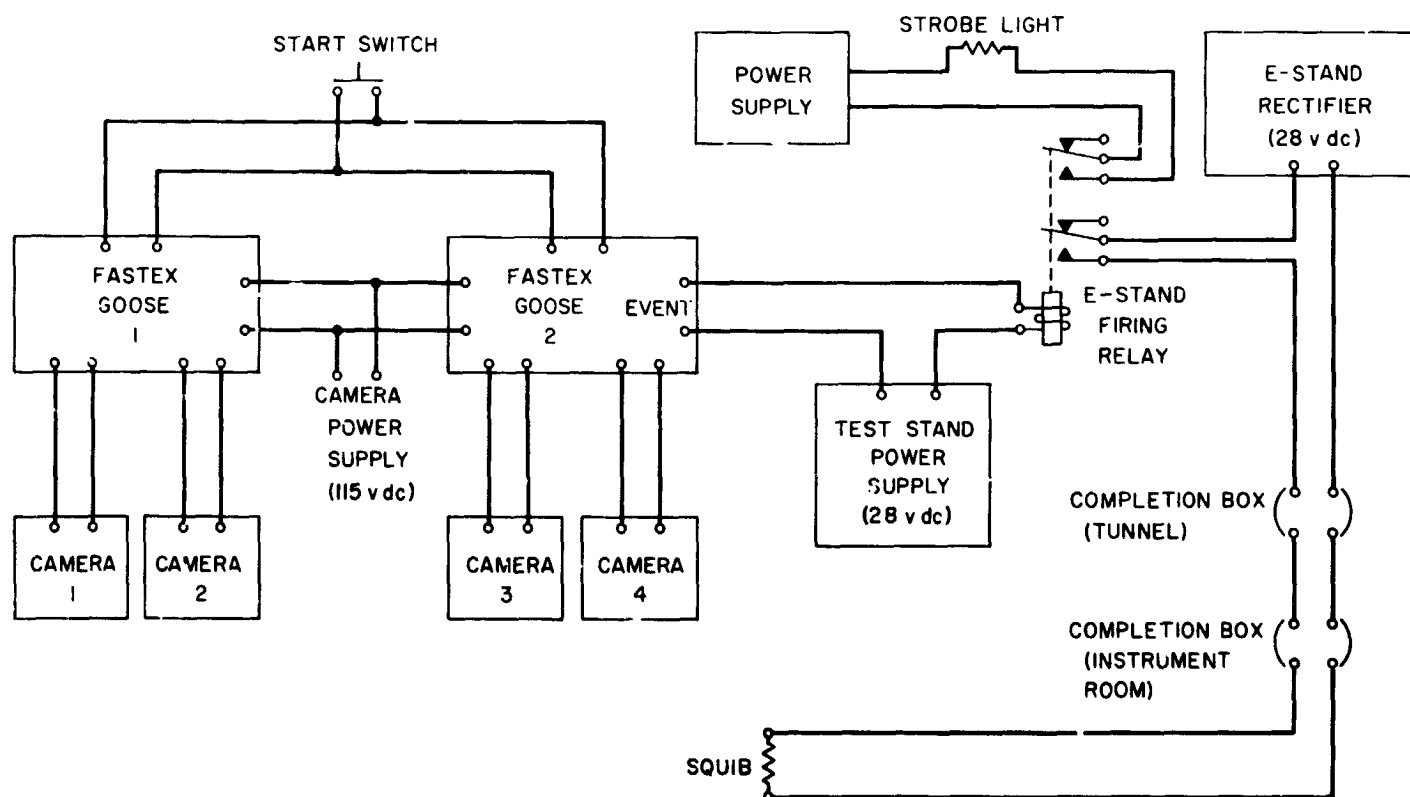


Fig. 7. Electrical diagram of high-speed camera and igniter ignition circuit

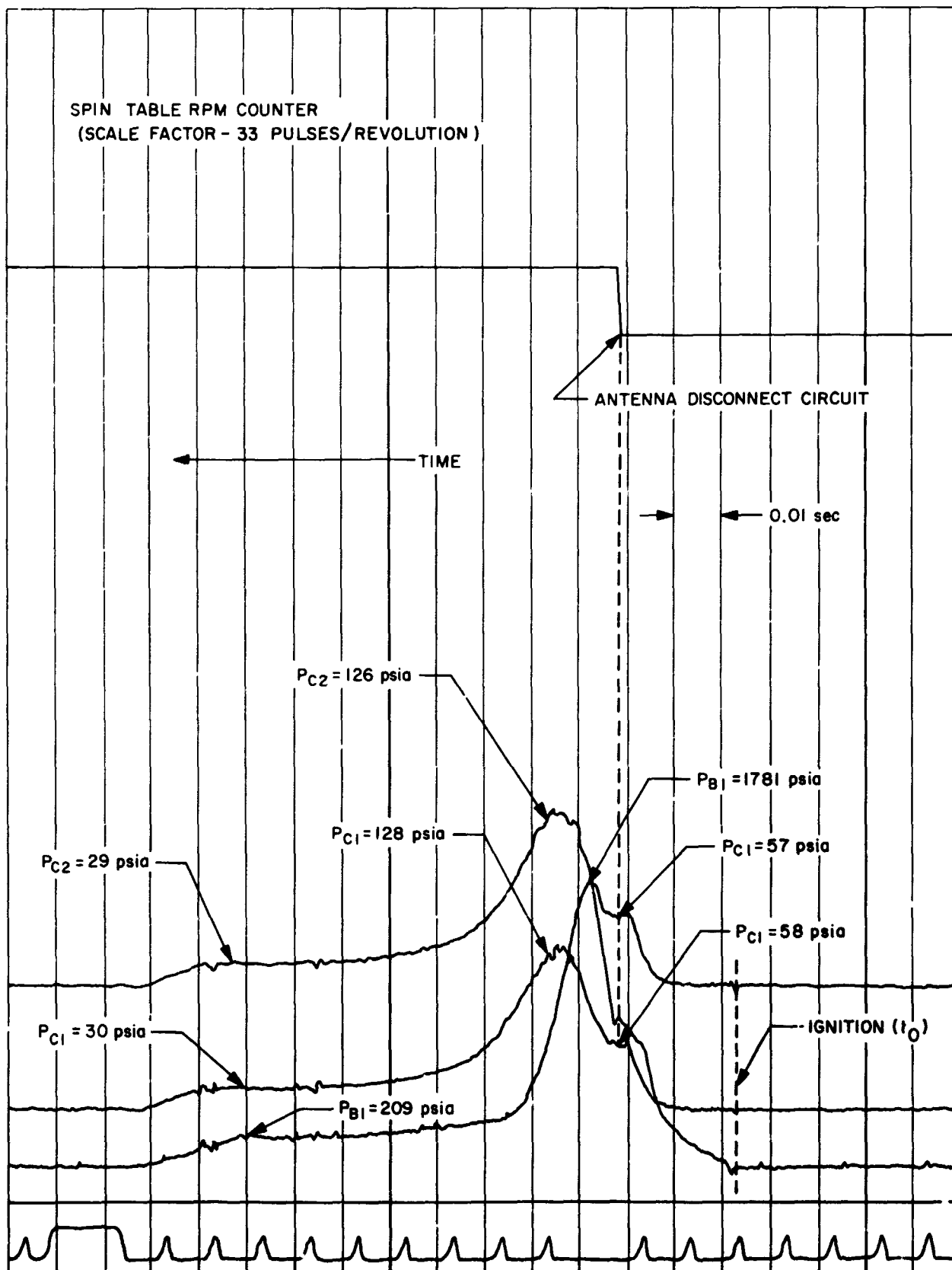


Fig. 8. Motor ignition characteristics—omnidirectional antenna Test No. 2

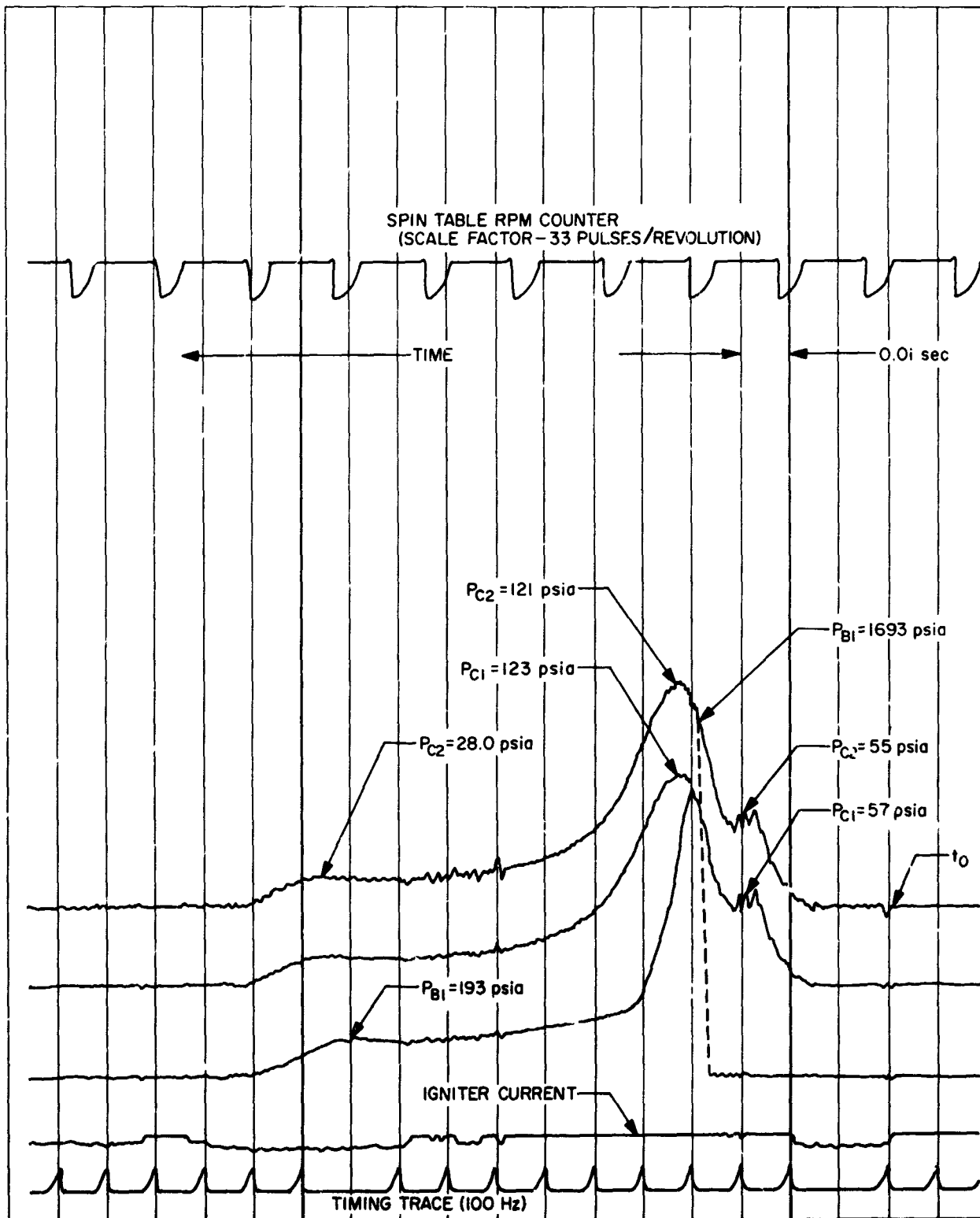


Fig. 9. Motor ignition characteristics—omnidirectional antenna Test No. 1

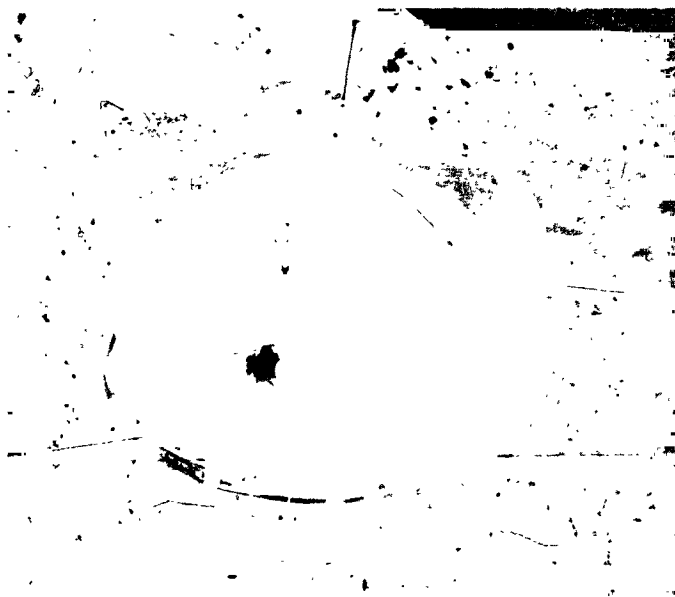


Fig. 10. Omnidirectional antenna-dome combination after test

and slightly damaging an antenna bracket. Fig. 10 shows the post-fire condition of the antenna at impact. The antenna disconnect circuit opened approximately 25 msec after the first application of squib current. At this time, the antenna had traveled approximately $\frac{1}{4}$ in. from its original position. At release, the motor chamber pressure was 58 psia, which closely corresponds to the nozzle diaphragm rupture pressure. The antenna is probably released from the motor within milliseconds after the nozzle diaphragm ruptures. Therefore, the antenna is never subjected to the peak chamber pressure generated during the motor's ignition phase.

In summary, the igniter impulse was sufficient to separate the redesigned antenna-dome combination from the motor without structural damage. As demonstrated by visual and photographic coverage, the antenna was released and maintained a free flight path along the motor's centerline well beyond the spacecraft's VHF antenna. The retention of the coaxial cable brackets was the only difficulty encountered during the test.

The third and fourth omnidirectional antenna tests were conducted on March 9 and 10, 1967. Test No. 3 only employed the ignition impulse to release the antenna. This test was almost identical to Tests Nos. 1 and 2. Test No. 4 employed a live motor (full duration) as a final confirmation of antenna design and release mechanism. The detailed result of Tests Nos. 3 and 4 will be reported in the next SPS. Initial results from the last two tests indicate that the antenna design and its release mech-

anism are reliable and no future antenna tests are anticipated.

C. Lockseal Adaptation to Syncom and Application Technology Satellite Rocket Motors, W. L. Dowler

1. Program Summary

This investigation was conducted to determine the weight increase of the *Syncom* and *Application Technology Satellite* (ATS) rocket motors if thrust vector control were provided. The Lockheed Propulsion Company's Lockseal thrust vector control system was the device used in the investigation. The study consisted of determining the weight of the components aft of the cylindrical shell for the rocket motors, both with and without Lockseal, and auxiliary hydraulic equipment. The adaptation of the motors to the Lockseal required the addition of several components as well as the thickening of the aft closure so that the closure could withstand the forces and torques required to actuate the nozzle through the desired angular deflections.

The results of the study indicated that the inert weight increases would be 19.03 lb for the *Syncom* motor and 28.69 lb for the ATS motor. These results show that the installation of the Lockseal (TVC) system on the ATS motor is more efficient, from a weight standpoint than on the *Syncom* motor. The estimated mass fraction for the *Syncom* motor changed from 0.85 without a thrust vector control system, to 0.67 with the Lockseal. The mass fraction of the ATS motor decreased from 0.90 to 0.87.

2. Adaptation of Lockseal to Syncom and ATS Rocket Motors

The designs of the *Syncom* motor, the ATS motor, and the Lockseal were reviewed. This preliminary evaluation showed that only motor components aft of the cylindrical section required modification. It was also determined that: (1) the location of the nozzle pivot point should be deep inside the chamber; (2) the Lockseal should be as near as possible to the pivot point, since the torque required to rotate the nozzle is proportional to the fourth power of the distance from the pivot point to the center of the Lockseal; (3) the Lockseal should be manufactured with specially prepared silicone rubbers capable of withstanding high pre-flight sterilization temperatures of 275°F. The selected silicone elastomer is designated Rubber Compound No. S-4 by Lockheed. This silicone elastomer also has a low shear modulus which minimizes the torque required to rotate the nozzle.

Table 4. Thrust vector control system parameters^a

Parameters	Syncom	ATS
Nozzle deflection (2 axes), deg	± 5	± 5
Total number of operating cycles (each axis)	40.0	40.0
Actuator stroke, in. ($\pm$)	0.75	1.25
Actuator distance from center of rotation, in.	6.13	11.5
Maximum actuator force, lb	450.0	345.0
Nozzle moment of inertia, ^b lb in. sec ²	6.29	33.6
Actuator system pressure, lb/in. ²	1500.0	1500.0
Nozzle angular velocity, max., deg/sec	0.6	0.6
Distance between attachment points, ^c in.	6.75	9.25

^aNote. In addition, the components of the servo-actuator system are designed to operate under adverse environmental conditions as follows (1) high vacuum of deep space, (2) temperature limits between -40 and $+170^{\circ}\text{F}$ after being subjected to six dry heat sterilization cycles at 135°C for 60 hr each and six surface exposures to Freon gas at 50°C for 24 hr

^bAbout center of rotation of nozzle.

^cEnds to be equipped with rod end bearings

The thrust vector control requirements included in Table 4 and the motor physical and geometrical data were used to compute the Lockseal configurations; the resulting motor configurations designed from these data and calculations are shown in Fig. 11 and 12. Servo-actuators were designed to move the nozzles through the desired rotation angle. The installation of these servo-actuators, as presently designed, was responsible for the major modifications to rocket chamber components.

The Lockseal adaptation to the *Syncom* rocket engine is shown in Fig. 11 and the weight distribution among components before and after the Lockseal adaptation is listed in Table 5. Fig. 13 is a plot of the torque as a function of the nozzle deflection angle. The aft closure was equipped with two sets of ears 90° to each other on the knee of the aft closure to hold one end of the actuators. Originally, the closure was designed to withstand an internal pressure only slightly larger than the design chamber

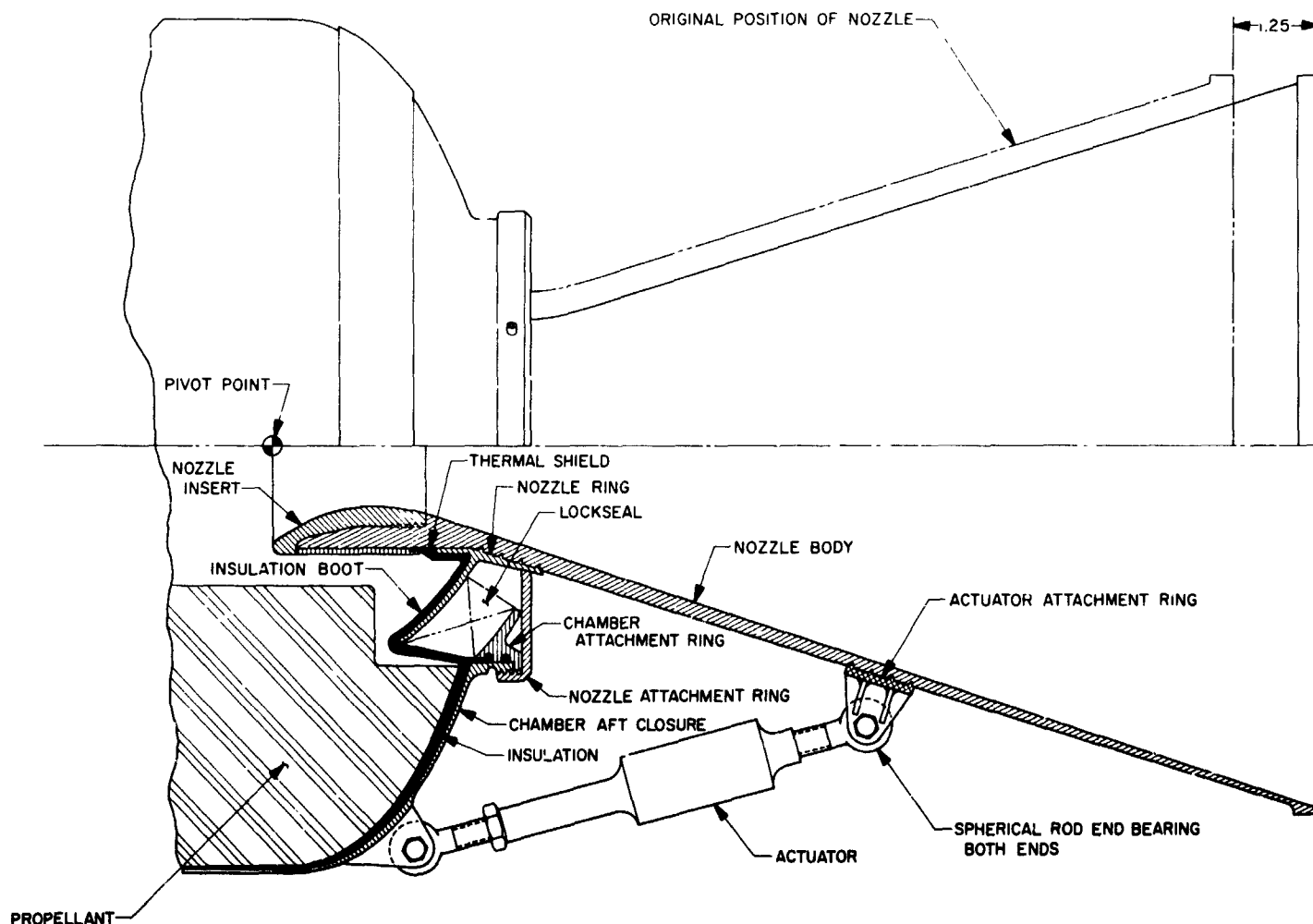


Fig. 11. Modification for adaptation of Lockseal thrust vector control system to Syncom rocket motor

pressure of the unit; this resulted in a thin and light-weight component. In the Lockseal adaptation, the shell must withstand the additional forces and torques of the actuators and Lockseal, and must, therefore, be made thicker. The required thickness was determined from a simplified model, consisting of a flat circular plate fixed around its edges and acted upon by a moment at the center. To minimize the weight of the aft closure, the combined stress produced by the moment and the internal pressure was made approximately 5% less than the yield strength of the material. This resulted in a wall thickness of 0.026 in., an increase of 0.012 in. at the thinnest section. With the greater wall thickness and the addition of ears, the increase in weight is approximately 0.50 lb.

The inside contour of the nozzle was not altered; however, the outside surface was modified to accept the aft end of the Lockseal and metal rings for seal and actuator mounting. The gas path is altered because of the Lockseal so that only the inside surface of the nozzle is exposed to the high temperature gases. This permits thinning of the nozzle wall with a consequent weight reduction of approximately 0.1 lb.

The Lockseal consists of two 6Al 4V titanium rings separated by a series of aluminum and silicone rubber washers. The forward ring is bonded to the forward end of the nozzle body with epoxy cement. The aft nozzle ring is machined with a flange which is retained in position against the chamber aft boss by a nut. A peripheral

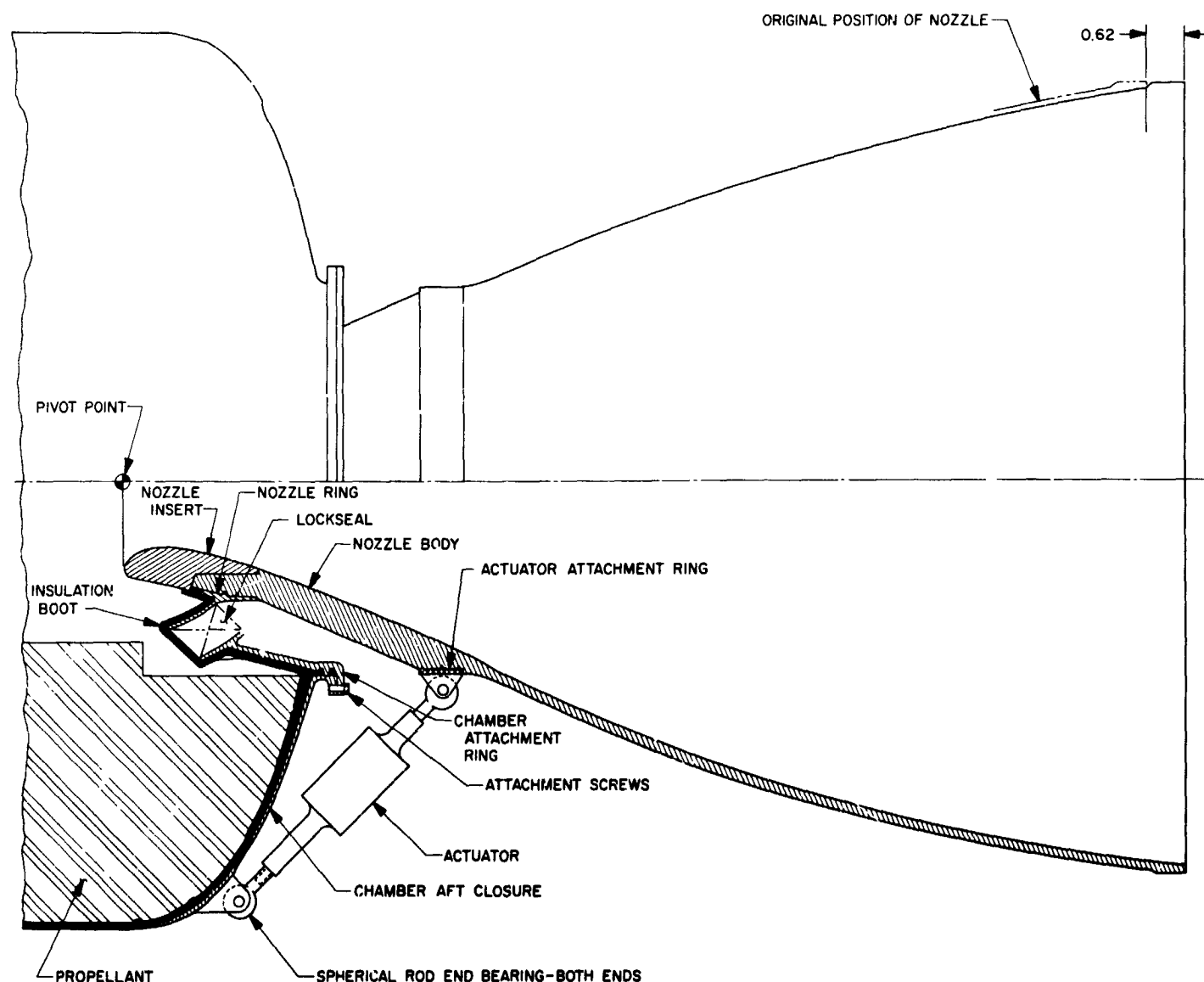


Fig. 12. Modification for adaptation of Lockseal thrust vector control system to AT5 rocket motor

Table 5. Syncom motor weight

Motor weights	Original design, lb	Lockseal design, lb
Propellant	60.39	59.24
Total motor assembly	70.86	88.74
Weight changes		
Nozzle insert	0.22	0.22
Nozzle body	2.47	2.37
Insulation boot	0.34	0.56
Chamber attachment ring	—	0.64
Nozzle attachment nut	—	0.39
Lockseal components	—	0.78
Actuator attachment ring (on nozzle)	—	0.64
Chamber aft closure	0.48	0.98
Nozzle ring	0.88	0.53
Servo actuator system ^a	—	16.31
Propellant weight change	—	(-1.15)
Total weight	4.39	22.27
Net difference	—	+17.88
Mass fraction	0.852	0.668

^aIncludes: N₂ container, squib valve, regulator, hydraulic reservoir, 2 actuators, checkout ports, burst diaphragms, hydraulic lines and fittings, hydraulic fluid, and collection reservoir.

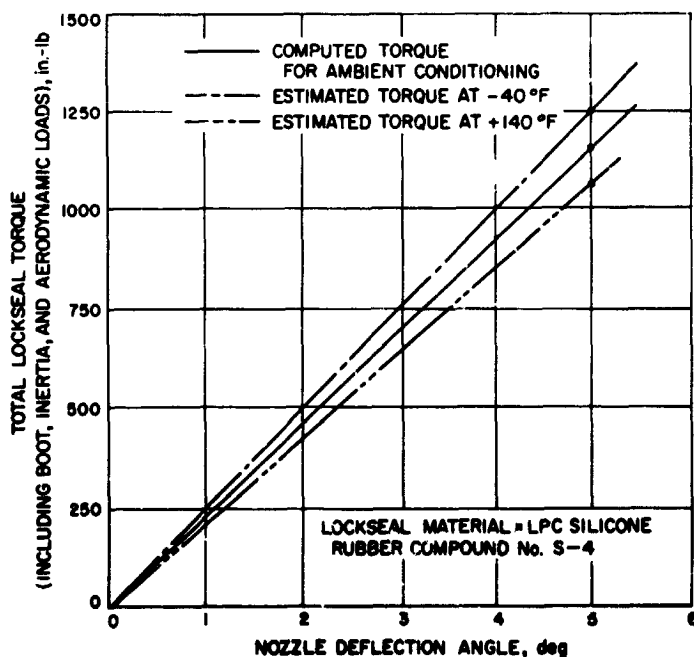


Fig. 13. Total Lockseal torque versus nozzle deflection angle for Syncom rocket motor design

O-ring in the aft ring prevents gas leakage between the chamber aft boss and the Lockseal ring.

The servo-actuator system consists of a pneumatic reservoir, a squib valve, a pneumatic regulator, a hydraulic reservoir, pitch and yaw actuators, and an oil return reservoir. This system is shown in Fig. 14, and values for the design parameters are included in Table 4. The estimated weight of the servo-actuator system for the Syncom motor was 16.31 lb.

A hydraulic system was selected to keep the overall weight to a minimum; however, electromechanical and pneumatic systems were considered. Electromechanical actuators were rejected because of the need for a larger power source. Pneumatic actuators were objectionable since these would result in soft systems because of the compressibility of the actuating fluid.

Additional components required to complete the thrust vector control system include a heat shield to protect the components of the Lockseal and various retaining pins securing the actuators to their respective attachment points. The thermal shield consists of an insulating boot which prevents the hot erosive gases of the rocket engine from impinging upon the components of the Lockseal. A thickness of 0.19 in. was estimated necessary to protect the Lockseal components during the rocket engine firing time of 45 sec. This insulation weighs approximately 0.55 lb as compared to the existing insulation which weighs 0.34 lb.

Retaining pins, acting in shear, will secure the two actuators to the chamber and the nozzle ears. The four pins which tie the spherical rod end bearings of the actuators to the ears weigh approximately 0.048 lb.

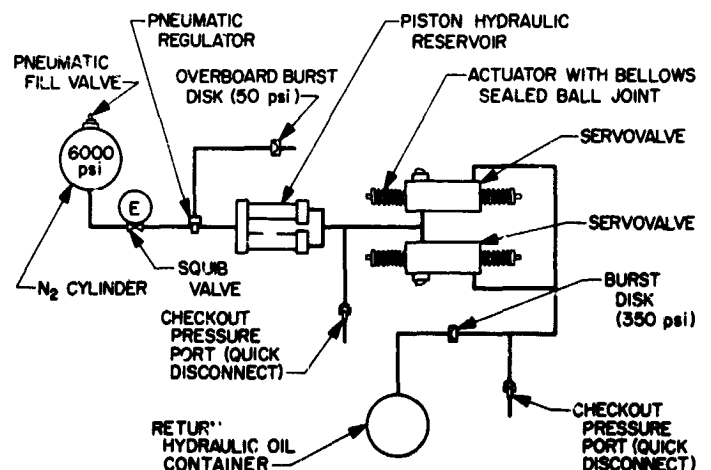


Fig. 14. Actuator system schematic

The weight of the bracketry required to fasten the auxiliary components of the servo-actuator system was not included in this weight analysis because much of it could possibly be provided by adjacent spacecraft structure. It is believed, however, that the weights of the servo-actuator components are conservative and will probably account for the greater part of the weight of the bracketry required if this is not the case.

It was necessary to remove an annular-shaped piece of propellant to allow the gimbaled nozzle to pivot freely. The weight of the propellant removed is 1.15 lb.

The ATS motor required modifications similar to those of the *Syncom* motor. Several trial designs of the Lockseal for the ATS motor resulted in high torques. Therefore, it was necessary to install the Lockseal inside the chamber to achieve lower torque. This configuration, which required the insertion of a conically shaped spacer between the attachment boss on the aft closure and the Lockseal, is shown in Fig. 12. The weights of components in the Lockseal adaptation on the ATS motor are tabulated in Table 6. Fig. 15 indicates the torque required as a function of the nozzle deflection angle.

Table 6. ATS motor weight

Motor weights	Original design, lb	Lockseal design, lb
Propellant	760.0	751.63
Total motor assembly	840.7	861.02
Weight change		
Nozzle insert	2.33	2.33
Nozzle body	38.87	33.79
Nozzle ring	2.61	2.04
Chamber aft closure	4.83	6.29
Chamber attachment ring	—	6.84
Insulation	—	2.95
Lockseal	—	4.00
Screws and washers	0.29	0.29
Actuator attachment ring	—	0.94
Servo actuator system ^a	—	18.15
Propellant weight change	—	(-8.37)
Total weights	46.93	69.25
Net difference	—	+20.32
Mass fraction	0.904	0.873

^aIncludes: N₂ container, squib valve, regulator, hydraulic reservoir, 2 actuators, checkout ports, burst diaphragms, hydraulic lines and fittings, hydraulic fluid, and collection reservoir.

The nozzle weight is reduced by 3 lb because it is exposed to hot gas on one surface only. The ring connecting the Lockseal to the nozzle is slightly lighter than the original design; however, this weight is offset by the weight of the additional conical ring which joins the Lockseal to the chamber aft closure. The 6Al 4V titanium attachment ring weighs approximately 6.85 lb. The chamber aft seal is effected, as in the *Syncom* motor, by an O-ring between the attachment ring and the inside surface of the aft closure boss. The Lockseal is attached to the aft chamber by 36 screws in the flange, as in the original design.

The actuators are mounted between the aft closure and nozzle as in the *Syncom* motor. The aft closure must be thickened from 0.033 in. to 0.053 in. to withstand the additional forces developed by the actuator and the Lockseal. This thickness was estimated by considering the aft closure as a flat plate with fixed edges and acted upon by a moment at the center. The increase in thickness results in a weight increase of from 4.83 to 6.29 lb. The weight computed from this model results in conservative values for the thickness and, therefore, for the weight of the component.

3. Conclusions

The *Syncom* and ATS rocket motor designs can be modified to accept a Lockseal thrust vector control system.

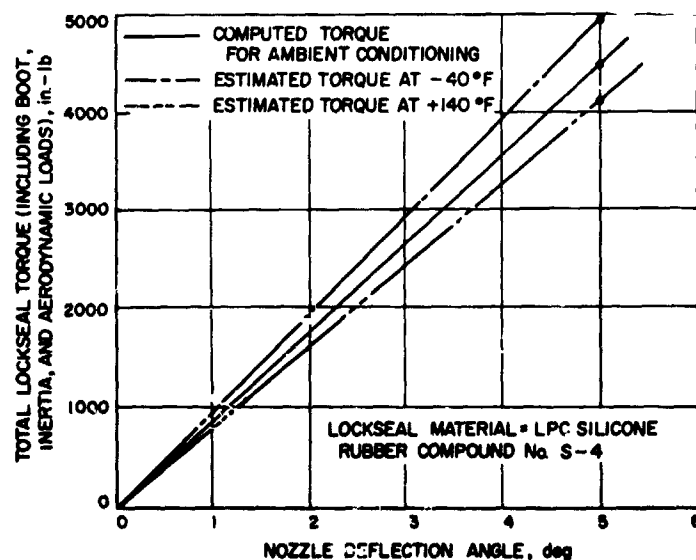


Fig. 15. Total Lockseal torque versus nozzle deflection angle for ATS rocket motor design

The weight of the servo-actuator system, which is essentially constant, causes the major increase in the motor inert weight. This results in a significant decrease in the mass fraction for the *Syncom* motor.

To install the Lockseal, the propellant charge must be reduced to provide clearance for the movable components. The *Syncom* and the *ATS* rocket nozzles had to be

moved rearward by 1.25 in. and 0.62 in. respectively so as to minimize the increase in weight. This rearward shift changes the center of gravity and the moment of inertia of the rocket engines. Existing hardware, except the aft end, may be used with this thrust vector control adaptation for both units. The aft ends, however, require replacement with items having thicker walls to withstand increased forces and torques.

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XI. Polymer Research

PROPULSION DIVISION

N67-29152

A. Permeability of Polymer Films and the Solubility Parameter, R. F. Landel

In the search for bladder materials, rather scant attention has been paid to some pertinent facets of elastomer behavior. The bladders should be reasonably flexible and have a low permeability. In general, these are incompatible requirements. Permeability P is defined as the rate of mass flow/cm²/cm thickness/unit time. It is actually the product of the diffusion coefficient D and the solubility S of the vapor in the material

$$P = DS \quad (1)$$

The quantities S and D can be examined separately.

The diffusion coefficient is determined by the internal mobility of the chains. Small molecules thread their way between the chains just as one solvent diffuses through another. The only way to reduce the diffusant mobility by modifying the polymer structure is to reduce the chain mobility, e.g., by raising the glass temperature or increasing the crosslinking in the elastomer. But both of these make the material harder and more leatherlike. Thus, any polymer whose flexibility originates in coop-

erative modes of motion of essentially liquid-like character will of necessity be permeable. Any search for new materials whose flexibility stems from this type of molecular motion should be rejected until some new principle of molecular dynamics is discovered to provide a theoretical basis for the search.

Fillers can reduce the diffusion rate by increasing the path length. Effectively, the sample has a smaller cross-sectional area for the vapor to diffuse through. Thus, leaf-like fillers such as mica and flake glass can be expected to be more beneficial on a volume per volume basis than more compact shapes. Van Amerongen (Ref. 1) found the diffusion of N₂ in natural rubber to be scarcely affected by various carbon blacks but was reduced by a factor of 4.9 by the inclusion of 20% by weight of mica. The reduction factors for H₂, O₂, and CO₂ were 4.5, 5.0, and 7.7, respectively.

Crystallites will also act as fillers in this context; therefore a partially crystallized polymer would be better than an amorphous system and a highly crystalline one better than a partially crystalline one. Thus, low density polyethylene, 65% crystalline, is eight times as permeable to N₂ or H₂O as high-density polyethylene, 85% crystalline (Ref. 2).

Even when some permeability is acceptable, however, there are rational guidelines to material selection based on S . The solubility of a substance in a polymer is determined by the intermolecular forces. In the absence of actual chemical attack by the solvent, these can be characterized by the solubility parameter δ . This quantity

is a measure of the energy required to separate a pair of molecules or chain segments in the liquid state. When $\delta_{\text{solvent}} = \delta_{\text{polymer}}$, the polymer will dissolve if it is not crosslinked; if it is crosslinked, it will swell and attain its maximum swelling. Thus, to prevent swelling, which enhances the diffusion constant, one should strive for the greatest possible mismatch in δ .

Table 1. Calculation of δ

Equations (Refs. 3, 4, and 5).		
(1) $\delta = \left[\frac{25\gamma}{(V_m^L)^{1/2}} \right]^{1/2}$		
where		
V_m^L = molar volume of liquid at 25°C		
γ = surface tension at 25°C, dynes/cm ²		
(2) $\delta = \left(\frac{\Delta H_v^* - (C_p^* - C_p^L)(T_b - T) - RT}{V_m^L} \right)^{1/2}$		
(3) $\delta = \left(\frac{\Delta H_v^* - RT}{V_m^L} \right)^{1/2}$ i.e., Eq. (2) without the heat capacity term.		
where		
ΔH_v^* = molar heat of vaporization at the boiling point		
ΔC_p = difference in molar heat capacities of vapor and liquid at the temperature T .		
V_m^L = molar volume of the liquid at temperature T , and T_b = boiling point, °K.		
Values of appropriate parameters		
Parameter	N ₂ H ₄	N ₂ O ₄
Heat of vaporization	9600 cal/g mole at 112°C	9110 cal/g mole at 21°C
Boiling point	114°C	18°C
C_p , liquid	25.67 cal/g mole/°C at 114°C	33.74 cal/g mole/°C at 18°C
C_p , vapor	13.96 cal/g mole/°C at 118°C	10.3 cal/g mole/°C at 77°F (25°C) (NO ₂ , 1.8 cal/g mole/°C at 18°C)
Surface tension	66.67 dynes/cm at 25°C	27.5 dynes/cm at 20°C
Density		
Anhydrous	1.007 g/cm ³	1.491 g/cm ³
Commercial	1.01	
Molecular weight	32.05 g/mole	92.02 g/mole
Molar volume	32.1 cm ³ /mole	61.8 cm ³ /mole

The value of δ for a polymer may not be known explicitly to a supplier of a given kind of polymer. However, he will usually have some indication of the swelling behavior in several common solvents, and the value of δ for these is generally known or easily estimated. Burrell and Immergut list the values for more than 240 solvents (Ref. 3). If the polymer swells in a solvent whose δ is close to that of the liquid to be retained in the bladder, it will probably be a poor choice and can be discarded without further testing.

Because the solubility aspects seem to have been neglected, we have calculated approximate values of δ for N₂H₄ and N₂O₄ from the equation and data given in Table 1. The results, given in Table 2, are not in particularly good agreement and are unusually high, compared to common solvents. On the other hand, the value for ammonia is 16.3, quite comparable to that calculated for hydrazine. Moreover, a plot of the values for low molecular weight mono amines as a function of the number of carbon atoms, when extrapolated to zero carbon content, also indicates that a very high value for N₂H₄ is to be expected (Fig. 1). Unfortunately, the δ values for the corresponding diamine series is not available, otherwise a similar plot could be made to obtain a more realistic confirmation of the calculated value.

To illustrate that such a plot is valid, and that end groups should contribute in a regular fashion, the δ values for *n*-paraffins and *n*-alcohols are also shown in Fig. 1. In the former case, the extrapolation shows that the expected value for hydrogen should be less than 4. The actual value is 3. From the curve for the alcohols, the

Table 2. Approximate solubility parameters for N₂H₄ and N₂O₄ at 25°C in (cal/cm³)^{1/2}

Reference (from Table 1)	N ₂ H ₄	N ₂ O ₄
Eq. (1)	15	11
Eq. (2)	18.1	11.3 (at 19°C, the b.p.)
Eq. (3)	16.8	

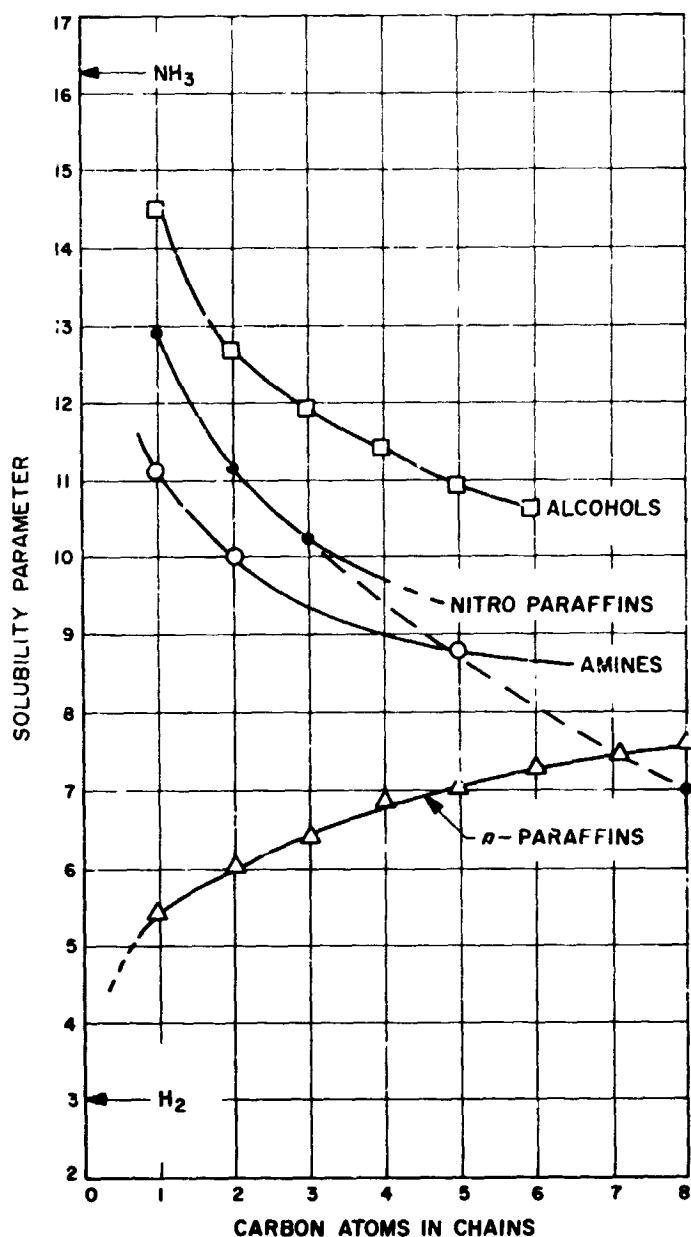


Fig. 1. Solubility parameters as a function of carbon content of *n*-paraffins and their primary hydroxy, amino, and nitro derivatives

expected value for water would be > 17 , and the actual value is 23.4.

Fig. 1 also contains data for the paraffinic nitro derivatives. (The value for 1-nitrooctane appears to be too low, since it lies below that of pure *n*-octane.) Extrapolation of the curve to zero carbon content indicates that the expected magnitude of δ for N_2O_4 is greater than 13, instead of the calculated value of 11 to 11.8. On the other hand, it should be realized that the structure of the NO_2 radical in N_2O_4 is different from that in nitro-compounds—the

main value of this plot, in the case of N_2O_4 , is a confirmation of the high calculated value. Hence, the values of 17.0 for N_2H_4 and 11.5 for N_2O_4 appear to be reasonable after all.

Table 3 gives a comparison of δ for several materials and the behavior of these with N_2H_4 , N_2O_4 , and 50/50 UDMH/ N_2H_4 .¹ The values of δ for UDMH have not yet been calculated. Thus, considering the N_2H_4 and UDMH/ N_2H_4 , it can be seen that the rubbers with low δ values tend to be more resistant. This suggests that nitroso rubber (a copolymer of tetrafluoroethylene, the monomer of Teflon, and trifluoronitrosomethane) with a δ value of 5.2 should be especially resistant to both N_2H_4 and N_2O_4 . In fact, it is not; nitroso rubber reacts with hydrazine.

Even amines, which are much weaker bases than hydrazine, will attack this polymer (and Viton, too) and can be used as crosslinking agents. This illustrates a weakness of the use of the solubility parameter as a screening tool—a large mismatch in δ is a necessary but not sufficient condition for the utility of an elastomer.

In this sense these elastomers are similar to the unsaturated hydrocarbon rubbers in the presence of N_2O_4 —according to their δ values they should be resistant to swelling, but in actual fact they are degraded. Such chemical reaction is quite a different phenomenon from simple dissolution and one in which δ values play no direct role. Similarly, both the Viton and nitroso gumstocks are stable in the presence of N_2O_4 , but the currently used amine curing systems introduce hydrocarbon-containing crosslinks and these, not the rubber, are oxidatively unstable. The recent chromium crosslinking method was created to avoid this problem.

There are some swelling anomalies in Table 3. Neither the butyl nor the nitroso rubbers, for example, ought to swell in N_2O_4 , according to the δ values, but they do. While some of the swelling may be the result of attack at the crosslinks, the polyisobutylene has no crosslinks at all and so should not have liquefied (presumably it was not chemically attacked). The reason for this may be that δ is also a function of the polarity and hydrogen-bonding capacity of the liquid. Thus, solvents should also be classified according to polarity, and if a true measure of the solvent resistance of a polymer is to be obtained, it should be tested with different classes of solvents. It appears,

¹The data in these tables were collected from various sources by D. Lawson or directly measured by S. P. Vango of JPL.

Table 3. Comparison of solubility parameters and resistance of rubbers to N_2H_4 , N_2O_4 , and 50/50 UDMH/ N_2H_4

Polymer	δ_s (cal/cm ³) ^{1/2}	N_2H_4	N_2O_4		50/50 UDMH/ N_2H_4	
		Swelling after RT ^b for 3 days; appearance after vacuum drying for 60 hr	Short-term exposure ^a	Long-term exposure	Short-term exposure	Long-term exposure
Poly(isobutylene)	8.1	0.7%; unaffected	70–80°F for 2 hr; swells 60%; liquefied in 16 hr ^c			
Butyl rubber (resin cure)	~8.1		RT ^b for 2 hr; about 60% swell			
Natural rubber (crepe)	8.35	17%; color change from cream to white	70–80°F for 30 sec; broke up RT for 16 hr; liquefied ^c			
Poly(butadiene) rubber	8.45	0%; unaffected in some cases; precipitate extracted in others (effect of different curing agents)	RT for 16 hr; decomposed ^c		160°F for 1 day; precipitate extracted. Swelling not reported	160°F for 30 days; up to 30% swell, 10–80% loss of tensile strength depending on formulation
Styrene-butadiene (23.5% styrene) rubber	8.60	7%; antioxidant dissolved	RT for 16 hr; liquefied ^c			
Poly(styrene)	9.2	0%; no change	RT for 3 hr; dark liquid ^c			
Neoprene rubber, GNA	9.25	20% swell; no change	70–80°F for 4 hr; decomposed RT for 16 hr; decomposed (dissolved?) ^c		70–80°F for 2 days; softens 55–60°F for 30 days; swells, plasticizer extracted	70–80°F for 9 days; softens
Thiokol	9.0–9.4		RT for 16 hr; liquefied ^c			
Buna N	9.4–9.6	7%; some reaction	55–60°F; dissolved < 1 day RT for 16 hr; decomposed ^c		70–80°F for 1 day; results not clear	
Poly(vinyl chloride), plasticized				55–60°F for 30 days; plasticizer is extracted 55–60°F for 90 days; softens 160°F for 1 day; no change 160°F for 7 days; crumbles	70–80°F for 1 day; discolored, softened	
Poly(vinyl chloride), unplasticized	9.6	10%; changed from white to yellow	RT for 16 hr; changed color ^c	70–80°F for 42 days; hardens slightly ^c		

^a Liquid phase exposure, except where noted.

^b RT means room temperature.

^c Gas phase exposure.

Table 3. (contd)

Polymer	δ , (cal/cm ³) ^{1/2}	N ₂ H	N ₂ O ₄		50/50 UDMH/N ₂ H	
		Swelling after RT ^b for 3 days; appearance after vacuum drying for 60 hr	Short-term exposure ^a	Long-term exposure	Short-term exposure	Long-term exposure
Nitroso rubber	5.2	Reacts	70–80°F for 7 days: swells 50–175%, depending on formulation. Elastomeric properties degraded			
Viton A	9.3	Reacts	55–60°F for 1 day; swells	70–80°F for 30 days; decomposes		
Viton B		Reacts	70–80°F for 1 day; swells, then shrinks	55–60°F for 30 days; swells		

from a cursory glance at the literature, that this has not been done for nitroso rubbers, nor have N₂H₄, UDMH, N₂O₄, or diborane been evaluated as to their polarity class. This ought to be done for a better understanding of the mode of attack by these liquids on candidate bladder materials.

Teflon is a material whose δ value is also very low, 6.2 (Ref. 6). It is also crystalline, which ought to hinder diffusion; the combination of these two factors ought to impart excellent properties to it. However, Teflon products are formed by sintering a powder. Unfortunately the interstices of such sintered materials never close completely, leaving pores which cause the high permeability observed.

It has, therefore, been suggested that simple plugging of the pores would reduce the permeability. A surface plugging, obtained by wiping on a film of silicone grease, has not been found to reduce the permeability,² though a silicone would not be expected to impede the diffusion materially. A more effective way of plugging the pores might be to precipitate a material in the pores, just as early osmometer membranes were made by precipitating potassium ferricyanide in clay cups — the two reactants were placed on either side of the cup, they diffused through it and reacted to form a precipitate when they met. The precipitate then became the semipermeable membrane. By leaving the cup in the reactants for long periods of time, the membrane could be made quite impenetrable. This may not be practicable in the present case, since the membrane would have to be semiflexible.

²Measurements by S. P. Vango, JPL.

B. Speculation on the Nature of the Virtually Crosslinked Block Copolymers, R. F. Fedors

1. Introduction

Recently a very interesting class of elastomeric block copolymers composed of alternate segments of styrene and butadiene has been introduced commercially.³ The outstanding feature of these materials depends on the fact that in the absence of any chemical crosslinking, they are capable of mimicking, in a certain temperature range, the behavior of typical chemically crosslinked elastomers. This effect will be described by the term "virtual crosslinking." These materials can be processed and fabricated (e.g., molded, extruded, etc.) at elevated temperatures on normal rubber machinery in the usual fashion. As the temperature is reduced, after the fabrication step, a temperature is reached below which the polymer will function as a virtually crosslinked material.

These polymers are not unique in their ability to function as virtually crosslinked elastomers, plasticized poly(vinyl chloride) (Ref. 6), to cite an example, also behaves in this manner. However, the block copolymers would appear to be of special interest because of their possible unique versatility, i.e., the possibility of varying both the composition and structure of the block segments. It will be evident from this short introduction, that the block copolymer has wide possible application including perhaps that of a binder material for solid propellant (SPS 37-14, Vol. IV, p. 107). In addition, the mere existence of the virtual crosslink effect is of intense interest

³Shell Chemical Co., Thermolastic materials.

in itself, both from a theoretical and experimental standpoint.

Very little information has been released concerning the composition or structure of these block copolymers, and so no mechanism has as yet been published to account for the virtual crosslink effect. For these reasons, it was thought to be of interest to present here some preliminary comments on a possible model for the virtual crosslink effect and some of its consequences.

2. Requirements for the Virtual Crosslink Effect

Although little data have been published, it is known that the commercial polymers are triblocks of the form ABA where A denotes a segment having a relatively high glass transition temperature T_g (such as styrene), and B denotes a segment having a relatively low T_g value (such as butadiene).

An idea of the mechanism of the virtual crosslink effect for these materials may be obtained by considering the nature of the processes occurring during fabrication. When the processing temperature is above T_g of segment A, and thus automatically above T_g of the B segment also, the polymer will flow under stress in a manner not unlike that to be expected for a mixture of the corresponding homopolymers.

Furthermore, if the A and B segments are compatible or mutually soluble, the block copolymer will consist of a single thermodynamic phase over the temperature range of interest. On the other hand, if A and B segments are incompatible or mutually insoluble, a two-phase system will result in which the A and B segments will tend to aggregate to form domains whose size will depend to a first approximation on molecular weight. It may be noted here that polymer compatibility is a function of molecular weight, temperature, and the extent of intermolecular interaction. It turns out that polymer-polymer insolubility is far more common than is solubility (Ref. 7).

As the temperature is decreased to values below the expected T_g of segment A, in the case of a single phase, no abrupt changes in properties are expected, since the single phase has only one T_g value. This kind of polymer will have the ability to flow at all temperatures down to the T_g of the mixture or single phase, a value which will be intermediate between the T_g values of the components. On this basis, compatible segments are not predicted to lead to the occurrence of virtual crosslinking. It is of interest here that another elastomer composed of 25 mole % styrene and the

remainder of butadiene and described as a block copolymer does not display virtual crosslinking (Ref. 8). Evidently the segment lengths are too short to permit mutual insolubility.

However, the case of the two-phase (or more generally, multi-phase) system will be characterized by two T_g values (or multiple T_g values). In this situation, as the temperature is reduced to a value near T_g of the A segment, these segments will undergo a second-order transition and will now exist as a high modulus glass. For this temperature and temperatures down to T_g of the B segment, these glassy domains are predicted to function as crosslink sites for B segments in polymers of the type ABA. That is, since each B segment is capped at each end by segments which are in the glassy state, the possibility of these domains functioning as crosslink sites is present. The "functionality" of the crosslink will depend on the domain size. Thus, the functionality will be numerically equal to the number of A segments present in the domain. To the extent that the possibility exists for varying the A domain size either by mechanical or thermal treatment, then it should be possible to vary crosslink functionality.

It is thus predicted that diblocks of the type AB and inverted triblocks of the type BAB cannot function as virtually crosslinked elastomers because in each of these cases, only one end of the B segment is attached to a possible crosslink site, i.e., to the glass-forming A segment. Support for this contention is provided by the fact that elastomeric materials prepared from mechanically blending of high molecular weight homopolymers of butadiene and styrene do not possess the virtual crosslink property.

3. Filler Effects

Since incompatibility implies that the number of contacts between A and B segments will tend to be a minimum, it is expected that the A and B segments will tend to exist in A-rich and B-rich domains. The size, geometric shape and number of these domains will be a strong function of both the mechanical and thermal history. For temperatures below T_g of the A segment, however, these domains should be relatively insensitive to temperature, since any change in size would necessarily depend on diffusion of A in the glassy state, which is not likely to occur in reasonable time periods. Because of the great difference in the modulus of a glass as compared to the modulus of an elastomer, most of the deformation in the temperature range for which the virtual crosslink effect is present, will take place in the B segment domains. Thus, the A domains should function in a manner similar to that of small-particle-size particulate fillers such as carbon black. These

latter filler effects should not be observable in the non-crosslinkable *AB*-type and the inverted *BAB*-type block copolymers, however, and should be manifested as an increase in viscosity. Thus, the glass domains are thought to perform two independent functions: (1) they act as virtual crosslinks for the *B* segments, and (2) they behave as small, nonagglomerated particles.

4. Criteria for Block Segments

The segment properties for copolymers of the type *ABA* which were presumed to influence the behavior of these materials have been discussed in a qualitative fashion. In what follows, an attempt will be made to discuss in a more quantitative manner several of the variables which determine the behavior of the system, such as segment compatibility, molecular weight and volume ratios of *A* to *B* segments.

5. Conditions Required for Insolubility

In order to study some of the variables which determine solubility, it is assumed that the liquid lattice theory developed by Flory and Huggins (Ref. 7) is valid. It is recognized that although the theory is not, in general, in quantitative accord with experimental results, it is also true that the theory provides a satisfactory qualitative explanation for the behavior of polymer solutions. Use of more elaborate theory, while narrowing the divergence between theory and experiment, also loses simplicity and hence is not warranted at the time.

The change in chemical potential, $\Delta\mu_1$, of component 1 in solution relative to the pure component can be written as (Ref. 7):

$$\Delta\mu_1 = RT \left[\ln(1 - v_2) + \left(1 - \frac{V_1}{V_2}\right) v_2 + \chi_{12} v_2^2 \right] \quad (1)$$

and likewise for component 2,

$$\Delta\mu_2 = RT \left[\ln v_2 + \left(1 - \frac{V_1}{V_2}\right) (1 - v_2) + \chi_{12} \frac{V_2}{V_1} (1 - v_2)^2 \right] \quad (2)$$

where R is the gas constant, T the absolute temperature, v_2 is the volume fraction of component 2, V_1 is the molar volume of component 1, V_2 is the molar volume of component 2 and χ_{12} is the interaction coefficient between components 1 and 2. This latter quantity is related to the heat of mixing, i.e., the increment of energy associated with the formation of intermolecular contacts between

components 1 and 2 in solution relative to the pure components. In using Eq. (1) and (2), the fact that component 1 and component 2 belong to the same molecule is ignored.

An inspection of Eq. (1) and (2) demonstrates that the effect of chemical structure of the polymer influences $\Delta\mu_1$, and $\Delta\mu_2$ only through the quantity χ_{12} , which can be written as (Ref. 9):

$$\chi_{12} = \frac{M_1}{\rho_1 RT} (\delta_1 - \delta_2)^2 = \frac{M_1 (\Delta\delta)^2}{\rho_1 RT} \quad (3)$$

where δ_1 and δ_2 are the cohesive energy densities of components 1 and 2 respectively, ρ_1 and ρ_2 are the densities of components 1 and 2 respectively, and M_1 is the molecular weight of component 1. The equation shows that χ_{12} is directly proportional to the molecular weight of component 1 and to the square of the cohesive energy difference. The δ value is a convenient measure for the intensity of intermolecular interaction in the pure component and as such can be obtained in several ways, for example, for low molecular weight materials from their heats of vaporizations (Ref. 10). Several compilations of values for both low molecular weight and high molecular weight materials are available (Refs. 10, 11, and 12). There is also a useful method due to Small for estimating cohesive energy densities of polymers from a table of additive constants requiring only a knowledge of the polymer structure (Ref. 9). In any event, a knowledge of δ values along with Eq. (1), (2), and (3) are sufficient to completely determine the phase relationships for a two-component system. The predicted phase diagram for a two-component system is shown in Fig. 2 in a plot of the reciprocal χ_{12} versus v_1 , for several values of molar ratio, V_1/V_2 . It is evident that the curves for fixed values of V_1/V_2 become increasingly asymmetrical as the ratio increases, i.e., as the disparity in the molar volumes of the two components increases. At a fixed difference in the cohesive energy densities, $\Delta\delta$, decreasing the molecular weight M_1 increases polymer compatibility, while at constant molecular weight, M_1 , compatibility increases as $\Delta\delta$ decreases, i.e., as the components become more similar in chemical structure.

It is evident from Fig. 2 that absolute insolubility will only occur when either M_1 or $\Delta\delta$ become infinite. However, as a practical matter, it is probable that some small measure of solubility could be tolerated while still maintaining reasonable mechanical properties. When $RT\rho_1/M_1(\Delta\delta)^2$ has a value of about 0.1 or less the volume fraction of the *A* segment soluble in the *B* segment will be 0.01 or less and if it is assumed that this measure of solubility is negligible, then values of approximate minimum molecular

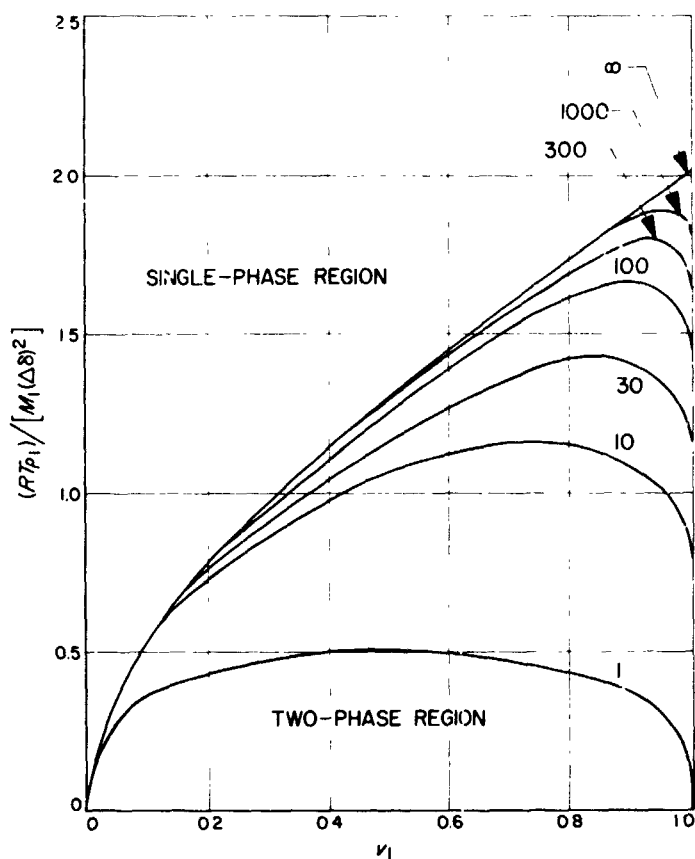


Fig. 2. Phase diagram for a two-component system. Numbers correspond to the ratio V_1/V_2

weights for the segments can be calculated. As a practical example, consider segment A to be styrene and segment B to be butadiene, and take styrene as component 1 and butadiene as component 2. For styrene, $\delta_1 = 8.83$ and for butadiene, $\delta_2 = 8.44$ which leads to a value of $\Delta\delta = 0.39$ (Ref. 10). Using Eq. (3), a minimum estimate for M_1 is obtained from,

$$M_1 (\text{minimum}) \approx \frac{(1.04)(2)(373)}{(0.15)(0.1)} = 5000 \quad (4)$$

In Eq. (4) T was set equal to 373°K, the approximate T_g for polystyrene and ρ_1 was taken as 1.04 g/cm³. The minimum value of $M_1 = 5000$ implies that the minimum molecular weight of each styrene segment is of the order of 2500 for an ABA-type block copolymer. An estimate of the minimum molecular weight for the butadiene segment obtained in a analogous manner, is about 6000. Hence, the predicted lowest molecular weight for the ABA system which guarantees negligible solubility is $A = 2500$ and $B = 6000$. It should be pointed out that the minimum molecular weight numbers are subjected to considerable

uncertainty because the individual δ values are not precisely known.

If mutual insolubility were the only factor involved in determining the suitability of a given monomer pair, then Eq. (4) would provide sufficient information. However, also to be considered is the requirement of a sufficiently high T_g for segment A so that ABA triblock will function as a virtually crosslinked elastomer in a useful temperature range.

6. Influence of Molecular Weight on T_g

In Fig. 3 are shown data for the glass transition temperature of fractionated polystyrene as a function of the molecular weight. The filled circles represent data from Ref. 13 while the unfilled circles represent data from Ref. 14. It is evident from Fig. 3 that although there is a very rapid increase in T_g with molecular weight in the low molecular weight range, i.e., roughly for molecular weights up to 10 to 12,000, for higher molecular weights, the relative increase in T_g is rather small. For example, with a molecular weight of 12,000, $T_g = 89^\circ\text{C}$, whereas for a molecular weight of 40,000, $T_g = 97^\circ\text{C}$ —an increase in only 8°C for a three-fold increase in molecular weight. In the case of polystyrene homopolymer, above a molecular weight of about 40,000 the bulk viscosity increases as the 3.4 power of molecular weight (Ref. 15). Since bulk viscosity is an important processing parameter, with lower bulk viscosities implying lower processing temperatures, the most suitable molecular weight would be the lowest

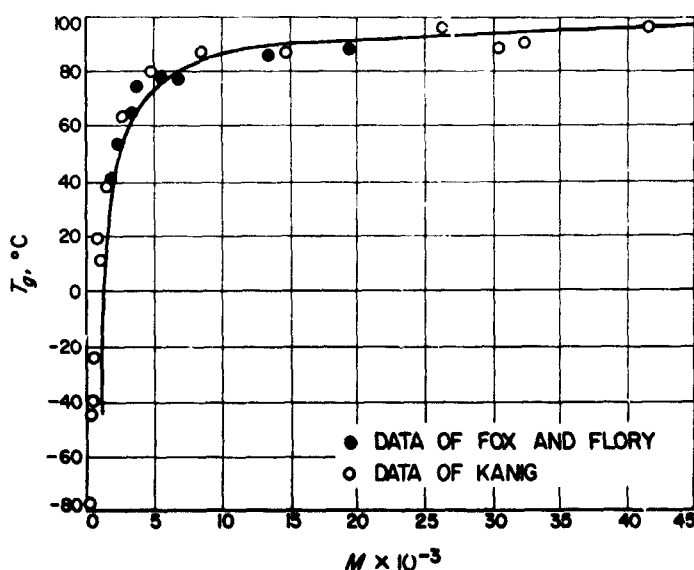


Fig. 3. Dependence of glass transition temperature on molecular weight for fractionated polystyrene

value which provides a sufficiently high T_g . For the case polystyrene, a molecular weight of 12,000 and a T_g in the undeformed state of 89°C would appear suitable. This molecular weight is to be contrasted to a minimum molecular weight value calculated on the basis of solubility of 2500 which would correspond to a T_g value of about 50°C. A gain of about 40°C in T_g can be obtained by increasing molecular weight from 2500 to 12,000.

For other polymer types, the necessary T_g versus molecular weight data may not be as extensive as is the data on polystyrene. In this case, estimates for T_g can be obtained by using any one of several available relationships which relate T_g to molecular weight. For example, using free volume considerations, Bueche (Ref. 16) has derived the following equation:

$$T_g = T_g(\infty) - \frac{K}{M} \quad (5)$$

where $T_g(\infty)$ is the glass transition temperature for polymer of infinite molecular weight, and K is a constant for a given polymer. If T_g is known for two molecular weights, K and $T_g(\infty)$ can be evaluated and the results used to calculate T_g for other molecular weight values. Eq. (5) is not expected to be valid for low values of molecular weight. As an indication of the usefulness of the equation, the full curve shown in Fig. 3 is the prediction of Eq. (5) with $K = 10^5$ moles/g and for $M > 4000$, the fit to the experimental data is excellent.

In instances where bulk viscosity plays a decisive role, a decrease in viscosity can be obtained by employing a glass-former having a higher T_g value. In this case, a given T_g value, say 90°C, can be obtained using a lower molecular weight segment. For example, $T_g(\infty)$ for α -methylstyrene is about 170°C. Assuming Eq. (5) holds using the same value for K , the following expression can be obtained:

$$T_{g1}(\infty) - T_{g2}(\infty) = K \left(\frac{1}{M_1} - \frac{1}{M_2} \right) \quad (6)$$

Using the appropriate values, i.e., $T_{g1}(\infty) = 100^\circ\text{C}$, $T_{g2}(\infty) = 170^\circ\text{C}$, and $M_1 = 12,000$, M_2 , the molecular weight of the equivalent α -methylstyrene segment, turns out to be roughly 2000, or a factor of 6 lower than the value obtained using polystyrene. There is a limit to how far the molecular weight can be reduced by this procedure since in any case, the molecular weight must be

at least as high as the minimum value necessary to ensure insolubility. As will be indicated below, a lower value of molecular weight for the A segment implies a lower value for the B segment also.

7. Influence of Strain on T_g

Another important factor which bears on the question of the appropriate T_g value for the glassy segment is the question of the effect of strain on T_g . Based on free volume considerations, Ferry has derived an equation for the shift in time scale due to an imposed strain (Ref. 17):

$$\log a_\epsilon = - \frac{\frac{\epsilon}{2.303f_0}}{\frac{\beta f_0}{\beta(1-2\mu)} + \epsilon} \quad (7)$$

where a_ϵ is the shift factor, ϵ is the strain, f_0 is the fractional free volume at $\epsilon = 0$, β_f is the compressibility of free volume, β is the total compressibility, and μ is Poisson's ratio. Since μ is less than or equal to $1/2$, Eq. (7) predicts that an imposed strain will always decrease the effective T_g . In order to estimate the reduction in T_g , a_ϵ is set to equal the time-temperature shift factor a_T , given by:

$$\log a_T = - \frac{\frac{T - T_g}{2.303f_g}}{\frac{f_g}{\alpha_f} + T - T_g} \quad (8)$$

Setting $\log a_\epsilon = \log a_T$, there is obtained:

$$\left(\frac{\beta_f}{\beta} \right) \frac{f_0^2}{(1-2\mu)\epsilon} + f_0 = \frac{f_g^2}{\alpha_f(T - T_g)} + f_g \quad (9)$$

where f_g is the free volume at T_g when $\epsilon = 0$, and α_f is the expansion coefficient of the free volume. Assuming $(\beta_f/\beta) \approx 1$, which implies that compressibility is due only to the variation of free volume with pressure, and since the temperature range of interest is $T \approx T_g$, f_0 can be taken equal to f_g , which yields:

$$-\Delta T_g \approx \frac{(1-2\mu)\epsilon}{\alpha_f} \quad (10)$$

For glassy polystyrene, $\mu = 0.33$ and $\alpha_1 \approx 4 \times 10^{-4}$ per $^{\circ}\text{C}$ (Ref. 18), hence,

$$-\Delta T_g = 850 \epsilon \quad (11)$$

which predicts a decrease in T_g of 8°C for each 1% strain imposed on polystyrene. Although the small strain modulus of a glass is roughly 10^4 to 10^5 times greater than the small strain modulus of an elastomer, it is not inconceivable that the glass would undergo strains of a few percent when the overall strains on the system are of the order of 1000%. This is especially true if the glass domains behave as filler particles, because in that capacity the domains could function as stress concentrators. Hence, strain could cause a reduction of about 20 to 30°C in the effective T_g of the glass. This consideration would imply that the T_g of the glassy segment must be designed to be about 20 to 30°C higher than the maximum temperature for which the polymer is designed to operate. Thus, for the case of polystyrene, with the molecular weight chosen as 12,000, the maximum temperature for which the polymer will function as a virtually crosslinked elastomer is 60 to 70°C .

8. Domains as Filler Particles—Molecular Weight Effects

The volume fraction of segment A, ϕ_A , can be shown to be:

$$\phi_A = \frac{2M_A \rho_B}{2M_A \rho_B + M_B \rho_A} \quad (12)$$

If segment A is insoluble in segment B, then ϕ_A should represent the volume fraction of "filler." If this analogy is valid, then a virtually crosslinked block copolymer can be compared in a formal fashion to "bound rubber," which is an insoluble gel composed of carbon black and rubber which is produced when active carbon blacks are mixed into rubber under the influence of high shearing forces. There is much evidence to show that in bound rubber the polymer chains are attached to the carbon black particle by means of chemical bonds (Ref. 19), and in this sense the carbon black particle might be considered as part of the polymer chain.

If the glassy domains function as filler in a fashion similar to carbon black, then experience with vulcanized rubber shows that the optimum volume fraction of filler should be about 0.25. Assuming this to be the case, then it is easy to demonstrate that the molecular weights of the A and B segments cannot be varied indiscriminately

while still maintaining useful elastomeric properties for the polymer. Eq. (12) shows that once ϕ_A is fixed, this automatically provides the relationship between M_A and M_B . For example, in the case of polystyrene, $M_A = 12,000$, $\rho_A = 1.04$, and $\rho_B = 0.96$ and taking $\phi_A = 0.25$, M_B is calculated to be about 83,000. Thus the total molecular weight of the ABA molecule would be about 110,000. For the case of α -methylstyrene, assuming $M_A = 3000$, the minimum molecular weight required to ensure insolubility, then M_B is calculated to be 21,000 or about 30,000 for the ABA molecule. It might be expected that a block copolymer with a molecular weight of 30,000 would be processible at far lower temperatures than one with a molecular weight of 110,000.

In this connection, Baranwal (Ref. 20) studied the effect of filler particle size on the stress-strain behavior of an SBR rubber using spherical polystyrene particles as filler. The particle sizes ranged from 357 to 1430 Å. He was able to show that both tensile strength and ultimate elongation increased with increasing volume fraction of polystyrene filler. Volume fractions of polystyrene in the range 0 to 0.25 were investigated. He was also able to study the effect of chemically linking the polystyrene filler to the rubber molecule. Although it is not clear from his data what effect chemically linking of the filler had on either the tensile strength or the ultimate elongation, nevertheless, the study shows that polystyrene can function as a "reinforcing" filler.

9. Domains as Filler Particles—Effect of Domain Size

It is of interest to attempt to determine an approximate size for the glass domain. In each phase, the chains or segments will tend to assume their unperturbed dimensions, for which the root-mean-square end-to-end distance $(\bar{r}_0^2)^{1/2}$ may be used as an appropriate measure of the size of a random coiling polymer chain. In using this measure of coil size, the tacit assumption is that the A and B segments behave as separate molecules. The values of $(\bar{r}_0^2)^{1/2}$ can be obtained experimentally in several ways, for example, from intrinsic viscosity measurements at the theta temperature. For polystyrene, $(\bar{r}_0^2)^{1/2} \approx 0.65 M_A^{1/2} \text{ Å}$ and for polybutadiene, $(\bar{r}_0^2)^{1/2} \approx M_B^{1/2} \text{ Å}$ (Ref. 21). These are the end-to-end chain distances to be expected in the bulk phase, i.e., in the absence of interaction between A and B segments. In the case of polystyrene, it will be assumed that this end-to-end distance, $(\bar{r}_0^2)^{1/2} \approx 0.65 M_A^{1/2} \text{ Å}$ characterizes the polystyrene coil at temperatures arbitrarily close to T_g . It has been demonstrated by Flory (Ref. 22) that chain segments of a randomly coiled molecule extend over dimensions equal to twice the $(\bar{r}_0^2)^{1/2}$ value.

Hence, the effective diameter of a coil can be taken as $2(\bar{r}_0^2)^{1/2}$. In the case of the block copolymer, if the glass domain is spherical in shape, then the maximum diameter should be approximately $4(\bar{r}_0^2)^{1/2} = 4(0.65)(12,000)^{1/2} \text{ \AA} \approx 300 \text{ \AA}$. This value for particle diameter is the same order of magnitude as the particle diameter for commercially available carbon blacks. For example, SAF black, diameter = 140 to 170 \AA; HAF black, 260 to 350 \AA in diameter; and FEF, 310 to 580 \AA in diameter.

In addition to the formation of spherical domains, it is likely that sheet-like or plate-like domains are also present. In this case, the maximum thickness of the sheet or plate will be roughly $4(\bar{r}_0^2)^{1/2} \approx 300 \text{ \AA}$, as was the case with the spherical domain. However, for the sheet, growth in the lateral directions should be possible so that lateral dimensions may be much larger than the thickness of 300 \AA. It is expected that there is likely to be a distribution in both size and geometrical shape for the glass domains. There will be a tendency to make the surface free energy of the domains a minimum which implies that the domains will tend to be as large as possible, subject to the condition that the domain be essentially a pure phase.

10. Domain Functionality

If the size of the domain is known, then the functionality of the crosslink can be obtained. For the case of the spherical domain,

$$f = \frac{V_{\text{domain}}}{V_{\text{segment}}} = \frac{\frac{4}{3} \pi [2(\bar{r}_0^2)^{1/2}]^3}{\frac{M_A}{N \rho_A}} = 57.6 M^{1/2} \quad (13)$$

or $f \approx 600$ for polystyrene having a molecular weight of 12,000. In Eq. (13), N is Avogadro's number. It is interesting to note that f is proportional to the square root of the molecular weight. For typical chemically crosslinked vulcanizates such as natural rubber cured with sulfur, the crosslink functionality is thought to be about 4. On this basis the calculated f value for the glass domain is exceptionally high, and this fact may have bearing on the behavior of the block copolymers. For example, it is not known what effect both a crosslink of finite size and one of high functionality have on mechanical properties.

In the above discussion it has been tacitly assumed that the segments were monodisperse, i.e., of uniform

molecular weight. In case this is not valid, i.e., the molecular weight is not uniform, then the block copolymer is expected to exhibit poorer mechanical properties. There are three cases to consider: (1) if the B segments were uniform but the A were not, then the lower molecular weight A segments, being soluble in the B segment, could not function as crosslink sites, and hence the fraction of the polymer which could support a load would be reduced; (2) if the A segments were uniform but the B were not, then the lower molecular weight B segments, being soluble in the A segment, would reduce the T_g of the A domains and thus reduce the maximum temperature for which the block would function as a crosslinked system; (3) if both the A and B segments were not uniform in molecular weight, then the above two factors would be operative simultaneously.

11. Application of Propellant Binders

The fact that properly designed block copolymers do not have to be chemically crosslinked make these materials of special interest for application as a binder material. Since the glassy domains function as filler, it is evident that for binder applications, ϕ_A should be as low as possible. However, the molecular weight of segment A is essentially fixed by the requirements of insolubility and sufficiently high T_g . According to Eq. (12), at fixed M_A the quantity ϕ_A can be decreased only by increasing M_B , the molecular weight of the B segment. However, increasing M_B at fixed M_A produces an increase in the weight of the ABA molecule which will tend to increase processing difficulties.

It can be shown that the relationship between the true volume fraction of filler ϕ_T , which assumes that both the glass domains and added oxidizer function as filler, and the apparent volume fraction of filler ϕ_{app} , which assumes that added oxidizer is the only filler present, can be written as:

$$\phi_T = \phi_{app}(1 - \phi_A) + \phi_A \quad (14)$$

Using Eq. (12) to express ϕ_A as a function of segment size, Eq. (14) can be written as

$$\phi_T = \phi_{app} + (1 - \phi_{app}) \frac{2M_A \rho_B}{2M_A \rho_B + M_B \rho_A} \quad (15)$$

Eq. (15) shows that ϕ_T is always greater than ϕ_{app} . (The two will be equal only when $M_A = 0$.) Furthermore, if ϕ_{app} is made equal to or approximately equal to ϕ_{max} , i.e., the maximum volume fraction of filler which the

binder can accommodate and still maintain a two-phase system. Then since $\phi_T > \phi_{mix}$, the system will necessarily be a three-phase system (binder, filler, and voids) and dewetting exists in the composite even before any deformation is allowed to occur.

If ϕ_{mix} is assumed to be about 0.70, and using $M_A = 12,000$ and $M_B = 83,000$, then Eq. (15) predicts that only 0.60 volume fraction of oxidizer can be added and still maintain a two-phase system.

12. Summary

A model for the virtual crosslink effect in triblocks of the type ABA, where A is a segment having a relatively high T_g value, and B is a segment having a relatively low T_g value has been presented. It is predicted that at least two requirements are necessary in order for the block copolymer to behave as a chemically vulcanized elastomer: (1) that the A and B segments be mutually insoluble, and (2) that the A segments have a higher T_g value than the maximum temperature at which the material will be expected to function. The size of the glass domain has been discussed along with the suggestion that the glass domains fulfill two functions: (1) as crosslinks and (2) as filler particles.

C. Use of the Trifluoromethyl Group as an F^{19} NMR Probe in Polymeric Systems, D. D. Lawson and J. D. Ingham

Since the chemical reactivity and mechanical behavior of polymers are determined by their molecular structure, methods of readily determining this structure, particularly for the most reactive sites of the polymer, are required. For prepolymers applicable as solid propellant or spacecraft materials the most important reactive sites are the end-groups, which influence curing and degradative reactions, and any chain or sidechain unsaturation, particularly in polybutadienes, which affects mechanical properties, processing, and stability. Although high-resolution proton magnetic resonance spectroscopy is applicable to the structure determination of many organic compounds, the multiplicity of protons in the different structural environments in prepolymers often results in extremely complex spectra, usually with overlapping signals that are difficult or impossible to interpret.

The methods described in this report involve the chemical insertion of a moiety containing fluorine at the relevant reactive sites in a polymeric system. The dual

effect is to avoid the broad signals that are normally observed in proton polymer spectra and to gain the increased chemical shift range of fluorine resonances. Although the chemical shifts, or positions of either proton or fluorine NMR signals, relative to appropriate internal standards, depend on the surrounding magnetic environment and chemical structure, the separation of fluorine resonances is generally greater than corresponding proton signals by a factor of ten or more. The logical NMR probe is the trifluoromethyl group, because it has three equivalent fluorines which generally yield a single narrow line for each structural environment. In previous work we described the introduction of the trifluoromethyl moiety in the form of the trifluoroacetyl group in poly(alkylene oxides) to determine the structure and configuration of terminal hydroxyl groups present (Ref. 23). This has been extended by the use of trifluoromethanesulfonyl chloride (I) which yields the sulfonate esters of the corresponding diols (SPS 37-40, Vol. IV, pp. 93-95). The addition of sulfonyl chlorides to olefinic double bonds is a well-known and interesting reaction that we have also considered as a method for introducing the trifluoromethyl group as a probe for unsaturated systems. Because of the complex products obtained with some model alkenes and (I) it was felt that a probe which yielded simple products was more desirable. It appears that trifluorothioacetic acid (II) reacts with double bonds by direct addition in a relatively straightforward manner to yield derivatives that give F^{19} spectra which are sensitive to their structural environment.

1. Reaction of Trifluoromethanesulfonyl Chloride With Double Bonds

Primarily we have been interested in double bonds associated with poly(alkenes). The use of (I) in the study of model alkenes has yielded F^{19} NMR spectra that are in some cases single lines, but in others are a series of complex spectra. Fig. 4 shows the range of spectra obtained by reacting (I) with a given alkene. There is some evidence that the adducts of (I) with alkenes may be a two or more step process, where the reagent adds to the alkene to give the anti-Markownikoff products, which, on standing at ambient temperatures, rearrange to the Markownikoff-oriented material (Ref. 24). Also, the adducts in some instances tend to eliminate hydrogen chloride to form unsaturated sulfides. Furthermore, it is possible that diastereoisomers are formed that are magnetically nonequivalent and give rise to two lines for each isomer pair. In the reaction of a polybutadiene polymer with (I), as shown in Fig. 5, hydrogen chloride is evolved, with the unsaturated sulfide most likely being

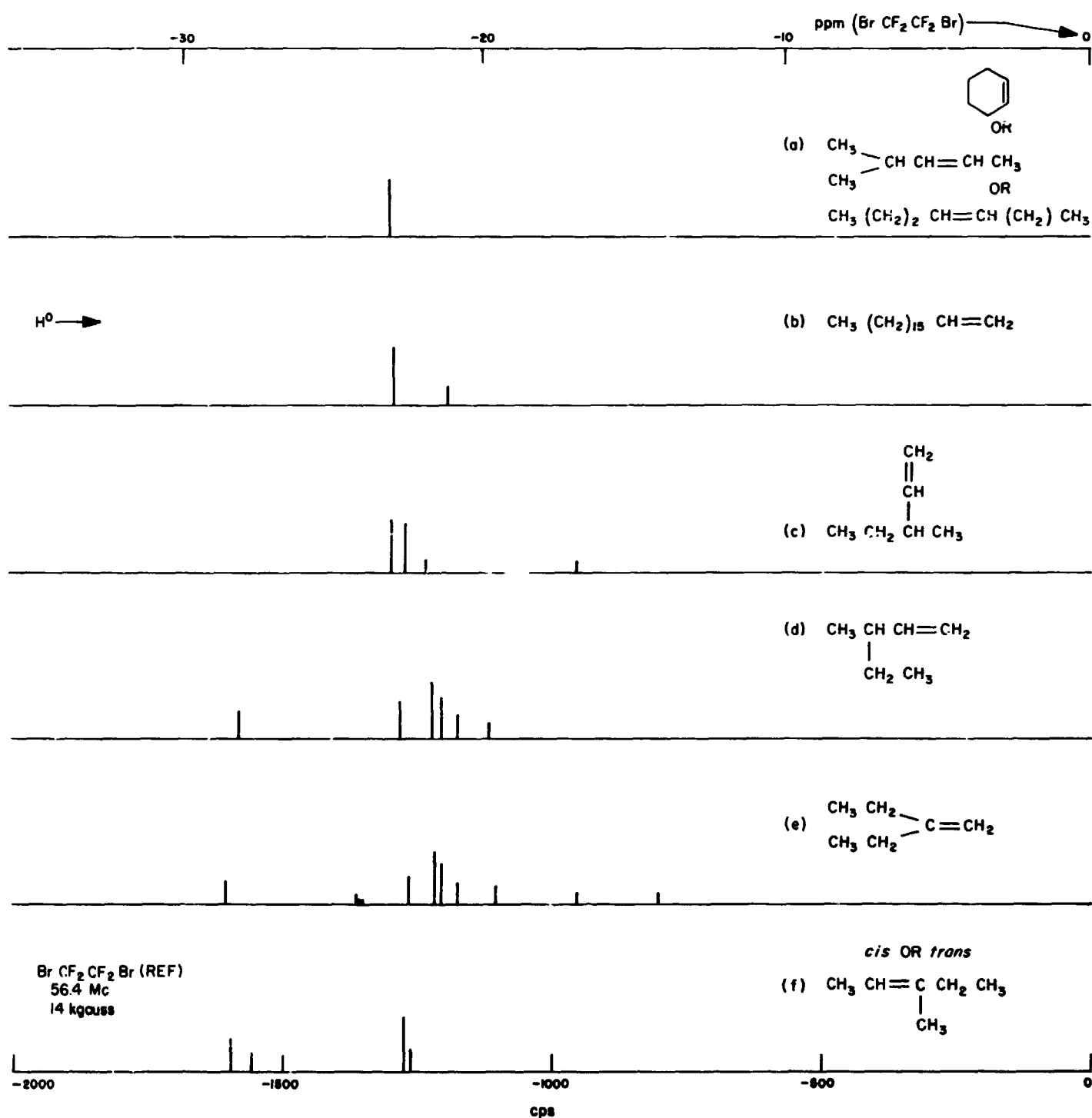


Fig. 4. F^{19} NMR spectra of the neat reaction mixture of alkenes and CF_3SCl at ambient temperature

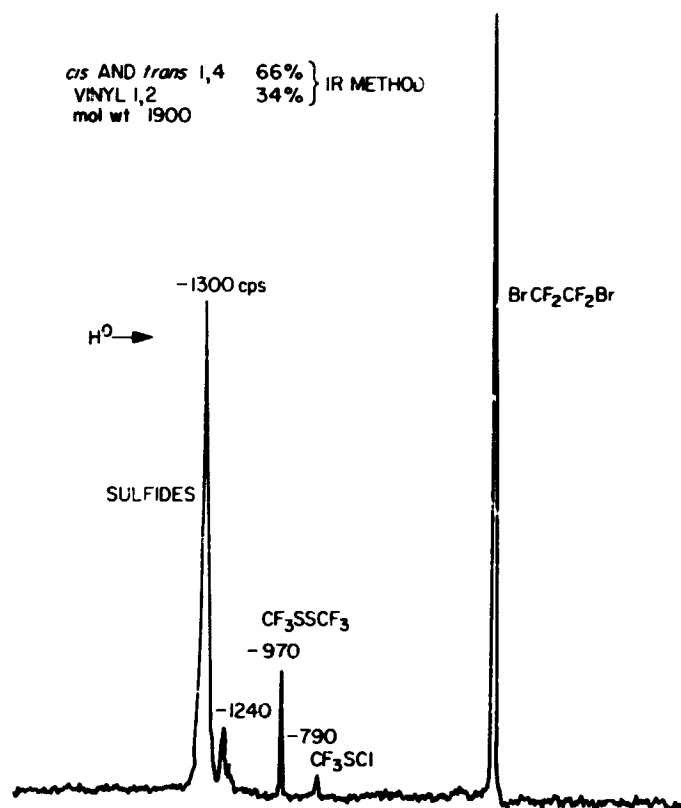


Fig. 5. F^{19} spectra of hydroxyl-terminated polybutadiene reacted with (II)

formed from the pendant vinyl groups. From this spectra the relative concentration of the chain double bonds or pendant vinyl groups in the polymer is not apparent.

In certain situations it may be possible to vary the conditions for reaction of (I) to study the hydroxyl and double bonds simultaneously or successively in the presence of each other. The reaction of (I) with a hydroxyl group is negligible in the absence of a base such as pyridine, and the reaction of the olefinic double bond can be controlled by suitably adjusting the temperature. An example, using 2-octene-4-ol, is shown in Fig. 6. The chemical shifts for the three derivatives are consistent with the observed values given in Fig. 7.

2. Reaction of Trifluorothiolacetic Acid With Double Bond Systems

The complex products that are obtained in the reaction of some alkenes and (I) led us to investigate the use of trifluorothiolacetic acid (II) as a method of introducing a trifluoromethyl probe into an alkene. In reaction 1, the addition of a thiol acid to a double bond is a free radical

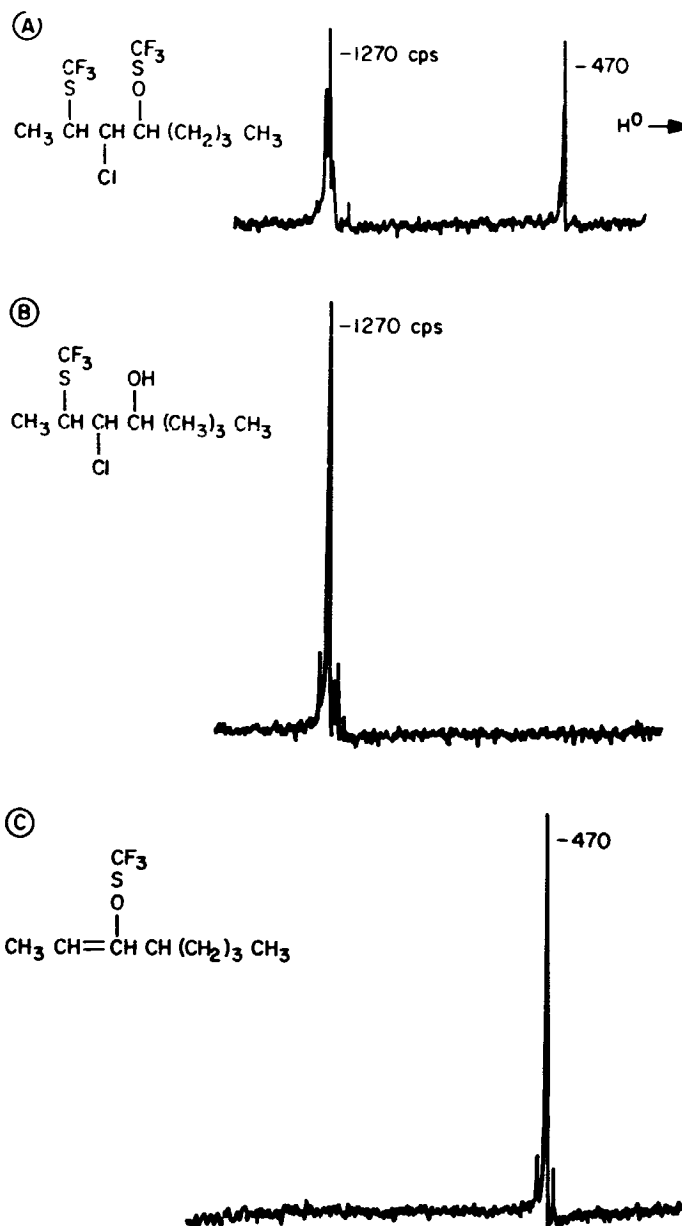
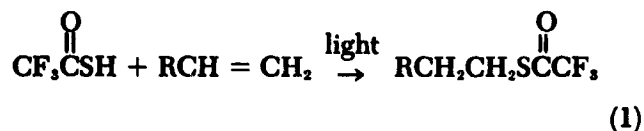


Fig. 6. Steps of reaction of 2-octene-4-ol with trifluoromethanesulfonyl chloride

process and yields products that are anti-Markownikoff-oriented.



In this reaction there is no possibility of the elimination of hydrogen chloride to form the unsaturated sulfides. The chemical shift range of the thioesters of the model

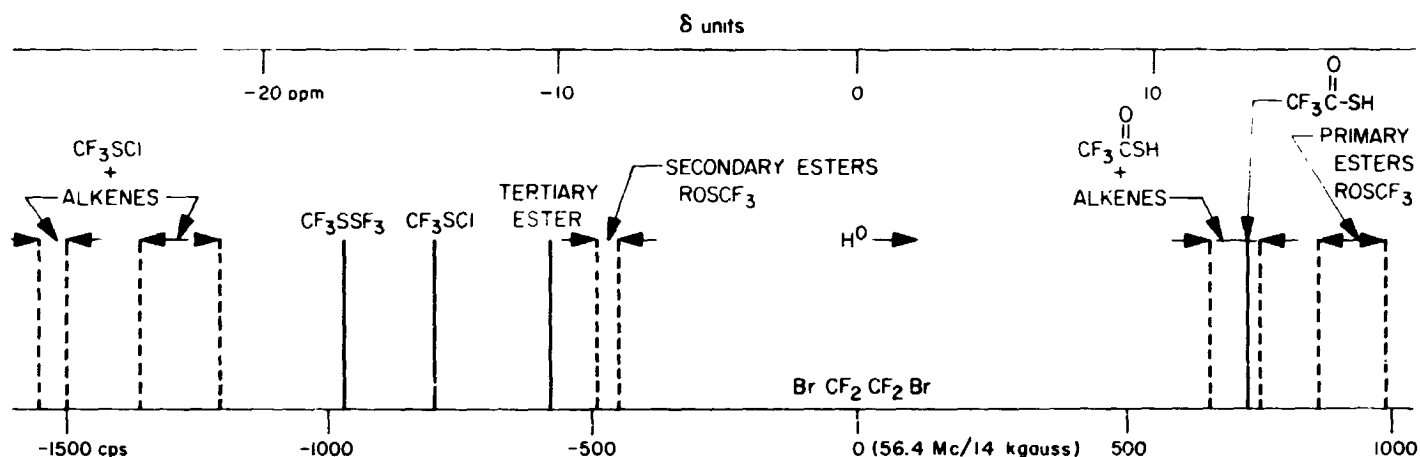


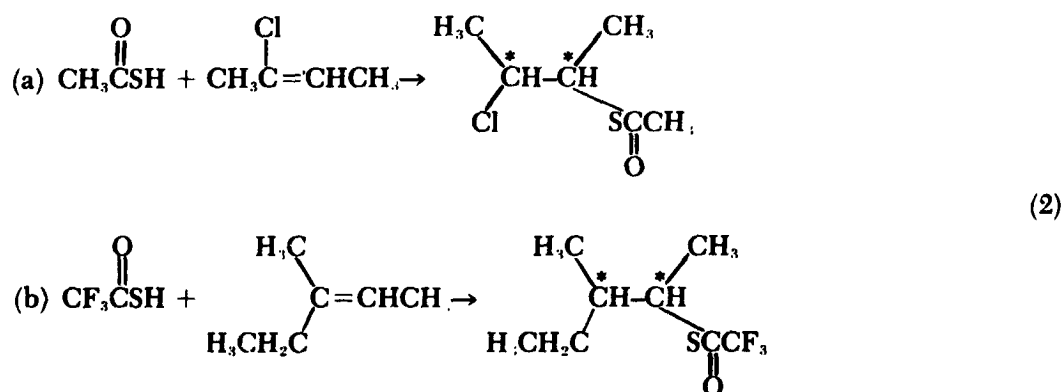
Fig. 7. Summary of chemical shifts of trifluoromethanesulfonyl chloride (I), trifluorothioacetic acid (II), and related derivatives

compounds and polymers is about 130 cps, as shown in Table 4.

In the case of trifluorothioacetic acid (II) reacted with a polybutadiene polymer the relative amounts of vinyl and chain double bonds can be estimated from the spectra as shown in Fig. 8. Apparently the polymer derivatives act as solvents and thus cause the NMR signals to appear at a lower magnetic field than the model compounds. An aromatic ring adjacent to a double bond as in styrene has a marked effect on the F^{19} NMR signal of the thioester derivative, shifting it downfield. It ap-

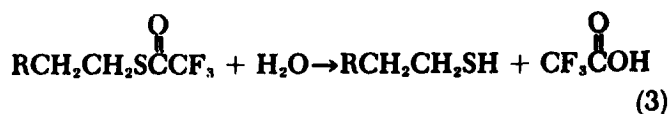
pears that 2,3-dimethyl-2-butene (tetramethyl ethylene) will not react with (I) even when exposed to ultraviolet light for prolonged periods. The failure to react can most likely be attributed to the predominant effect of steric hindrance.

Bordwell, et al., reacted thioacetic acid with *cis* and *trans* 2-chloro-2-butene and showed (reaction 2a) that diastereoisomers were formed (Ref. 25). The reaction of trifluorothioacetic acid with 3-methyl-2-pentene as shown in reaction 2b should yield the threo and erythro adducts in the same manner as 2-chloro-2-butene.



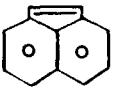
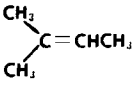
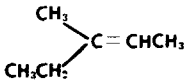
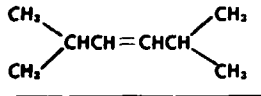
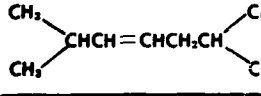
In Fig. 9 the room temperature F^{19} NMR spectra of the reaction mixture shows two thioester peaks that are separated by about 3 cycles. These are most likely the *dl* forms of threo and erythro thioesters (Ref. 26).

The thioesters that are formed from alkenes have a tendency to undergo hydrolysis to yield the mercaptan as shown in reaction 3.



Therefore, in cases where quantitative information is required, the use of water or compounds which can undergo an exchange reaction with the thioester should be avoided.

Table 4. Derivatives of alkenes and polymers with trifluorothioacetic acid (II)

Reactants	Cycles/sec from $\text{BrCF}_2\text{CF}_2\text{BR}$	Comments
$\phi^*\text{C}\equiv\text{C}\phi$	+620 ^b	1 to 1 ratio of $\bar{\text{C}}$ and I
	+660 ^b	Two much smaller lines ~ 10 cps downfield
$\phi\text{CH}=\text{CH}_2$ $\phi\text{CH}=\text{CHCH}_2$	+685 ^b	
CH_3 $\phi\text{C}=\text{CH}_2$	+695 ^b	
$\text{CH}_3(\text{CH}_2)_2\text{CH}=\text{CH}(\text{CH}_2)_2\text{CH}_3$  	+705 ^c	
$\text{CH}_3(\text{CH}_2)_2\text{C}\equiv\text{CH}$	+708 ^b	1 to 1 ratio of $\bar{\text{C}}$ and I
$\text{CH}_3(\text{CH}_2)_{15}\text{CH}=\text{CH}_2$	+712 ^c	
$\text{CH}_3\text{CH}_2\text{CHCH}_2\text{CH}_3$ 	+720 ^c	
$\text{CH}_3\text{CH}_2\text{CHCH}_3$ 	+725 ^c	
CF_3COSH	+738	Neat
	+745 ^c	
	744-746 ^c	3 lines
Cis-polybutadiene (95% 1, 4 addition)	+670 ^b	
High vinyl polybutadiene (91% 1, 2 addition)	+685 ^b	
Polybutadiene secondary hydroxyl-terminated mol wt ~ 2000 By IR method 66%, 1, 4 and 34% 1, 2 (vinyl)	+688 ^b +675	About 2-to-1 area ratio (Fig. 8)

^a ϕ = phenyl
^b Benzene solution
^c The neat reaction mixture of 1 mole of (II) and 2 moles of the olefin.

cis AND trans 1, 4 66%
 VINYL 1, 2 34%
 mol wt 1900

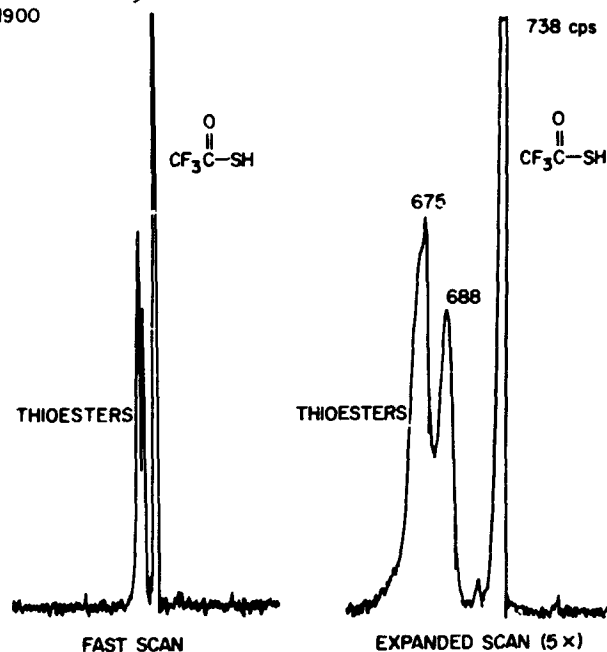


Fig. 8. Spectra of the reaction mixture of trifluorothioacetic acid and a hydroxyl-terminated polybutadiene

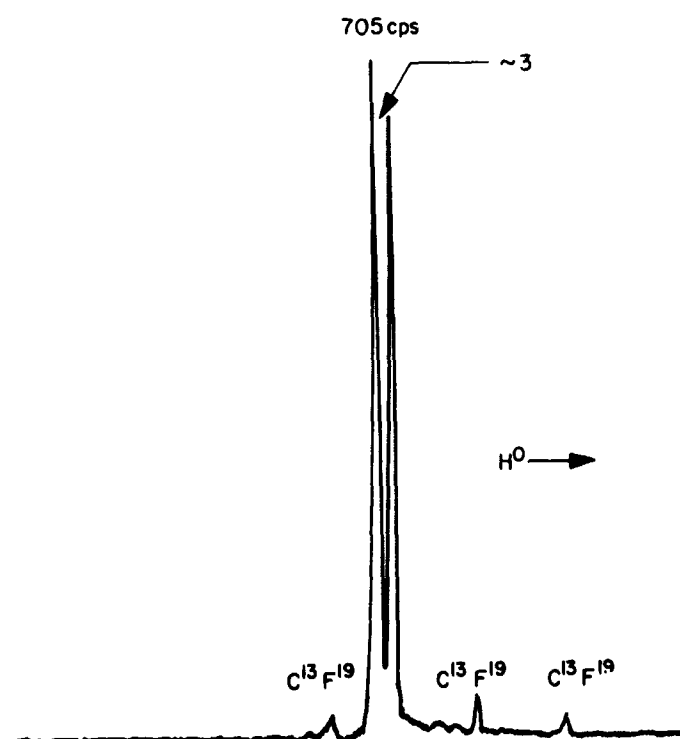


Fig. 9. F¹⁹ spectra of the adduct of 3-methyl-2-pentene and trifluorothioacetic acid (II)

A summary of the chemical shifts of (I), (II), and related derivatives is shown in Fig. 7. The total range of the chemical shifts observed is about 2500 cps at 56.4 Mc and a field of 14 kgauss.

D. Electronic Properties of TCNQ Complexes,

F. E. Stewart, A. Rembaum, and L. E. Creasy

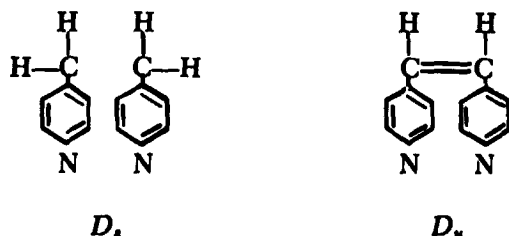
1. Introduction

The discovery of a pseudometallic behavior of heterocyclic organic salts (Refs. 27, 28, and 29) of general formula $D^+ (\text{TCNQ})^-$ TCNQ, in which D is an aromatic molecule generally containing a nitrogen atom, and TCNQ symbolizes the tetracyanoquinodimethane molecule, constitutes the most interesting development of the last decade in the area of electronic conductivity in organic systems.

This class of compounds exhibits room temperature conductivity ranging from 100 to about $10^{-5} \text{ ohm}^{-1} \text{ cm}^{-1}$, i.e., the highest conductivity of organic crystals synthesized to date. It was also shown recently (Ref. 28) that polymeric analogs of the TCNQ salts can be prepared. One such complex, copoly(styrene), 1-butyl-2-vinylpyridinium (TCNQ^-) TCNQ had a conductivity of $10^{-3} \text{ ohm}^{-1} \text{ cm}^{-1}$ and these polymeric complexes could be cast from solution as homogeneous films. The mechanism of electronic transport in the above monomeric or polymeric salts is not well understood.

In this report a study of the electrical properties of TCNQ complexes representing unit segments of nonconjugated as well as conjugated polymers is described.

The following two model compounds were chosen: a saturated donor 1, 2-bis-(4 pyridyl)-ethane, D_s ; and unsaturated donor 1, 2-bis-(4 pyridyl)-ethene, D_u .



Both structures D_s and D_u in form of their TCNQ salts exhibited high electrical conductivities only in the presence of neutral TCNQ, but no significant differences due to conjugation could be detected so far.

2. Experimental

Monomethyl iodonium salts were prepared by adding excess methyl iodide (8.52 g) to D_s or D_u (0.84 g) dissolved in benzene (50 ml). The TCNQ complexes were formed in a nitrogen atmosphere by adding Li TCNQ (0.42 g) in boiling ethanol (100 ml) to a hot solution of monomethyl iodonium salt (0.47 g) in ethanol (10 ml), according to a recently described procedure (Ref. 29). The elemental analyses of iodonium salts and TCNQ complexes were carried out by means of a F & M Model 185 CHN analyzer.

All visible spectra were determined with a Cary Model 14 spectrophotometer in spectral grade acetonitrile as solvent.

Conductivity measurements were made on 0.5-in.-D cylindrical pellets. Pellets prepared under pressures between 20,000 and 100,000 lb/in.² had essentially identical

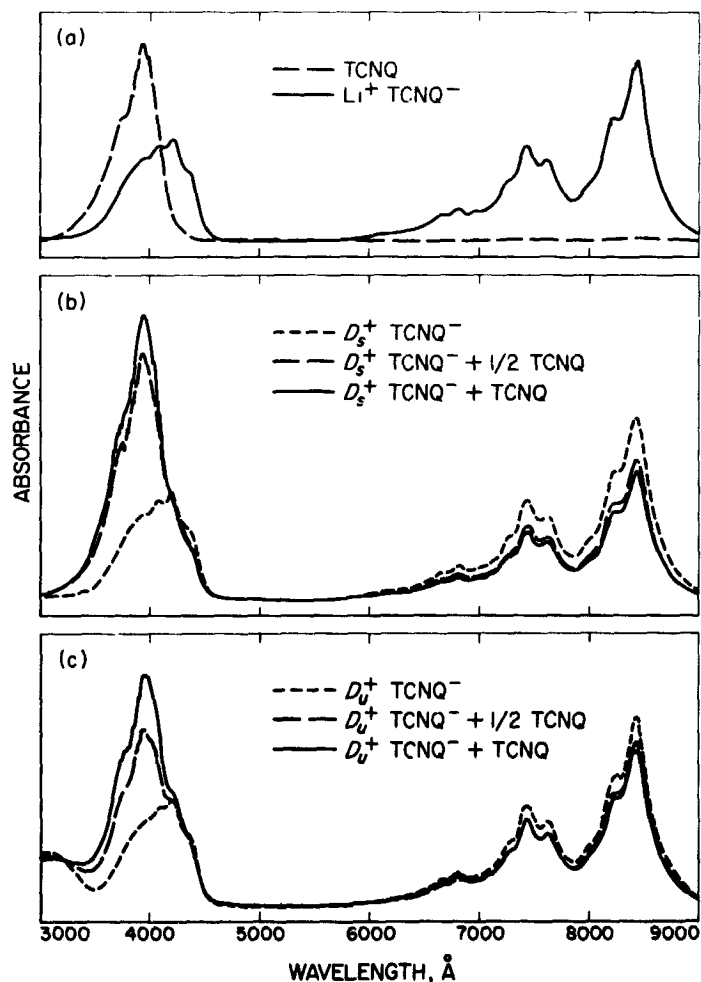


Fig. 10. Visible absorption spectra (10^{-5} M) in acetonitrile

conductivities. Electrical contact was made with vacuum-deposited gold electrodes. Using this technique, resistivity values could be reproduced within a factor of two. Silver paste or platinum disk contacts yielded much less reliable results. ESR measurements were made by a previously reported procedure (Ref. 30), on samples consisting of either small chips of compacted pellets, powdered complexes, or complexes dispersed in KCl. Pressure and grinding seemed to have no significant effect on the free spin concentration as derived from the ESR intensity.

3. Results and Discussion

a. Spectrophotometric data. Confirmation of the proposed structures of the synthesized complexes is provided by their visible absorption spectra. As shown in Fig. 10, the TCNQ radical anion absorbs strongly at 4200 Å ($\epsilon = 24,300$) and at 8420 Å ($\epsilon = 43,300$), while neutral TCNQ absorbs only at 3950 Å ($\epsilon = 63,600$). The intensity ratio of the 4200 and 8420 Å bands is approximately 0.57 for simple salts, and that of the 3950 and 8420 Å bands is approximately 2.0 for salts containing one neutral molecule of TCNQ per complex.

The spectrophotometric analysis agreed with the proposed radical anion-to-neutral acceptor ratio to within

about 5%. However, the elemental analysis (Table 5) agreed, on the average, with the theoretical percentages of C, H, and N to better than 1%.

b. Electrical properties. In Table 5 are listed the analytical results as well as room temperature resistivities and their activation energies E obtained by applying the formula $\rho = \rho_0 \exp(E/kT)$ from 77 to 300°K. Average values are quoted with deviations being less than 30% for ρ and 10% for E . Such deviations are presumably due to variations in intergranular resistances (Ref. 31). The addition of neutral TCNQ dramatically lowers the resistivity due to increased electron delocalization. The low activation energy of these latter complexes approaches that of metallic conduction behavior.

c. Electron spin resonance (ESR). Each of the four complexes studied here contains approximately Avogadro's number of unpaired electrons per mole at room temperature (Table 5). There is evidence that increased electron delocalization facilitates unpairing, since those complexes containing neutral TCNQ have considerably higher unpaired spin concentrations than those without neutral TCNQ. Relative spin concentrations quoted are considered accurate to 30%; however, absolute values are known only to within a factor of two.

Table 5. Electronic and magnetic properties of TCNQ-1 substituted pyridinium salts

Complexes	Calculated percentage				Experimental percentage				ρ at 25°C, ohm-cm	Resistivity activation energy E , ev	ESR intensity at 25°C, spins/mole $\times 10^{-23}$	Singlet- triplet separation energy J , ev	g value
	C	H	N	I	C	H	N	I					
D_4 Mono-iodonium salt	47.84	4.63	8.59	38.93	47.04 46.80	4.24 4.45	8.61 8.45	40.7					
D_4 Mono-iodonium salt	48.14	4.04	8.65	39.50	47.54 47.44	4.69 4.60	9.13 9.02	37.95					
$D_4^+TCNQ^-$	74.42	4.75	20.83	—	73.55 73.55	5.37 4.88	20.71 20.43	—	6.4×10^4	0.105	0.35	~ 0.03	2.0028
$D_4^+TCNQ^-$	74.80	4.27	20.94	—	72.68 72.90	4.64 4.39	21.19 21.09	—	6.4×10^4	0.145	1.5	~ 0.045	2.0028
$D_4^+TCNQ^-$ +TCNQ	73.15	3.79	23.06	—	72.12 72.28	3.66 3.74	24.89 24.84	—	3.4	0.035	11.7	0	$g_{11} = 2.0033$ $g_{\perp} = 2.0024$
$D_4^+TCNQ^-$ +TCNQ	73.39	3.48	23.14	—	74.32 74.20	3.61 3.65	23.96 23.67	—	3.8	0.035	8.0	0	$g_{11} = 2.0036$ $g_{\perp} = 2.0028$

Simple TCNQ salts follow a singlet-triplet model for paramagnetism in which the ESR intensity I is proportional to $T^{-1} [\exp (J/kT) + 3]^{-1}$. The singlet-triplet separation energy J may be calculated from the formula $J = 1.61 kT_m$, where T_m is the temperature at which I is

a maximum. We find for the D_s TCNQ and D_u TCNQ simple salts that T_m is considerably above ambient temperature and in the range in which these salts begin to decompose. Therefore an alternate procedure for determining J was attempted.

In the temperature range for which $J \gg kT$, the product $I \times T$ is proportional to $\exp (-J/kT)$, and a semilog plot of $I \times T$ versus T^{-1} will be a straight line the slope of which is $(J/k) \log e$. From such a plot (Fig. 11) the measured value of J was found to be not sufficiently large in comparison with kT to make the above approximation valid. We estimate that the measured value of J is too small by a factor of approximately two. It can only be concluded that J is probably somewhat smaller than 0.1 ev.

In the low-temperature region of Fig. 11 the triplet contribution to paramagnetism is small and the doublet impurity contribution predominates. In this region $I \times T$ is independent of temperature.

Complex TCNQ salts containing neutral TCNQ exhibit Pauli spin paramagnetism which is characteristic of a degenerate electron gas and operative in metals (Ref. 31). In this case the measured ESR intensity is independent of temperature. Such a paramagnetism was found for our complex salts (Table 5).

The shape of the ESR signal of the complex salts exhibits an interesting rotational anisotropy that has not

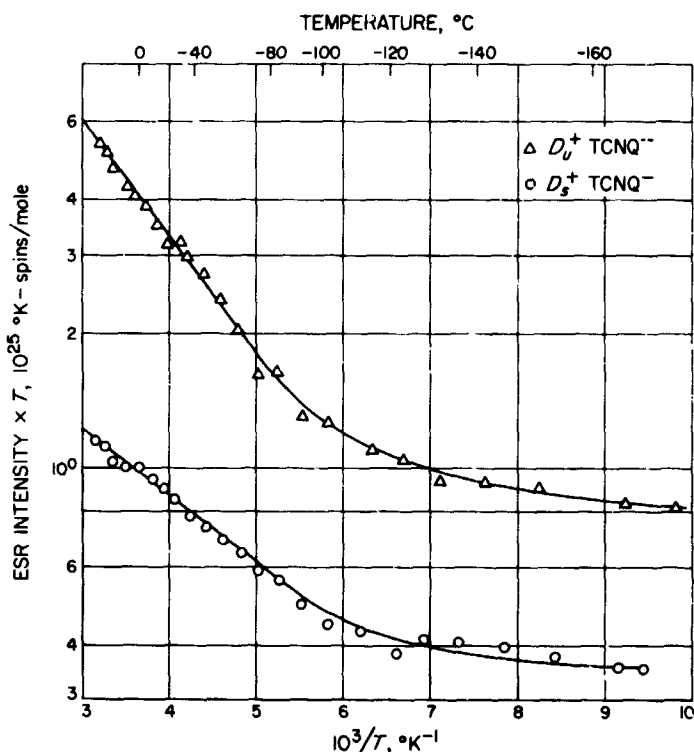


Fig. 11. Temperature dependence of ESR intensity for simple TCNQ salts

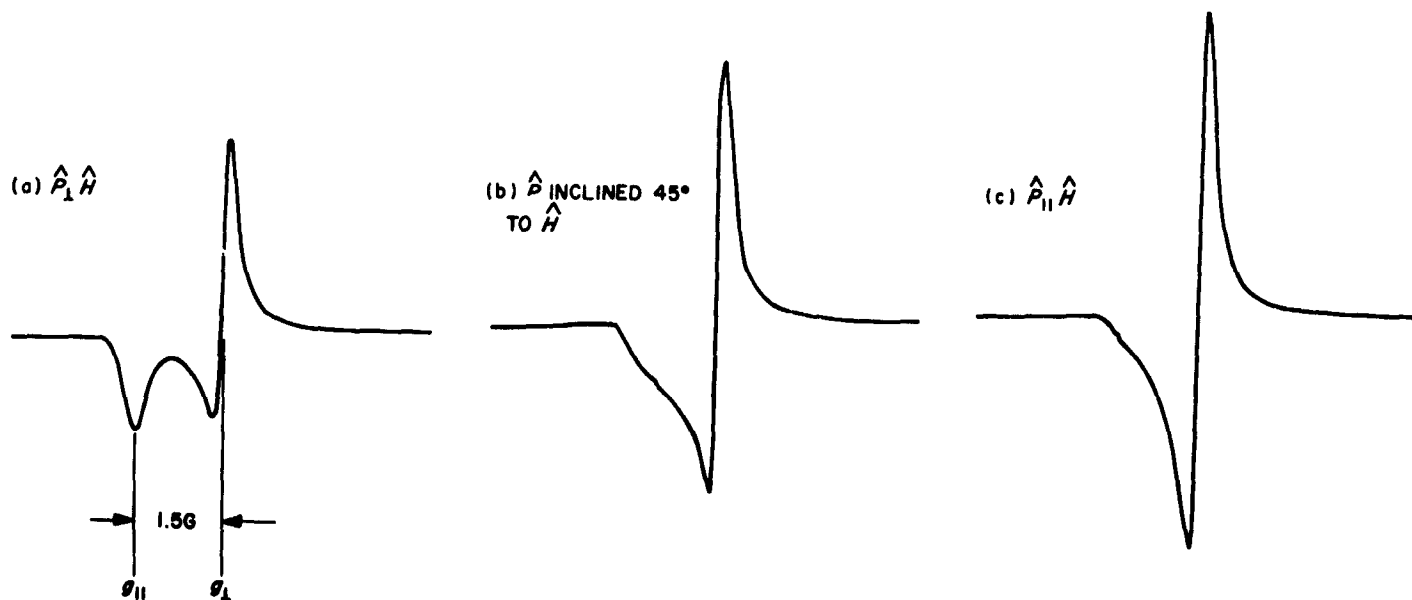


Fig. 12. Rotational anisotropy of a pellet of D_s TCNQ-TCNQ

been reported previously. If a pellet is oriented in the magnetic field in such a way that the direction in which the pressure was applied to it, $\hat{P}$, is kept perpendicular to the magnetic field $\hat{H}$, no line-shape change results; however, a change is observed when the pellet is rotated to make $\hat{P}$ parallel to $\hat{H}$ (Fig. 12). Such a change does not occur in the simple salts. Neither does it occur in the complex salts if they are in a powdered rather than in a compacted form. The ESR signal shape for $\hat{P} \parallel \hat{H}$ (Fig. 3a) is similar to that of a single crystal having axial symmetry and an anisotropic g value. Single crystals of organic TCNQ salts are known to have anisotropic g values but compacted pellets, usually composed of many microcrystals randomly oriented, are not known to exhibit rotational anisotropy. It would appear that under pressure the molecules of our complex salts assume a preferred orientation resulting in an axial symmetry. Investigations are in progress to establish a correlation between the anisotropy of a compressed pellet and its electronic conductivity.

E. Reactions of Carboxyl-Terminated Prepolymers With Polyaziridines and Polyepoxides,

S. H. Kalfayan and B. A. Campbell

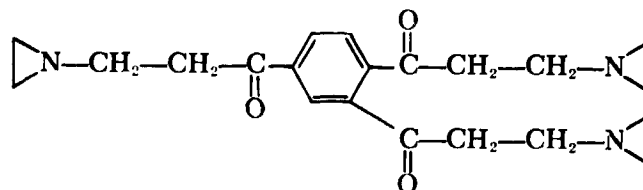
1. Introduction

The study of the reactions of carboxyl-ending polyether and hydrocarbon backbone prepolymers with aziridines and epoxides was continued. A novel trifunctional aziridine, 1,2,4-*tris* 2-(1-aziridinyl) ethyl trimellitate was evaluated as a crosslinking agent. Two carboxyl-ending poly(propylene oxides), products of the condensation of succinic and adipic acids with PPO 1000, discussed in Section XI-F in this SPS issue, were reacted with different aziridines and epoxides. A condensation product of diethylene glycol and the dimer fatty acid, Empol 1010, was also evaluated. These investigations are of a preliminary nature and are part of the overall objectives of studying the effect of backbone structure on the properties of an elastomer and investigating the products obtained for possible use as propellant binders.

2. Experimental

a. Materials. Many of the materials used were described previously (SPS 37-39, 37-40, 37-41, Vol. IV). The new ones are as follows:

- (1) 1,2,4-*tris* 2-(1-aziridinyl) ethyl trimellitate (Dow Chemical Co., NC 1455), structural formula:



- (2) Carboxyl-ending PPO 1000-succinic acid condensate (PPO-COOH-S), equiv. wt. 1200.
- (3) Carboxyl-ending PPO 1000-adipic acid condensate (PPO-COOH-A), equiv. wt. 870.
- (4) Carboxyl-ending diethylene glycol-Empol 1010 condensate, equiv. wt. 590.

NC 1455 was used as received. Stoichiometric amounts were employed in all cases.

b. Preparation and characterization of the condensates.

All three carboxyl-ending prepolymers were prepared by Fischer esterification in dry xylene. The milder *p*-toluenesulfonic acid was used as catalyst, instead of a mineral acid. Water from the condensation reaction was removed by fractional distillation as the reaction proceeded. The products were analyzed for acidity, hydroxyl content (acetylation method) and molecular weight (vapor pressure osmometry).

3. Results and Discussion

a. Reactions of PPO-COOH-S and PPO-COOH-A with aziridines and epoxides. Results for these reactions are given in Tables 6 and 7. Cures could be obtained with

Table 6. Curing of succinic and adipic acid condensates of PPO 1000 with polyaziridines

Prepolymer	Curing agent	Gel time at 100°C	Shore A hardness	
			24 hr at 100°C	48 hr at 100°C
PPO-COOH-S ^a	HX 740	>48 hr	thick fluid	thick fluid
	ZC 466	3¼ hr	10	10
	MAPO	1 hr	reverts to fluid	
	NC 1455	5 min	37	37
PPO-COOH-A ^b	HX 740	1½ hr	reverts to fluid	
	ZC 466	1¾ hr	17	17
	MAPO	1 hr	reverts to fluid	
	NC 1455	8 min	45	47
	NC 1455	20 min	25	37
	ZC 466			

^a Succinic acid condensate.

^b Adipic acid condensate.

Table 7. Curing of succinic and adipic acid condensates of PPO 1000 with polyepoxides

Prepolymer	Curing agent	Gel time at 150°C, hr	Shore A hardness		
			24 hr at 100°C	48 hr at 100°C	48 hr at 150°C
PPO-COOH-S ^a	ERL 0510 Shell X 801	>2 <17 >48	fluid fluid	fluid fluid	soft soft
PPO-COOH-A ^b	ERL 0510 Shell X 801	>4 <23 >48	fluid fluid	soft fluid	2 thick fluid

^aSuccinic acid condensate.
^bAdipic acid condensate.

Table 8. Curing of Empol 1010 condensate of diethylene glycol with polyaziridines and polyepoxides

Curing agent	Gel time,		Shore A hardness			
	at 100°C	at 150°C	24 hr at 100°C	48 hr at 100°C	24 hr at 150°C	48 hr at 150°C
HX 740	3¼ hr		soft	soft		
ZC 466	¾ hr		20	24		
MAPO	1 hr		soft	soft		
NC 1455	5 min		50	55		
NC 1455 } ZC 466 }	7 min		31	40		
ERC 0510	5½ hr	2½ hr	10	13	27	37
Shell X 801	>48 hr	>8 <22 hr	fluid	fluid	1	23
Epon 828	>48 hr	>32 <48 hr	fluid	fluid	—	soft

the triaziridine NC 1455 and the diaziridine ZC 466. The triaziridine MAPO gelled the prepolymers, but further heating at 100°C caused liquefaction. The diaziridine HX 740 did not gel the PPO-COOH-S and gelled the PPO-COOH-A, but the gel liquefied on prolonged heating at 100°C. These results seem perplexing.

The functionalities of PPO-COOH-S and PPO-COOH-A, calculated from molecular weights determined by vapor pressure osmometry (VPO), were 1.4 and 1.7, respectively. A triaziridine would be capable of curing prepolymers of low functionality, but a diaziridine would not be capable. The fact that NC 1455 gave cures and HX 740 did not, is explainable on the functionality basis; but the observation that MAPO-cured products reverted and ZC-466-cured ones did not, should have a different explanation. One possible explanation could be that both HX 740 and MAPO, because of their imide-type linkages, form oxazolines on heating in the presence of the acidic prepolymers. Oxazoline formation reduces the aziridine functionality and prevents cure. NC 1455 and ZC 466 are not capable of oxazoline ring formation.

It should also be mentioned that the accuracy of molecular weight determinations by the VPO method is greatly affected by the presence of low molecular weight impurities. The functionality values given above should not be considered as altogether accurate.

Satisfactory cures could not be obtained with both the triepoxides used.

b. Reactions of Empol 1010-diethylene glycol condensate with aziridines and epoxides. The results of these experiments are summarized in Table 8. The functionality of the prepolymer, calculated from the VPO molecular weight was 2.2. Gelation occurred with all the aziridines, but cures with MAPO and HX 740 resulted in soft products, showing the same tendency observed with the PPO condensates. The product obtained from ZC 466 was highly extensible. NC 1455 gave a harder product with low elongation. Products obtained from a mixture of ZC 466 and NC 1455 in 1:1 equivalent ratios, had intermediate properties.

Satisfactory cures were obtainable after 48 hr at 150°C from the triepoxides ERL 0510 and Shell X 801, but the products had low tear and elongation.

c. Reaction of NC 1455 with carboxyl-ending polybutadienes. Immediate gelation occurred when NC 1455 and carboxyl-ending polybutadiene mixtures were exposed to 100°C. The mixtures thickened at room temperature and pot lives were short (30 to 40 min). Gelation at room temperature occurred within 2 to 5 hr (Table 9). NC 1455 can be useful for curing compositions at low temperatures.

Table 9. Curing of carboxyl-ending polybutadienes with NC 1455

Prepolymer	Gel time at 100°C	Gel time, at room temperature, hr	Shore A hardness	
			24 hr at 100°C	48 hr at 100°C
Telagen CT	Immediate	2½	30	31
Butarez CTL	Immediate	2	46	46
HC 434	Immediate	5	55	55

F. Carboxyl-Ending Poly(Alkylene Oxides),

S. H. Kalfayan

1. Introduction

The comparative study of the effects of backbone structure on the properties of an elastomer is of interest. The polyether and hydrocarbon backbones are the ones presently under study. To obviate the consideration of the influence of the type of crosslink, it is necessary that both types of polymer possess the same kind of functional group and be extended or crosslinked by the same reagents.

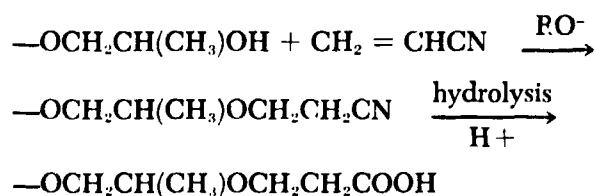
Carboxyl-ending, unsaturated hydrocarbon backbone prepolymers have been commercially available for some time and are suitable for the kind of study desired. These are butadiene-based polymers, of number average molecular weights M_n ranging from 1000 to 6000. Carboxyl-ending polyether backbone prepolymers have to be prepared. Several seemingly feasible methods, all using poly(alkylene oxides) (PAO) as starting materials, were tried. Of these methods the cyanoethylation of PAO followed by hydrolysis of the cyano-ending intermediate was reported previously in several SPS's. Two others, the haloform and the esterification methods, are discussed presently. The haloform method is applicable only to

PAO with secondary hydroxyl groups, with the secondary carbon atom attached to one methyl group. Poly(propylene oxide), PPO, of M_n 1000 fulfills this requirement. The esterification method, using dibasic acids, is applicable to both poly(ethylene oxides) (primary hydroxyls) and to PPO (secondary hydroxyls). A brief review of the cyanoethylation-hydrolysis method is also included in the discussion that follows:

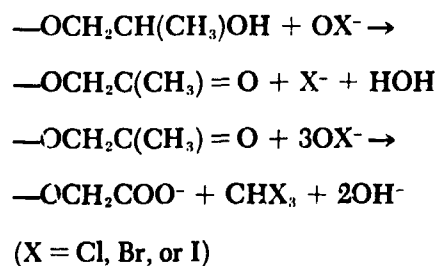
2. Discussion

The equations that represent the three methods employed to modify the OH end-groups of PPO 1000 to carboxyl groups are as follows:

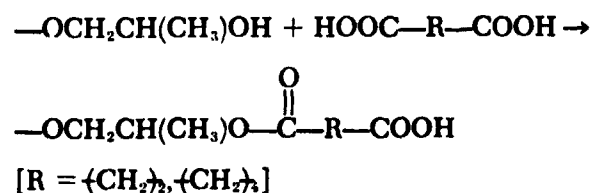
(1) Cyanoethylation and hydrolysis



(2) Haloform reaction



(3) Esterification



a. Cyanoethylation and hydrolysis. Extensive study of this method showed that cyanoethylation could be carried out up to 95% completion, and that complicating side reactions such as block polymer formation with acrylonitrile could be minimized. During hydrolysis, however, cleavage of the alkoxy group introduced by cyanoethylation took place, resulting in partial reversion to the original PPO. The purification of the dicarboxylic acid was tedious and incomplete. End-group analysis showed the presence of about 15–18% of hydroxyl-terminated material considered to be monocarboxylated PPO.

b. The haloform reaction. This reaction was carried out with disecundary dipropylene glycol (DPO) as well as with PPO 1000, using sodium hypoiodite, formed *in situ* from iodine and NaOH. The haloform (iodoform) reaction, a commonly used test in organic analysis for the identification of $\text{CH}_3-\text{C}=\text{O}$ group or compounds which can yield this group on reaction with the reagent, involves also the formation of the salt of an acid, as well as the iodoform. The conversion of DPO to the corresponding acid, based on the weight of iodoform obtained, averaged to 20%. That of PPO averaged to about 10%. When reaction mixtures were warmed to increase the rate and the yield, more iodine was consumed than accounted for by the amount of iodoform obtained, indicating the probability of iodination of the chain, or the formation of iodate, or both. The haloform reaction, applied as a preparative method for the carboxylic acids desired, shows little value.

c. Esterification. PPO 1000 was esterified with both succinic and adipic acids in dry xylene, using excess of the dibasic acids (2.5 equivalents of COOH to 1.0 equivalent of OH), in presence of toluenesulfonic acid as catalyst. The water formed was removed by fractional distillation as the condensation reaction proceeded. Seventy percent of the water could be removed during the first hour of refluxing. The results obtained for some of the runs are given in Table 10.

It must be noted that molecular weight M_n determinations by vapor pressure osmometry (VPO) can lead to large errors if low molecular weight impurities are present in the analyzed prepolymer. The presence of unreacted acid in the preparations of Table 10 is a possibility, although repeated extractions with water, to remove the suspected impurities and prolonged drying in vacuum were carried out with each run. The molecular weight values as well as the functionality values which were based on the former, should not be regarded as un-

suspect. The hydroxyl contents reported are the lowest concentrations detectable by the method used. The acetylation method was used for the first four runs. A colorimetric method was employed for the last run. These results indicate that probably all the OH terminals of PPO 1000 were reacted.

3. Conclusion

Of the three methods used to prepare carboxyl-terminated PPO, condensation with dibasic acids is the simplest and the most promising.

G. Method for Estimating the Temperature-Dependence of Foam Diffusion Constants, E. F. Cuddihy and J. Moacanin

It has been shown (Ref. 33, and SPS 37-34 through 39, Vol. IV) that the diffusion of gases in rigid, cellular foams obey Fick's Law and that the diffusion coefficient D_f can be predicted from the following expression

$$D_f = \left(\frac{3RT}{M} \right) (\rho_0/\rho) P_e \quad (1)$$

where

D_f = diffusion coefficient of the foam

P_e = permeation constant of the solid polymer gas system

ρ_0 = density of the solid polymer

ρ = density of the foam

T = absolute temperature

M = mol wt of the diffusing gas

R = gas constant

Table 10. Esterification of PPO 1000 with dibasic acids ($\text{OH}/\text{COOH} = 1/2.5$)

Run No.	Acid	Reflux time at 135-138°C, hr	Acidity, meq/g	OH content, meq/g	Molecular weight by VPO	Equivalent weight	Functionality, $\frac{\text{mol wt}}{\text{equiv wt}}$
1	Succinic	5	0.83	<0.1	—	1205	—
2	Succinic	8.5	0.86	<0.1	1660	1162	1.42
3	Adipic	4	1.07	<0.1	—	34	—
4	Adipic	10	1.15	<0.1	1482	870	1.70
5	Adipic	14	1.01	<0.05	1706	990	1.72

The validity of this equation was confirmed at room temperature from experimentally measured values of D_f as a function of ρ .

Using Eq. (1), it should also be possible to evaluate D_f at any temperature from the knowledge of the temperature dependence of P_e , which is known (Ref. 34) to obey an Arrhenius relation

$$P_e = P_{e0} \exp(-\Delta H_p/RT) \quad (2)$$

where ΔH_p is the activation energy of permeation. Substituting Eq. (2) into Eq. (1) and rearranging results in

$$\ln \frac{D_f}{T} = -\frac{\Delta H_p}{RT} + \ln \left(\frac{3R\rho_0 P_{e0}}{M\rho} \right) \quad (3)$$

Inasmuch as the ratio ρ_0/ρ is temperature independent, the last term is a constant and hence a straight line relationship between $\ln D_f/T$ and $1/T$ is predicted.

Table 11. Diffusion constants of a polyurethane foam^a at various temperatures

Temperature, °C	$D \times 10^4$ cm ² /sec	$D/T \times 10^9$ cm ² /sec °K	$1/T \times 10^3$
22	1.76	5.96	3.39
41	3.34	10.6	3.18
61	9.57	28.6	2.99
81	13.7	38.7	2.82

^aEmerson and Cummings, Inc., Eccofoam FPH/12/6H, density=4.5 lb/ft³

In Table 11 are given values of D_f obtained at four temperatures for a rigid, closed-cell (>95%) polyurethane foam of density 4.5 lb/ft³ (Emerson and Cummings, Incorp., Eccofoam FPH/12/6H). The experimental details are reported in SPS 37-35, Vol. IV. The data which are plotted in Fig. 13 as $\ln D_f/T$ versus $1/T$ show a reasonably good straight line relationship. The slope yields for ΔH_p a value of 7.4 kcal/mole. This value is in excellent agreement with the usual 6 to 12 kcal/mole range for the activation energies for diffusion processes in polymers (Ref. 34).

For the particular case of the permeation of CO₂ through a rigid polyurethane, no data were found in the

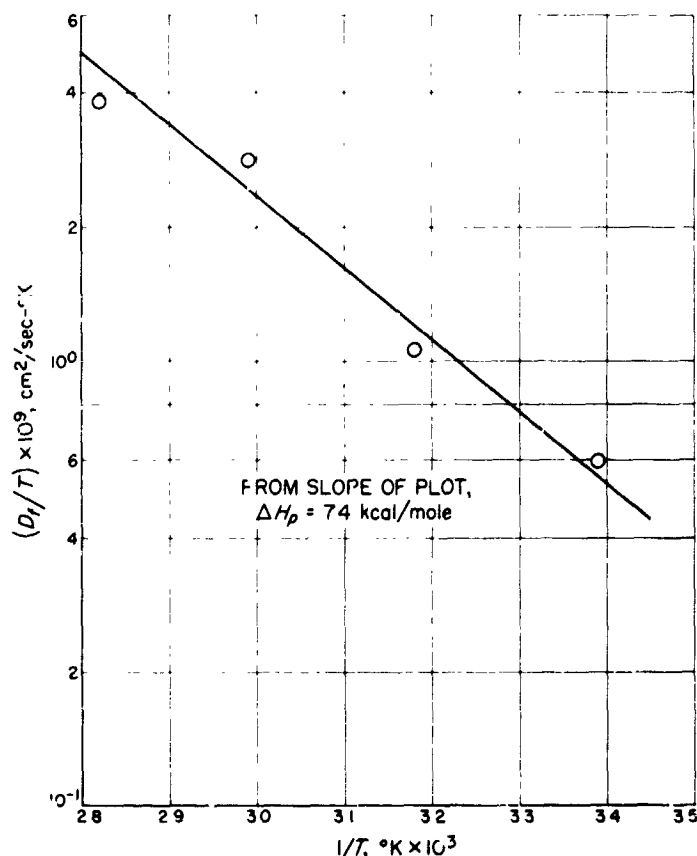


Fig. 13. Temperature dependence of the diffusion coefficient for a CO₂-filled rigid polyurethane foam, density 4.5 lb/ft³

literature. However, some data (Ref. 35) were found for a rubbery polyurethane, and these are given in Table 12. The activation energy for permeation ΔH_p was found to be 9.4 kcal/mole, and this compares favorably with the

Table 12. Results for CO₂ permeation through a rubbery polyurethane^a

Temperature, °C	$P_e \times 10^5$ (cm ³ STP) mm cm ² -sec-cmHg	$D \times 10^4$ cm ² /sec	Solubility, l.m ³ STP/cm ³ -atm
25	1.65	0.094	1.50
50	6.34	0.440	1.10
Activation energies, kcal/mole			
$\Delta H_p = 9.4$ (for permeation)			
$\Delta H_d = 11.7$ (for diffusion)			
$\Delta H_s = -2.3$ (for solubility)			

^aData for Volcprene polyurethane rubber (Ref. 35)

value of 7.4 kcal/mole. Further, it is expected that ΔH_p for the rigid polyurethane should be lower than that for the rubbery polyurethane, since it has been adequately verified (Ref. 34) that the activation energy of a polymer above its T_g is higher than that value obtained when the polymer is below its T_g . Hence the 9.4 kcal/mole corresponds to a polyurethane above T_g , whereas the 7.4 kcal/mole is for a polyurethane below T_g .

In conclusion, we have shown that the equation

$$D_f = \left(\frac{3RT}{M} \right) (\rho_0/\rho) P_{fo} \exp(-\Delta H_p/RT)$$

adequately describes the temperature dependence of the foam diffusion constant.

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XII. Research and Advanced Concepts

PROPULSION DIVISION

N67-29153

A. Breakdown Potentials for Binary Gas Mixtures of Nitrogen-Helium and Helium-Argon,

J. A. Gardner

1. Introduction

An experimental investigation in the area of gaseous breakdown of a binary gas mixture was undertaken in order to understand more fully the mechanisms involved during the starting phase of an MPD arcjet and an MHD accelerator. Devices of this type generally undergo high electrode erosion rates and periods of instability during the initial phases of operation. Improvements in these areas would provide longer operational lifetimes and higher reliability.

A possible approach to the problem of reducing the deleterious effects of electrode erosion that was taken is the addition of a secondary gas into the flow stream during the starting phase. It was assumed that the configuration of the electrode and other supporting hardware had been established from other constraints. Starting transients and periods of instability are generally associated with a time constant which is much smaller than that which would be associated with the gas flow; therefore, preliminary breakdown investigations were performed under static conditions. The effect of a secondary gas on

the breakdown potential has never been completely described analytically. Certain phenomena, such as the Penning effect, are known to reduce the breakdown potential of a mixture of gases to a potential which is lower than any of the constituents separately. This effect is known to occur whenever the gas, which is the major constituent, has a metastable level higher than the ionization potential of the impurity (minor constituent). Examples of such mixtures are neon-argon, helium-neon, and xenon-rubidium.

Experiments were conducted in order to evaluate breakdown voltage as a function of electrode separation, gas pressure, and constituent concentrations for mixtures of nitrogen-helium and helium-argon by establishing Paschen curves over a wide range of concentrations. Data for pure nitrogen, helium, and argon were also acquired. In order to establish if initial gas purity had an effect on breakdown voltage, both standard (commercial grade) and ultra-high-purity (spectroscopic grade) gases were utilized in the controlled mixtures.

2. Experimental Apparatus and Procedure

Various binary gas mixtures were provided by the mixing system shown schematically in Fig. 1. This gas mixing system was designed for mixing, with high constituent

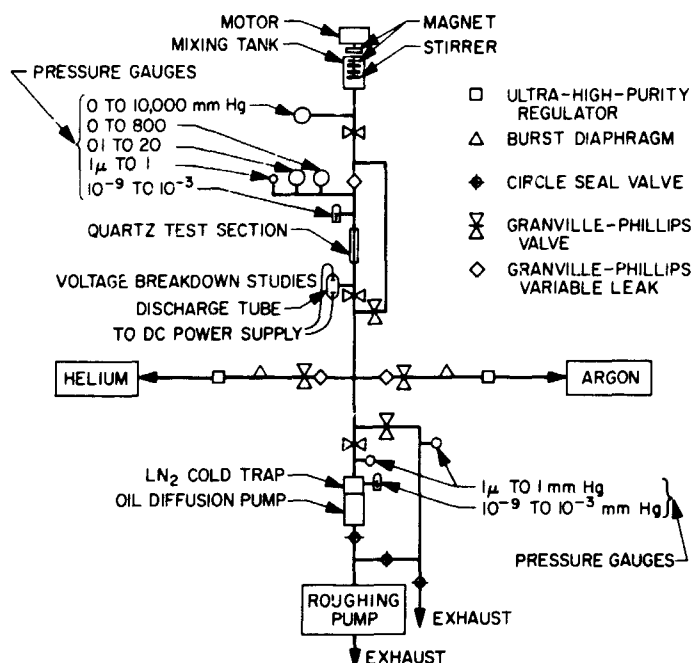


Fig. 1. Gas mixing system

concentration accuracies, any two gases within a concentration range of 0.01 to 100%. Stainless steel construction, in conjunction with ultra-high vacuum valves, minimum O-ring use, and Bourdon tube and aneroid-type pressure gauges were considered essential to ensure mixture purity. However, the selection of the Bourdon tube and aneroid-type pressure gauges restricted the range of pressures that were available. Dalton's law of partial pressures furnished the principle for actual concentration determination. Accuracy of mixture concentration was never less than 1% of the minor constituent within the range of concentrations mentioned above. Incorporated into the mixing system was the experimental discharge tube shown in Fig. 2. This tube consisted of two ultra-high-purity copper disks separated by a 1.9095-in. gap. The two copper surfaces facing each other were flat within 0.0003 and 0.0042 in. and were parallel within 0.0012 in., as determined from the disk centers. The high-voltage, direct-current power supply consisted of a primary controlled transformer used in conjunction with a full-wave rectifier and two 4- μ f capacitors for filtering. The high-voltage limit for this variable supply was approximately 7000 v.

The actual mixing procedure consisted of first evacuating the mixing system to a maximum pressure of 10μ , then purging with the minor constituent. This evacuating and purging procedure was repeated several times with the minor constituent in order to ensure and maintain

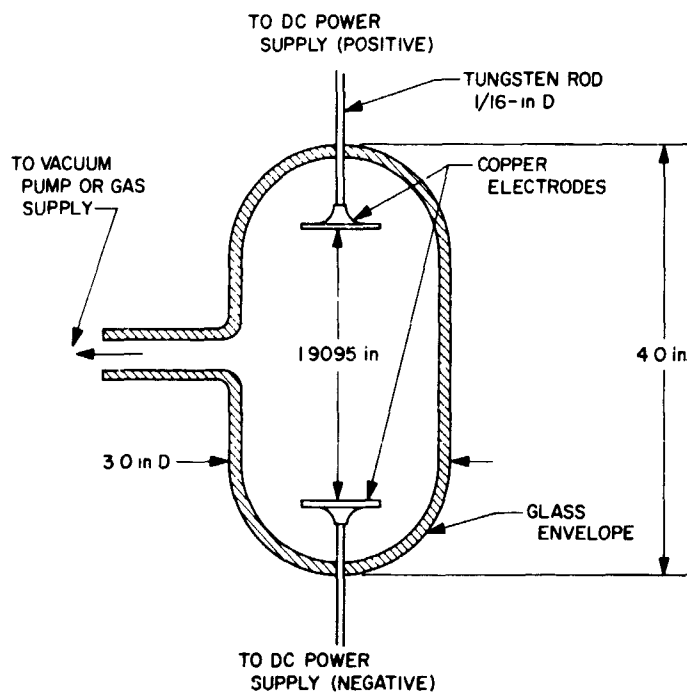


Fig. 2. Discharge tube

high purity levels for the minor constituent. After the minor constituent was placed into the mixing tank (Fig. 1) at the desired pressure, the remainder of the system was completely evacuated to less than 10μ in preparation for adding the major constituent to the mixing tank. The evacuating and purging procedure of the mixing system, excluding the mixing tank, was repeated several times for the major constituent to ensure purity. The final concentration or mixture ratio was computed from the partial pressures of the constituents. The gas mixture was then stored at high pressure in the mixing tank while undergoing forced mixing to provide a homogeneous mixture. This forced mixing was performed by a series of turbine blades (stirrer) located within the mixing tank. The turbine blade shaft was supported on ruby bearings with a magnet at the lower end of the shaft. On the exterior side of the mixing tank, adjacent to the magnet was located another magnet which was positioned on the shaft of a variable speed motor. These magnets provided a magnetic coupling between the turbine blade shaft and the motor shaft. This coupling eliminated the need for a rotating shaft and seal and their resulting contamination. This arrangement of motor and stirrer is shown schematically in Fig. 1. The leak rate of the entire system was approximately 5×10^{-5} ft³/min at 1μ for a total system volume of approximately 0.2 ft³. The mixing tank was helium leak-checked and found to have a leak rate which was negligible for the time periods and pressures concerned.

In order to avoid contamination of the final gas mixture, which was stored at pressures ranging from 1 to 10 atm in the mixing tank, the entire discharge tube and associated plumbing were thoroughly purged in a manner similar to the methods used in the mixing procedure. After thorough purging of the discharge tube and evacuating to a maximum pressure of $10\ \mu$, the test gas was allowed to enter the discharge tube until a pressure of 0.1 mmHg was obtained. This pressure was determined by the criterion of accurate pressure measurements using an aneroid-type pressure gauge. Certainly, other types of gauges are available for measuring pressure at these low regions, but to assure a high gas mixture purity and a reliable measurement, the aneroid type gauges were selected. The limited pressure range of these gauges was not detrimental for the purpose of this investigation but did restrict the determination of the Paschen curve over a wide range of pd (product of pressure and electrode separation). After breakdown information was obtained for the 0.1-mmHg pressure setting, the pressure was increased until breakdown could not be produced with the

available power supply. The actual experimental procedure for producing gaseous breakdown in the discharge tube was to gradually increase the primary voltage of the power supply until breakdown was observed. The breakdown potential was observed on an oscilloscope and recorded on polaroid film. The actual output voltage of the power supply was reduced by the use of a 10,000/1 precision voltage divider before connecting to the oscilloscope. These procedures are similar to those described in SPS 37-43, Vol. IV, pp. 221-223.

3. Results

Data were obtained with two grades of nitrogen and helium in order to demonstrate the effect of gas purity on breakdown potential. This effect is clearly shown in Fig. 3 as a departure of the two curves for the two grades of helium in the high and low regions of pd . This effect was not as apparent for the case of the two grades of nitrogen. The dependence of concentration for a mixture of nitrogen and helium is shown in Fig. 4(a). The cases of pure

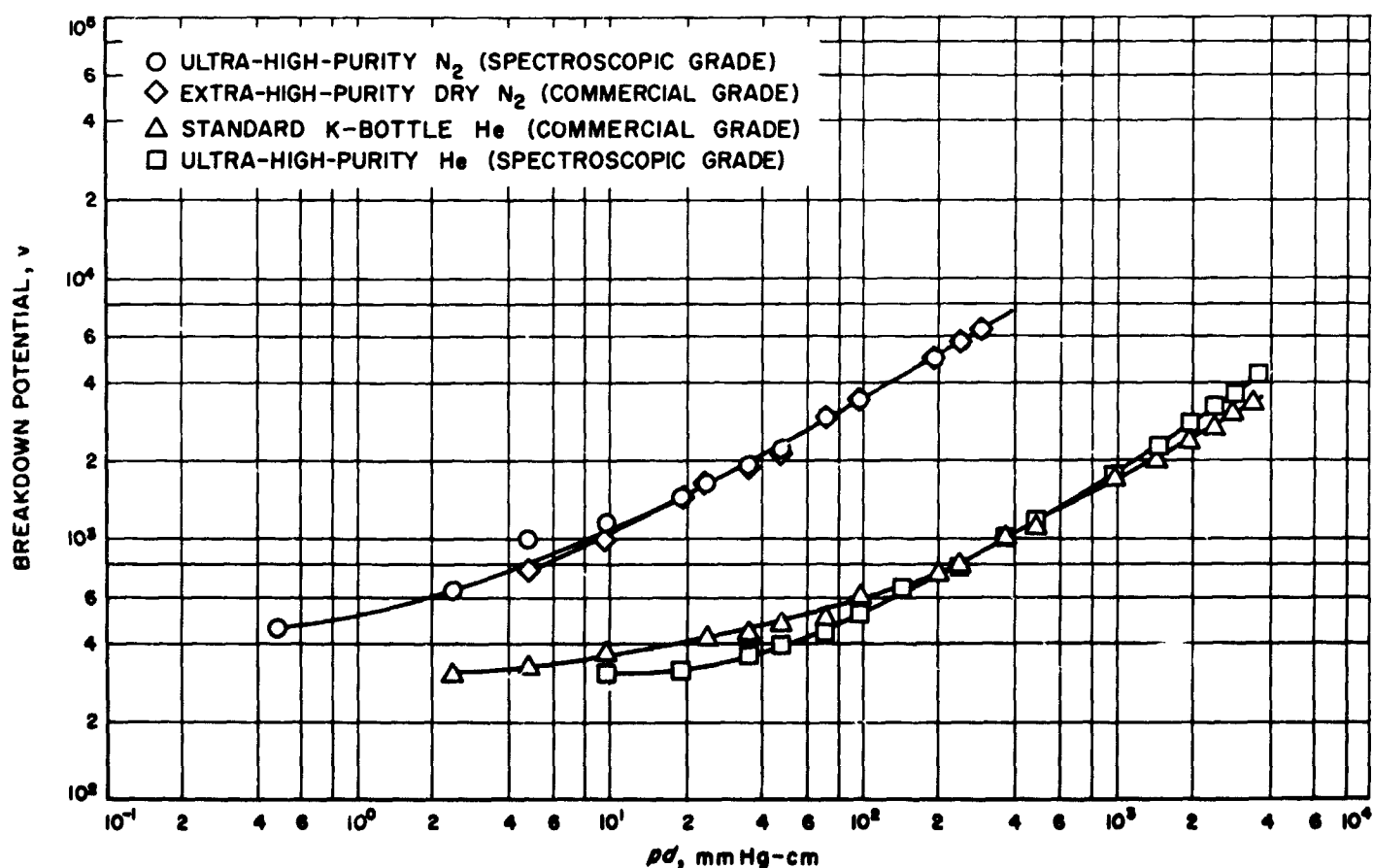


Fig. 3. Breakdown potentials for N_2 and He

nitrogen and pure helium establish the boundaries for this family of curves of breakdown potential as a function of electrode separation, pressure, and concentration. It should be noted that the mean curve of the breakdown potentials between the values for pure nitrogen and pure helium is in the neighborhood of a mixture of 90% helium and 10% nitrogen. These curves demonstrate that a small percentage of nitrogen in a nitrogen-helium mixture has a more dominant effect than a small percentage of helium in a nitrogen-helium mixture.

The experimental data for mixtures of argon and helium are presented in Fig. 4(b). Again the pure gases determine the boundaries for the family of curves for various concentrations, except in the lower regions of pd where it appears that the pure helium curve begins to level off and intercepts the mixture region. This departure from establishing the boundaries might even be expected for the case of nitrogen-helium mixtures in the lower pd regions if it is assumed that nitrogen and air are similar in breakdown characteristics. The intersection of the

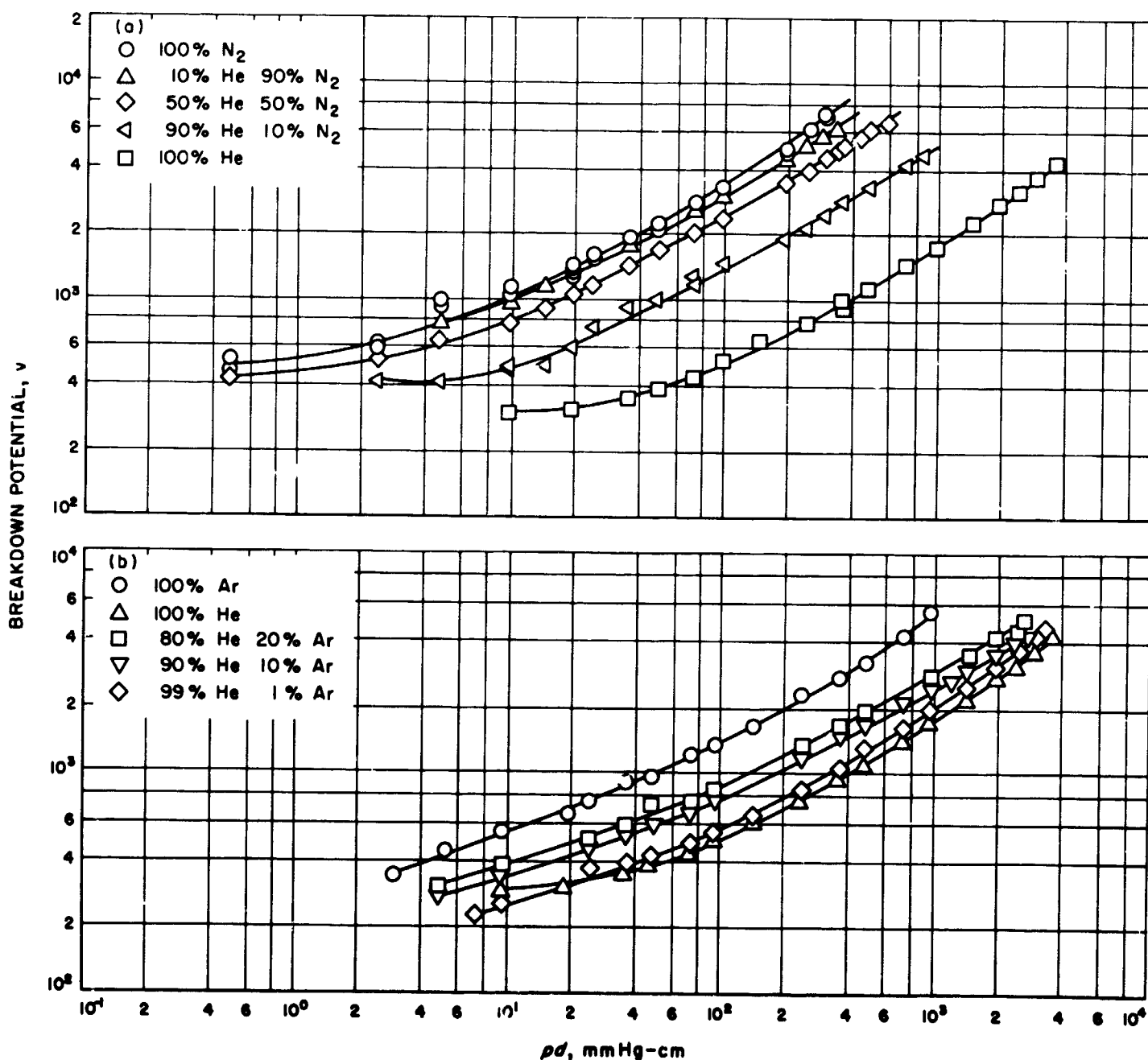


Fig. 4. Breakdown potentials for N₂ and He mixtures

Paschen curves for many pure gases and air is clearly seen in the data given by Druyvestyn and Penning (Ref. 1). A similar result to that exhibited in the nitrogen-helium mixture for the mean curve of the breakdown potential, for a constant product of electrode separation and pressure, is seen for the helium-argon mixture. The pure argon, throughout the pressure range covered in these experiments, always required a higher breakdown potential than the case of pure helium for a constant electrode separation of 1.9095 in. It is interesting to note this fact since, if it were assumed that the ionization potential controlled the breakdown potential, then helium would

have been expected to have the higher breakdown potential. The mean curve of breakdown potential, for a range in concentrations between the extremes is again about 10% argon in 90% helium. Consequently, a slight amount of argon in helium produces a noticeable effect; whereas, a small concentration of helium in argon does not exhibit such a dominating effect. To demonstrate this effect more clearly, two mixture ratios, 0.1 and 5% helium in argon, are shown in Fig. 5. Breakdown voltages were repeatable, for a constant product pd for a given concentration to within approximately 5%, as shown in a typical oscilloscope trace of several breakdown events in Fig. 6. This

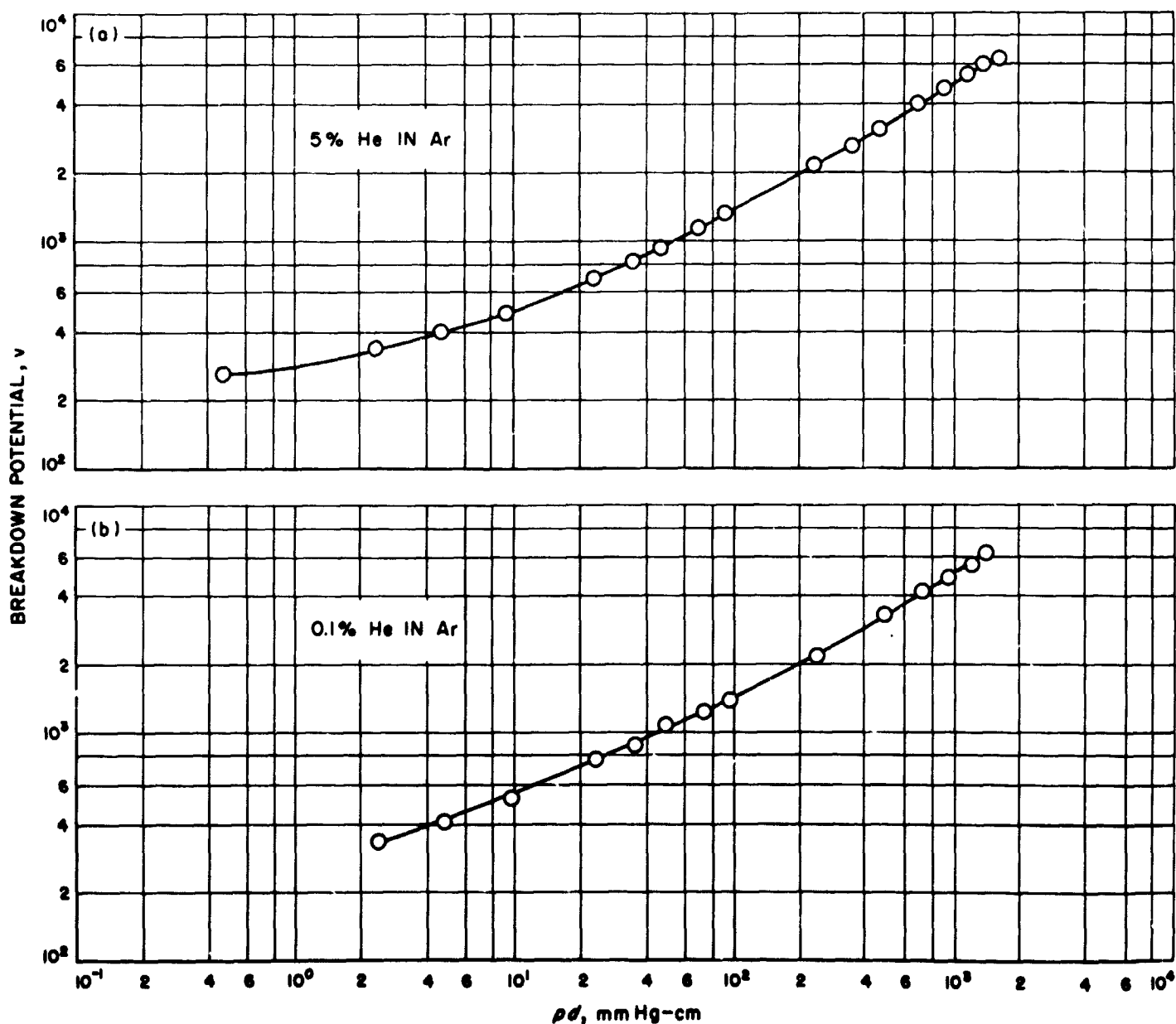


Fig. 5. Breakdown potentials for (a) 5% He in Ar; (b) 0.1% He in Ar

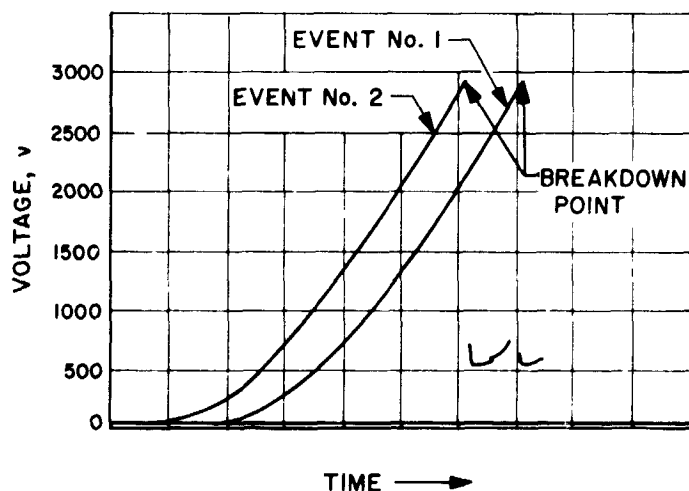


Fig. 6. Typical breakdown voltage-time curve showing two breakdown events for 100% spectroscopic grade N_2 at pd 72 mmHg-cm

figure represents the voltage-time characteristics of the power supply as the primary voltage is increased. At low values of pd on the Paschen curves presented here, the experimental error becomes dominated by pressure gauge errors of approximately 10%.

4. Summary and Conclusions

The effects of a secondary gas on the breakdown potential of the primary component of the resultant mixture were experimentally investigated for gaseous mixtures of nitrogen-helium and helium-argon. The results of this investigation show, for both the nitrogen-helium and helium-argon mixtures, that the breakdown potential does not have a linear relationship with concentration and is evidently not dependent on the ionization potential directly. It has been shown that the breakdown potential of a device operated on argon may be lowered by the addition of helium. This effect would also occur for a device if it were operated on nitrogen and a secondary gas of helium were added. The curves for the nitrogen-helium mixture would also be applicable to a system which has been helium leak-checked, and the effect of residual helium on a nitrogen system environment could be evaluated.

B. Liquid MHD Power Conversion, I. G. Hays, D. J. O'Connor, and G. M. Haskins

The long lifetimes required of electric-propulsion powerplants make systems without rotating parts attractive. A nonrotating Rankine cycle being investigated is

the liquid-metal magnetohydrodynamic system wherein lithium is accelerated by cesium vapor in a two-phase nozzle, separated from the cesium, decelerated in a magnetohydrodynamic generator, and returned through a diffuser and heat source to the nozzle.

A high-temperature corrosion experiment was completed which determined the effect of high flow velocities on the mass transfer of columbium-1% zirconium (Cb-1%Zr) alloy and yttrium oxide by lithium. Examination of the test section after 109 hr at 1141°C had previously revealed (SPS 37-43, Vol. IV, pp. 216-220) that complete dissolution of the yttria test specimens had occurred. The experiment was continued for an additional 391 hr at 1073°C in order that the Cb-1%Zr test specimens would be exposed for 500 hr. This period of time was required to obtain significantly measurable material loss for comparison with theory.

1. Mass Transfer Relations for Simple Dissolution

The mass transfer of wall material into a flowing stream when simple dissolution occurs is described by the conventional relation for fully developed turbulent flow in channels:

$$Sh = 0.023 Re^{0.8} Sc^{0.33} \quad (1)$$

The Sherwood number, which is analogous to the Nusselt number in heat transfer, is given by:

$$Sh = \frac{k_d d_h}{D_r} \quad (2)$$

where

k_d = mass transfer coefficient, cm/sec

d_h = hydraulic diameter, cm

D_r = diffusivity of columbium in lithium, cm^2/sec

The Reynolds number has its usual definition

$$Re = \frac{V d_h}{\nu} \quad (3)$$

where

V = lithium velocity, cm/sec

ν = kinematic viscosity, cm^2/sec

and the Schmidt number Sc is the ratio of ν to D_r .

If k_d is determined from Eq. (1) for given flow conditions and fluid properties, then the mass transfer rate from the wall in g/cm²-sec for the diffusion limited case is

$$\dot{m}_w = k_d \rho_s (C_w - C_b) \quad (4)$$

where

ρ_s = solvent density (g/cm³)

C_w = concentration of solute at wall $\left(\frac{\text{g solute}}{\text{g solvent}} \right)$

C_b = concentration of solute in bulk stream

If the concentrations correspond to the saturated concentration at the wall temperature and the saturated concentration at some other temperature in the system, then for small temperature differences Eq. (4) becomes

$$\Delta t_w = k_d \frac{\rho_s}{\rho_w} \left. \frac{\partial C_s}{\partial T} \right|_{T_w} (T_w - T_e) \tau \quad (5)$$

where

Δt_w = the thickness, cm, lost (or gained) from the wall in a time period τ , sec

ρ_w = density of wall material, g/cm³

C_s = saturated concentration at temperature T , $\frac{\text{g solute}}{\text{g solvent}}$

T_w = wall temperature, °C

T_e = effective concentration temperature in system, °C

Two properties of the columbium-lithium system required in these equations are not well known: D_r and C_s . The diffusivity can be estimated from the Einstein-Stokes relation with reasonable accuracy (Ref. 2):

$$D_r = \frac{kT}{6\pi\mu r_s} \quad (6)$$

where

k = Boltzmann constant

T = temperature, °K

μ = viscosity of solvent, poise

r_s = atomic radius at melting point of solute

$= (3 v_a / 4 \pi N_1)^{1/3}$, cm

v_a = atomic volume at melting point of solute, cm³

N_1 = Avogadro's number

The solubility of columbium in lithium can be estimated from the experimental data of Ref. 3. This data has a spread of greater than one order of magnitude, but the equation for the most reasonable fit is

$$\log_{10} C_s = -2.57 - 1310/T \quad (7)$$

for C_s in weight percent. Hence,

$$\frac{\partial C_s}{\partial T} = \frac{8.06}{T^2} 10^{1.310/T} \quad (8)$$

Over the range of temperatures tested

$$\frac{\partial C_s}{\partial T} \approx 4.8 \times 10^{-7} \% \text{ Cb in Li per } ^\circ\text{C}$$

or for weight fraction,

$$\frac{\partial C_s}{\partial T} = 4.8 \times 10^{-10} \frac{\text{g Cb}}{\text{g Li}} \frac{1}{^\circ\text{C}}$$

2. Prediction of Material Loss for Conditions of Experiment

For the 500-hr experiment the average velocities for the approach piping and the three test specimens were determined to be

$$V_a = 1.6 \text{ m/sec (5.25 ft/sec)}$$

$$V_1 = 19.6 \text{ m/sec (64.3 ft/sec)}$$

$$V_2 = 33.6 \text{ m/sec (110 ft/sec)}$$

$$V_3 = 48.5 \text{ m/sec (159 ft/sec)}$$

The average temperature as determined by pyrometer readings on a hohlraum was 1088°C. The lithium was taken to be saturated with columbium at the temperature of the Cb-1%Zr filter in the system at the base of the reservoir. The location of this filter is indicated in Fig. 7, which shows the test section, reservoir, and piping at 1073°C. The large filter surface area (80 cm²), small pore size (0.25 mm), and highly turbulent vortex flow in the reservoir should provide a sink for all columbium above the saturation concentration.

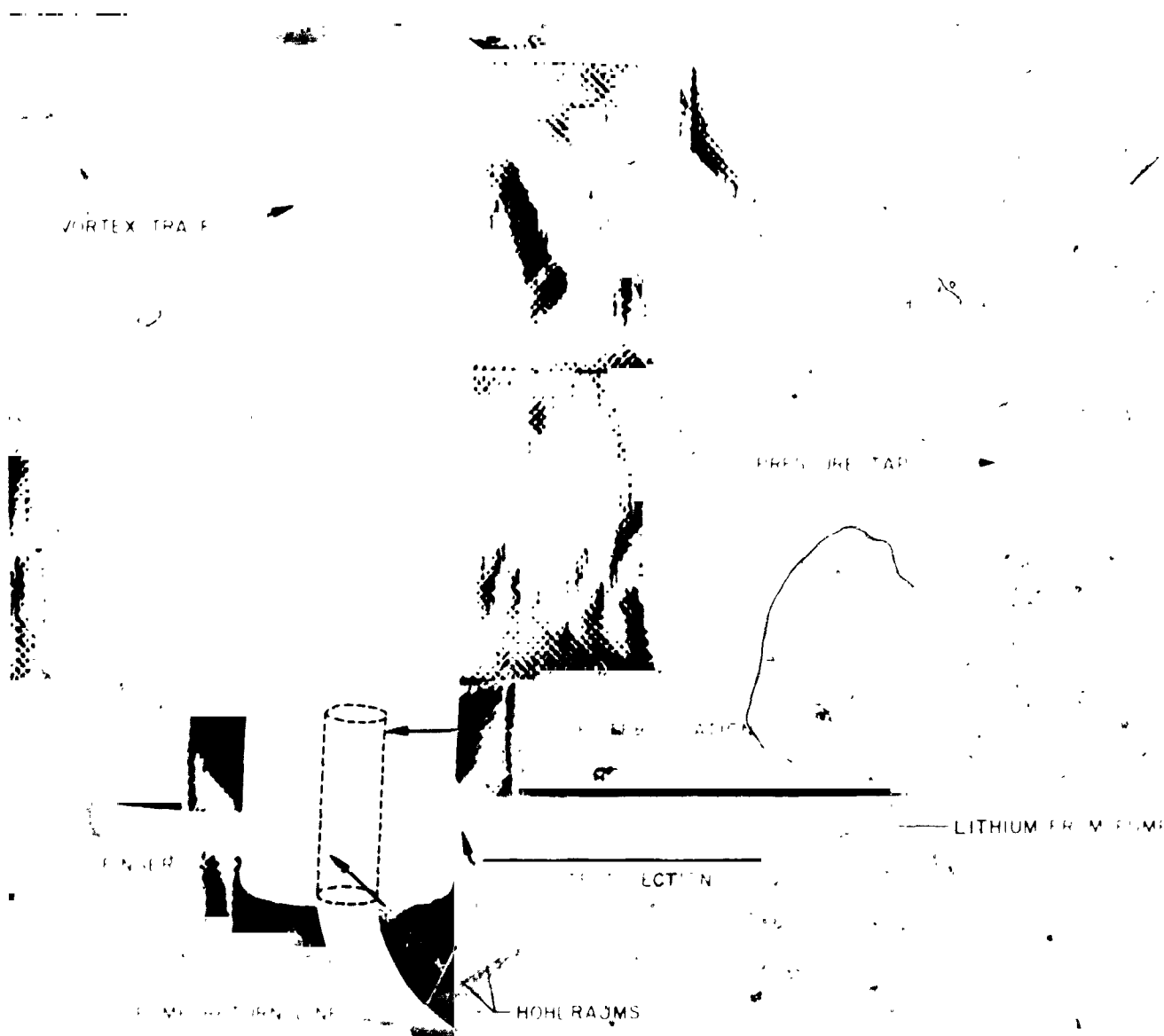


Fig. 7. High-velocity test section and reservoir at 1073°C

The temperature difference was obtained by calculating the heat radiated from the reservoir and then dividing by the product of the lithium mass flow rate and thermal heat capacity

$$\Delta T = \frac{\epsilon_1 \sigma A_1 (T_r^4 - T_c^4) + \epsilon_2 \sigma A_2 (T_r^4 - T_c^4)}{\dot{m}_a c_p}$$

where

ϵ_1 = emissivity of grit-blasted columbium = 0.32 from Ref. 4

σ = Stefan-Boltzmann constant

A_1 = area of reservoir that was grit-blasted

T_c = chamber temperature $\cong 530^\circ\text{K}$

T_w = wall temperature

ϵ_2 = emissivity of as-received columbium = 0.295 from Ref. 4.

A_2 = area of reservoir in as-received state

$\dot{m}_a$ = lithium flow rate = 202 g/sec

c_p = specific heat = 1.0 cal/g $^\circ\text{C}$

Substitution of the appropriate values gives $\Delta T = 7.7^\circ\text{C}$ for the 109-hr period and 7.5°C for the 391-hr period, or an average of 7.5°C . The following properties of lithium at the average temperature were determined from Refs. 5, 6, and 7, respectively:

$$\rho_s = 0.434 \text{ g/cc}$$

$$\mu = 0.15 \text{ centipoise}$$

$$v_a = 10.83 \text{ cm}^2/\text{g-atom}$$

Substitution of these values in Eq. (6) yields

$$D_v = 4.1 \times 10^{-4} \text{ cm}^2/\text{sec}$$

and

$$Sc = \frac{v}{D_v} = 8.7$$

Further use of D_v and Sc in Eqs. (1) and (5) gives the results shown in Table 1 for the 500-hr time period.

The maximum predicted mass loss would be 66.7 microns from the high-velocity wall. More detailed calculations of the increase in concentration of columbium in lithium around the test loop are in progress and are expected to decrease these values by a significant amount.

3. Test Results

The macroscopic appearance of the test specimens before and after 500 hr at an average temperature of 1088°C are shown in Figs. 8 to 14. The closeup pictures reveal that several changes occurred:

- (1) A crystalline deposit was formed in the high-velocity flow channel
- (2) Material removal was minor in the high-velocity, fully developed flow regions.
- (3) Material removal was heavy from the columbium downstream of a cavity left by dissolved yttria. This apparently resulted from local cavitation due to the sharp corner.
- (4) Moderately heavy cracking due to hydrogen embrittlement occurred on all surfaces exposed to lithium as a result of post-test removal treatment with water (Ref. 8).

The deposit was identified by chemical analysis to be yttrium. A photomicrograph showing the top of a crystal and the substrate in the highest velocity region is shown in Fig. 15. These crystals appear to have formed during the 10-hr cooling period after the test was completed from the yttrium previously dissolved in the lithium.

Table 1. Calculated dissolution parameters

Test specimen	d_h , cm	V , ml/sec	Re	Sh	k_a , cm/sec	Δt_w , microns	Δt_w , in.
Approach	1.904	1.6	8.80×10^4	4.25×10^4	0.091	3.0	1.2×10^{-4}
No. 1	0.440	19.6	2.50×10^5	8.05×10^4	0.75	23.9	2.9×10^{-4}
No. 2	0.362	33.6	3.62×10^5	1.24×10^5	1.40	46.5	1.8×10^{-4}
No. 3	0.308	48.5	4.32×10^5	1.52×10^5	2.02	66.7	2.5×10^{-4}

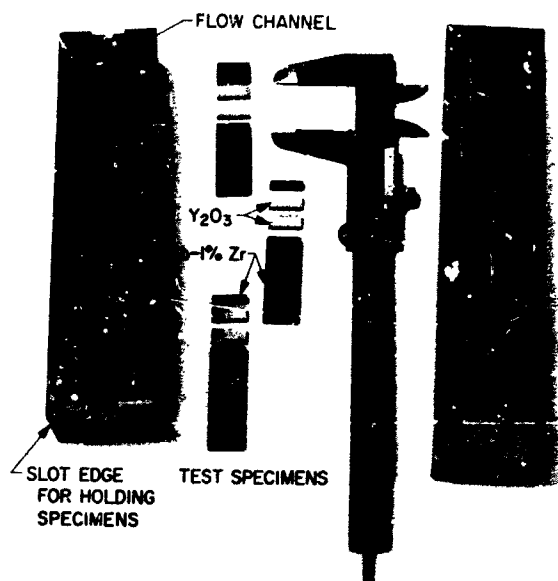


Fig. 8. Test section unassembled prior to welding

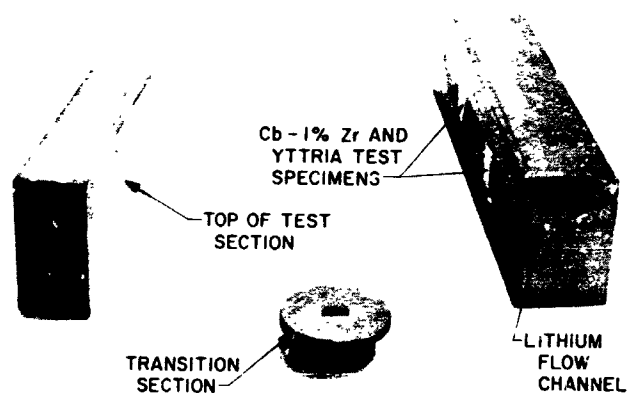


Fig. 9. Test section assembled prior to welding

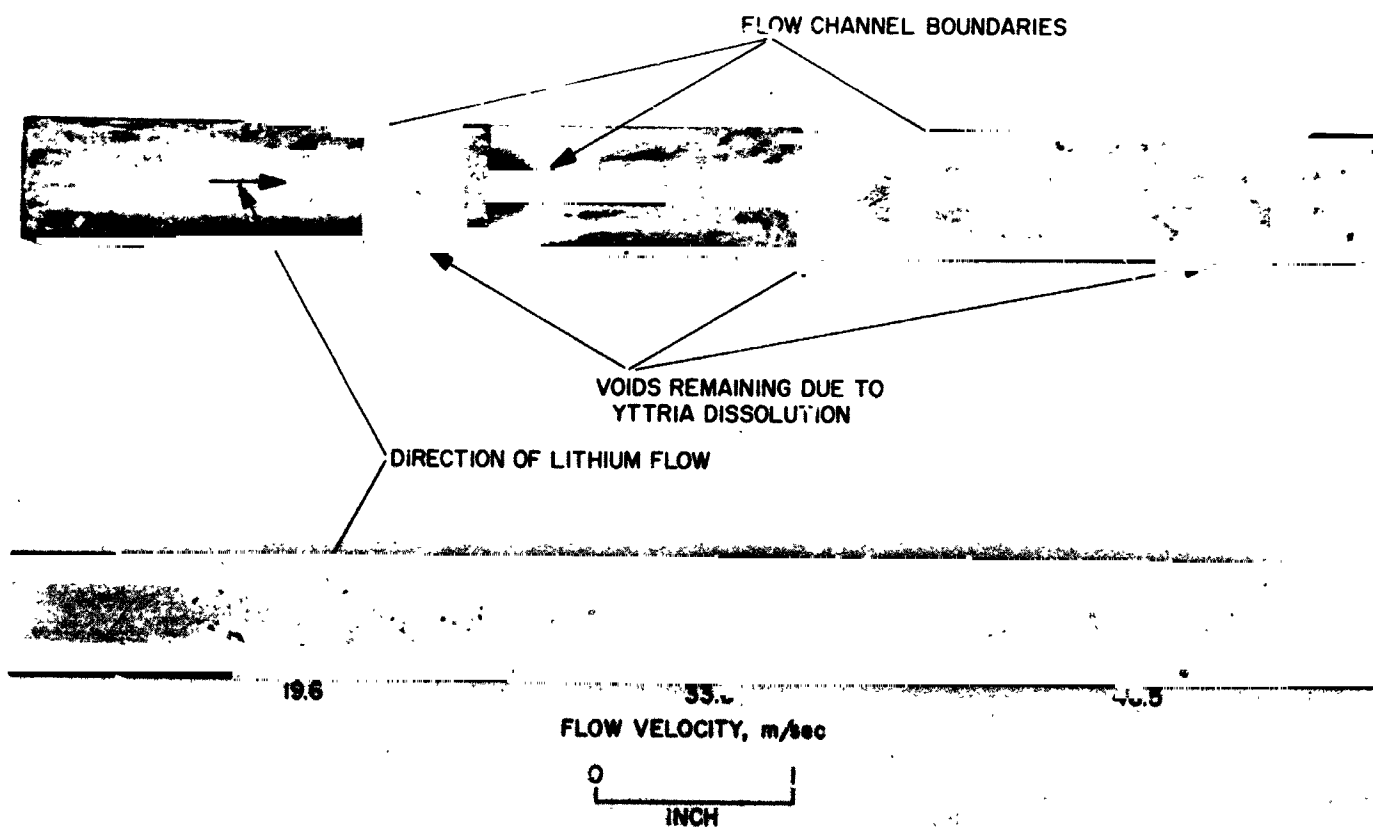


Fig. 10. Test section disassembled after 500 hr at 1088°C average temperature

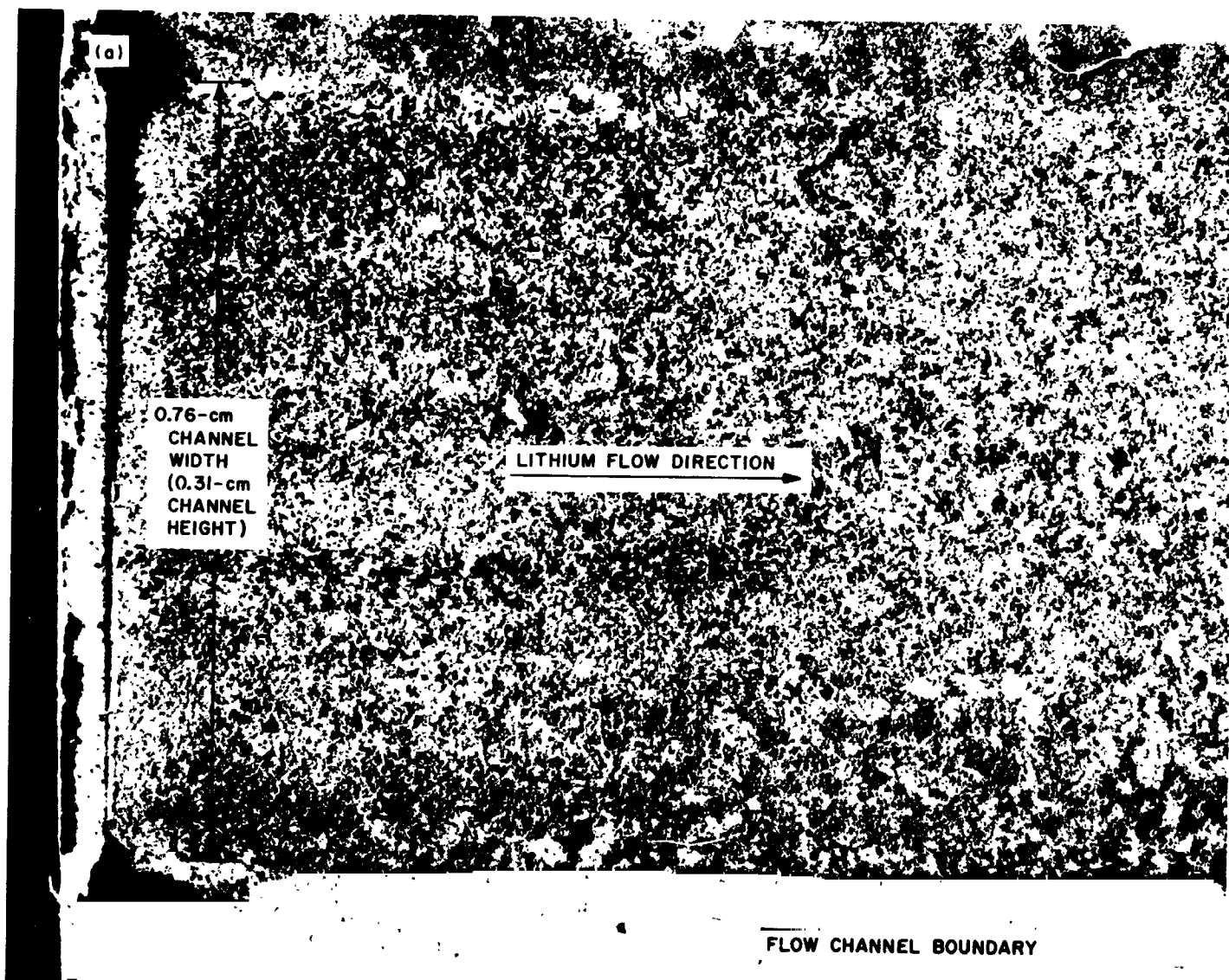


Fig. 11(a). First test specimen after 500 hr at 19.6 m/sec at entrance region

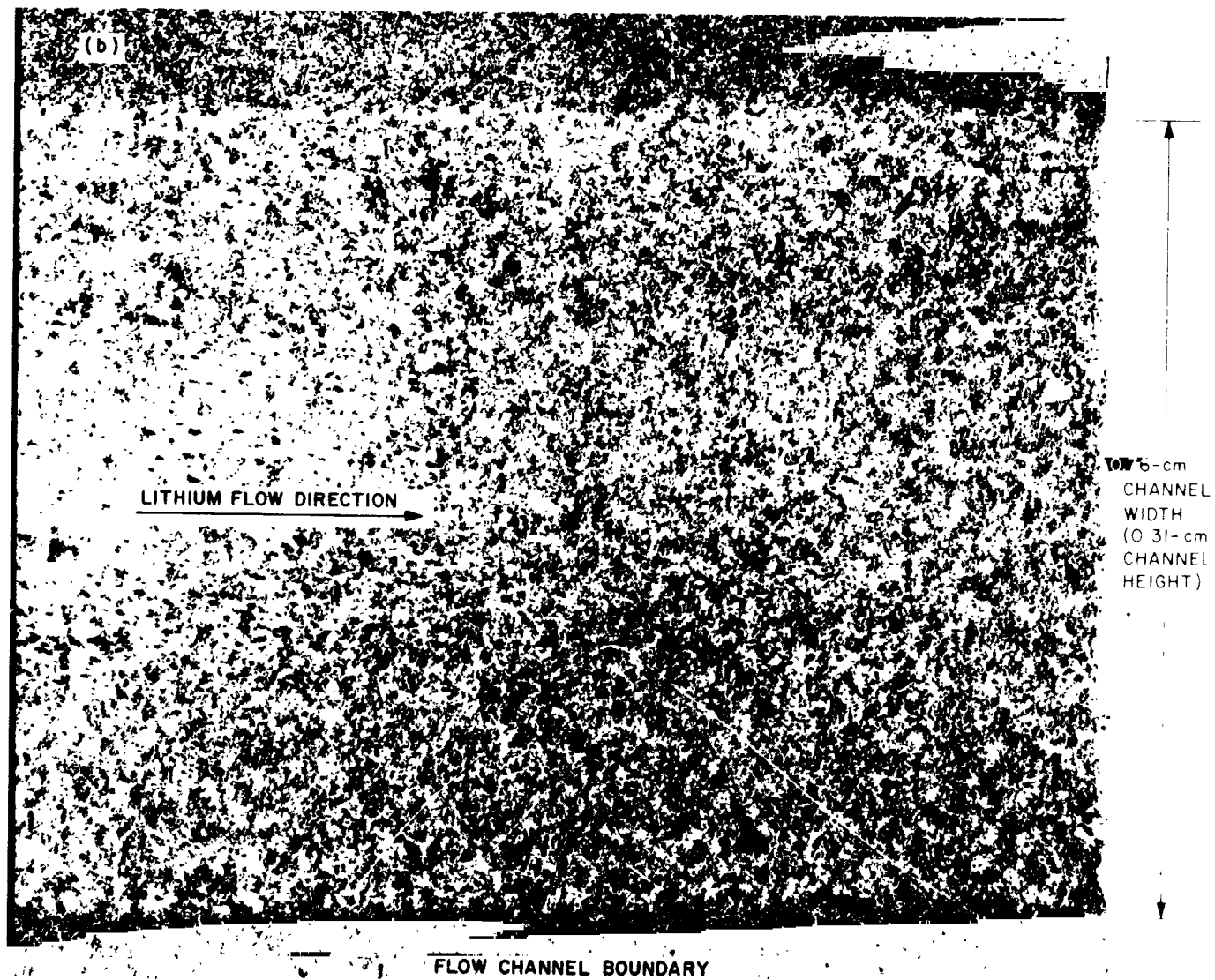


Fig. 11(b). First test specimen after 500 hr at 19.6 m/sec at exit region

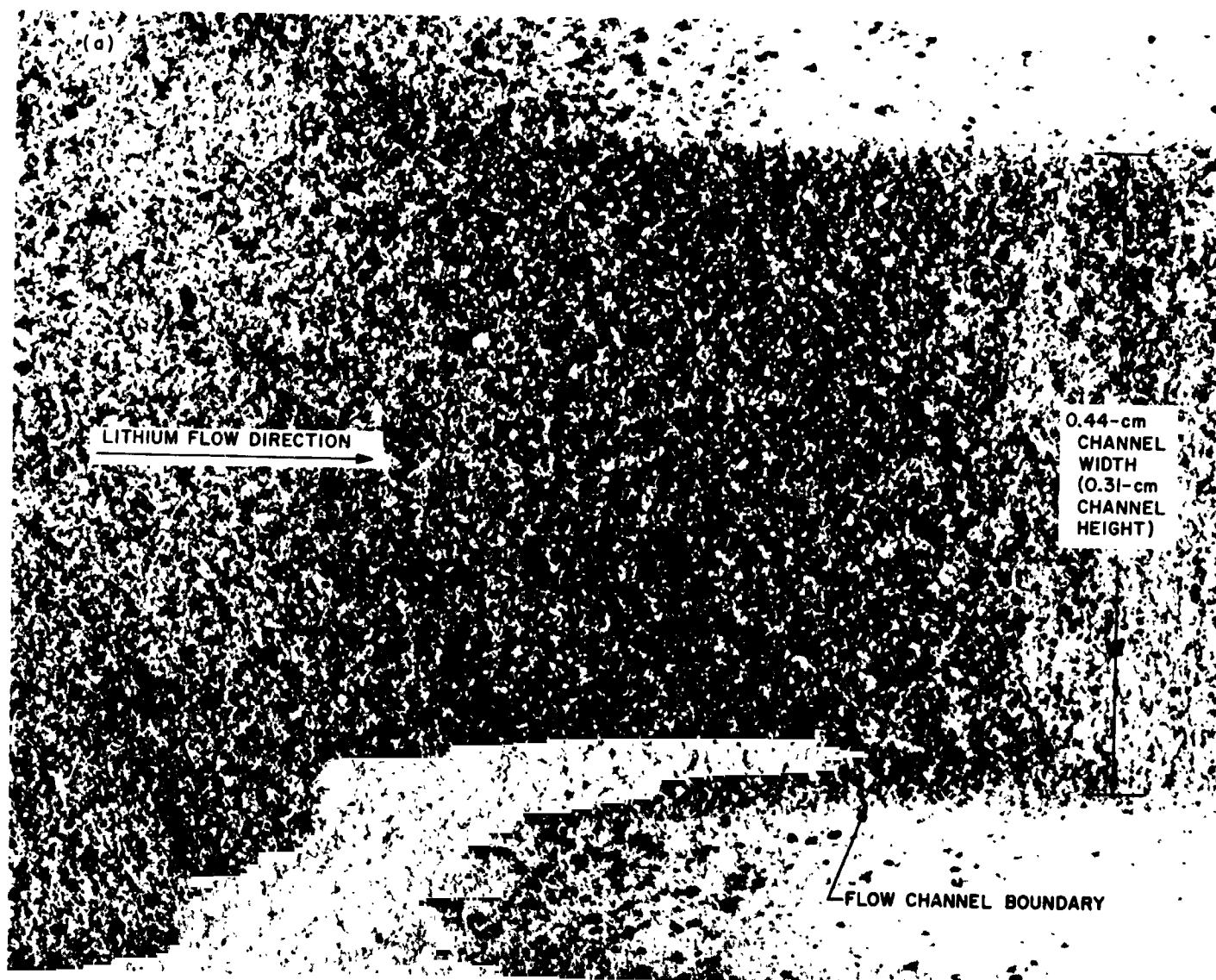


Fig. 12(a). Second test specimen after 500 hr at 33.6 m/sec at entrance region

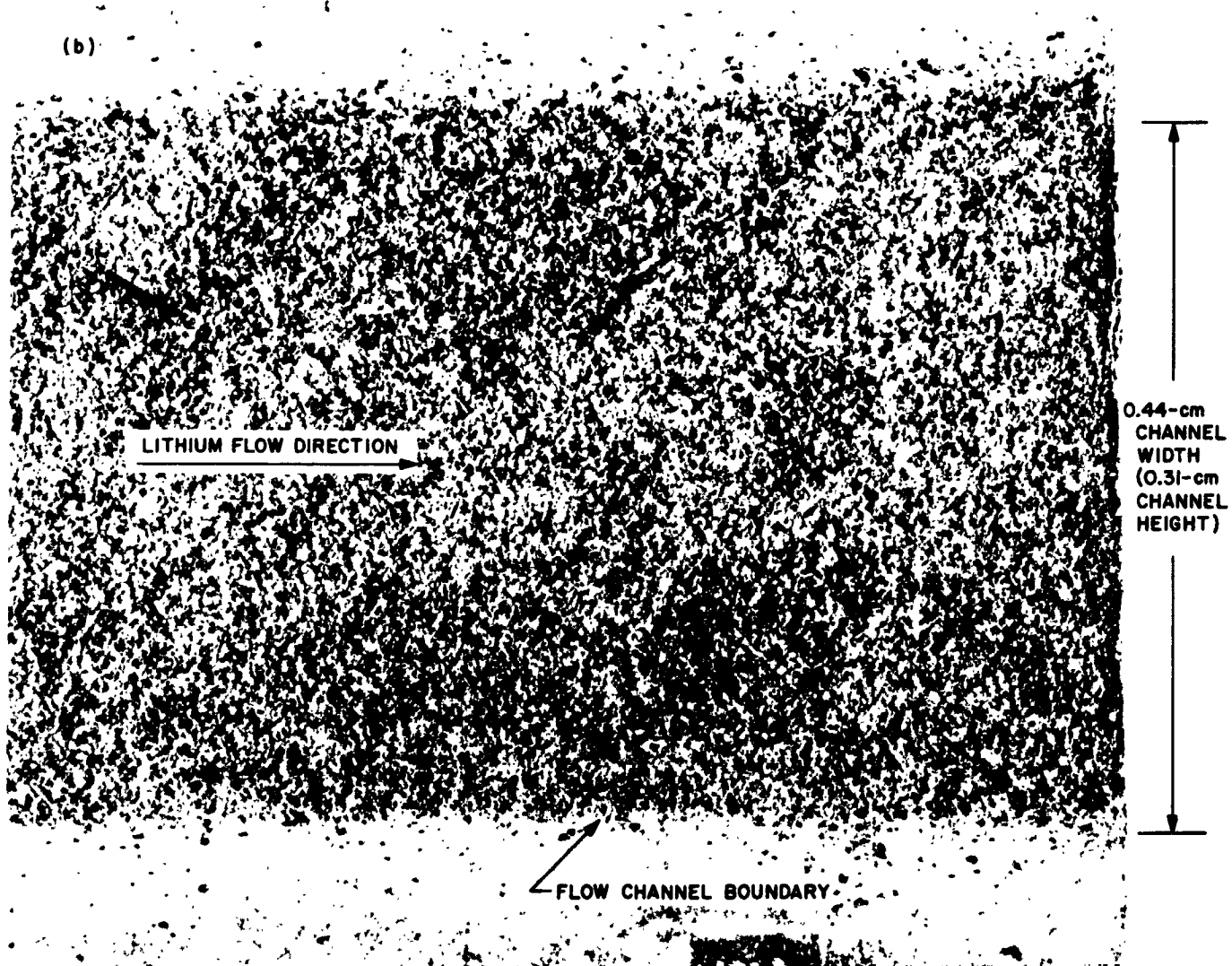


Fig. 12(b). Second test specimen after 500 hr at 33.6 m/sec at exit region



Fig. 13(a). Third test specimen after 500 hr at 48.5 m/sec at entrance region

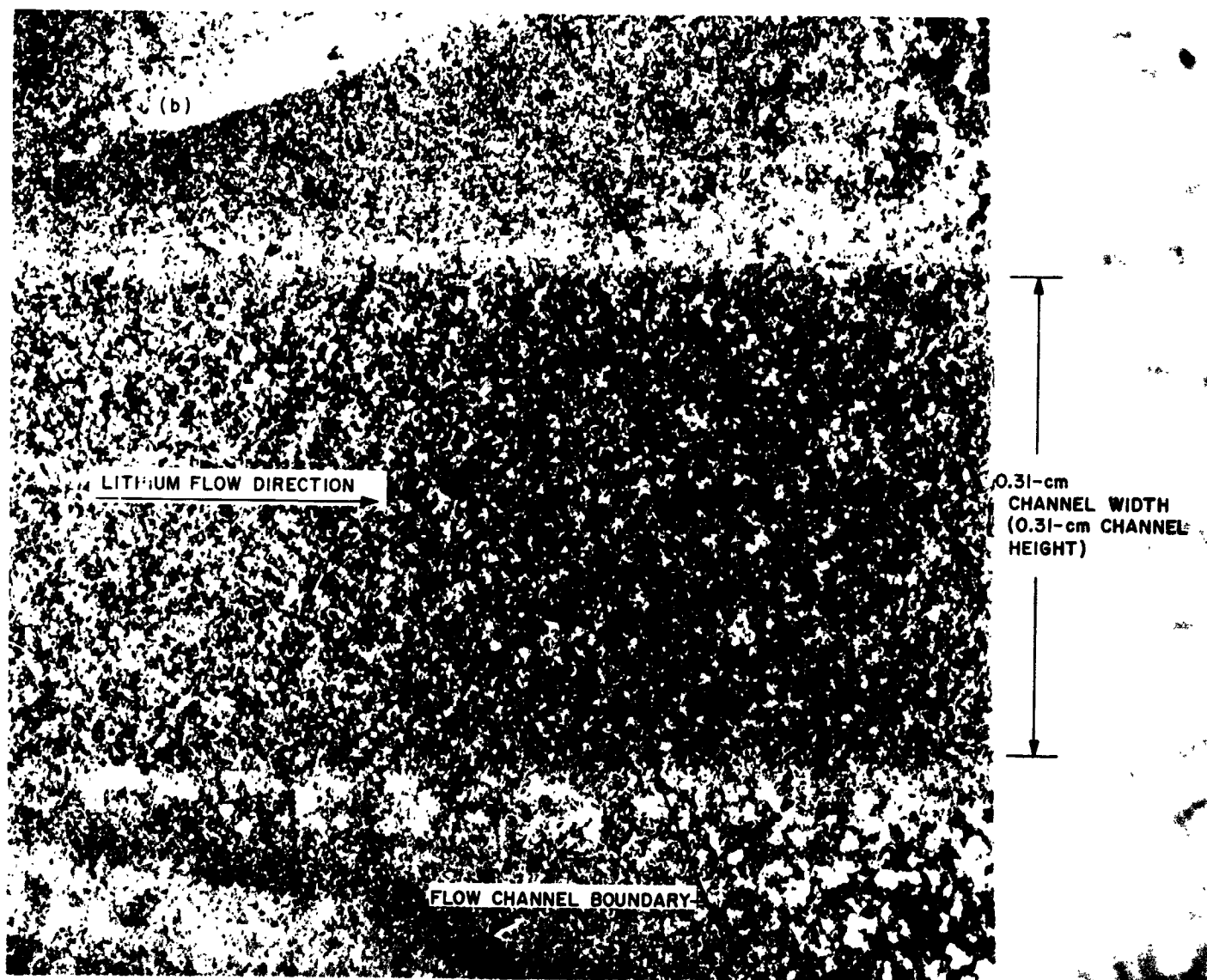


Fig. 13(b). Third test specimen after 500 hr at 48.5 m/sec at exit region

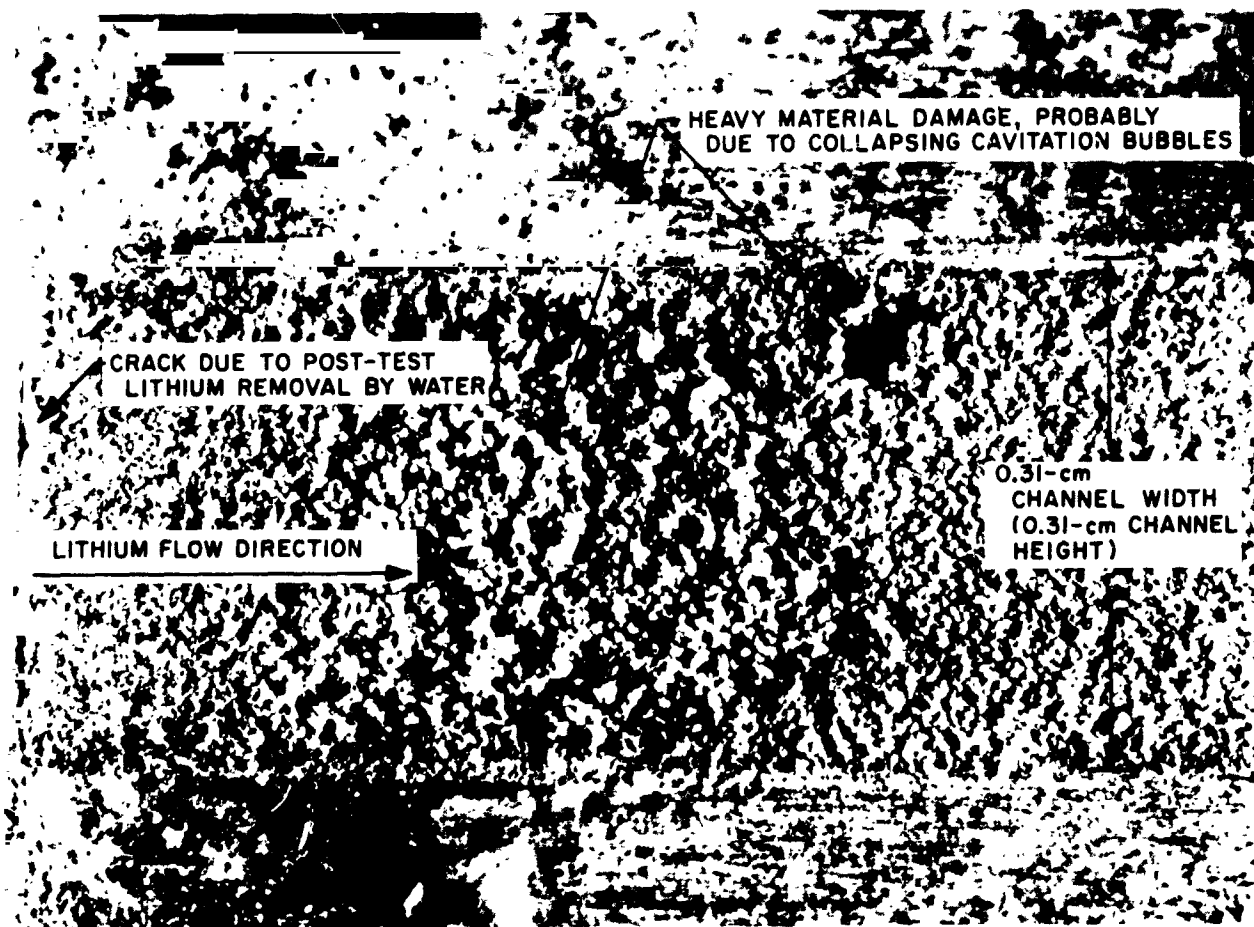


Fig. 14. Flow channel wall downstream of highest velocity step after 500 hr at 48.5 m/sec

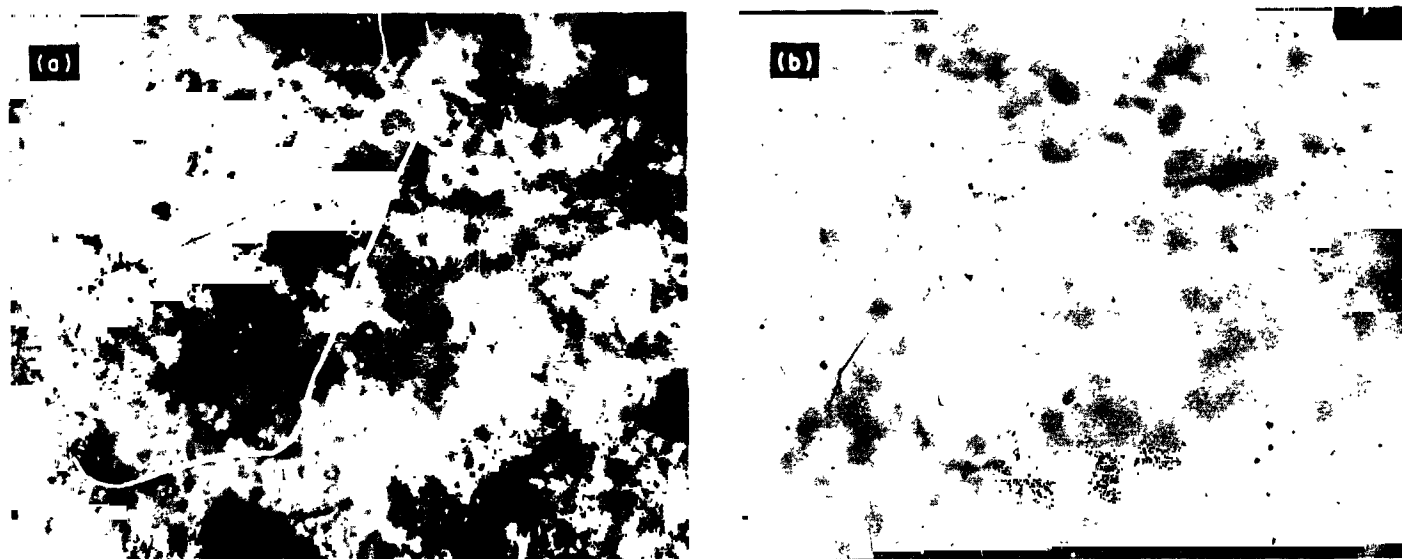


Fig. 15. Test specimen after 500 hr at 48.5 m/sec, $L/d_h = 10$ (250 ×) (a) surface; (b) crystal formation on specimen surface. Largest crystal ~ 54 microns high (Photo reduced to 82%)

Since the lithium was saturated with yttrium at the test temperature, slow cooling of the lithium while maintaining flow resulted in crystal formation on the wall at the high-velocity location. This conclusion is supported by observation that the grooving of the substrate surface, discussed below, was continuous from underneath the crystals to locations where no crystal was formed. A plot of the RMS roughness and average crystal height (measured by a surface profilometer) is shown in Fig. 16 as a function of the peak flow velocity. Both curves have a slope of about 0.8, close to that which would result if turbulent diffusion was the limiting growth mechanism. (The velocity ratios between the specimens remained constant during the cooling down process.)

Measurable material removal, although slight, definitely occurred in the fully developed flow regions. Figs. 15, 17, 18, and 19 are photomicrographs of the surfaces in the

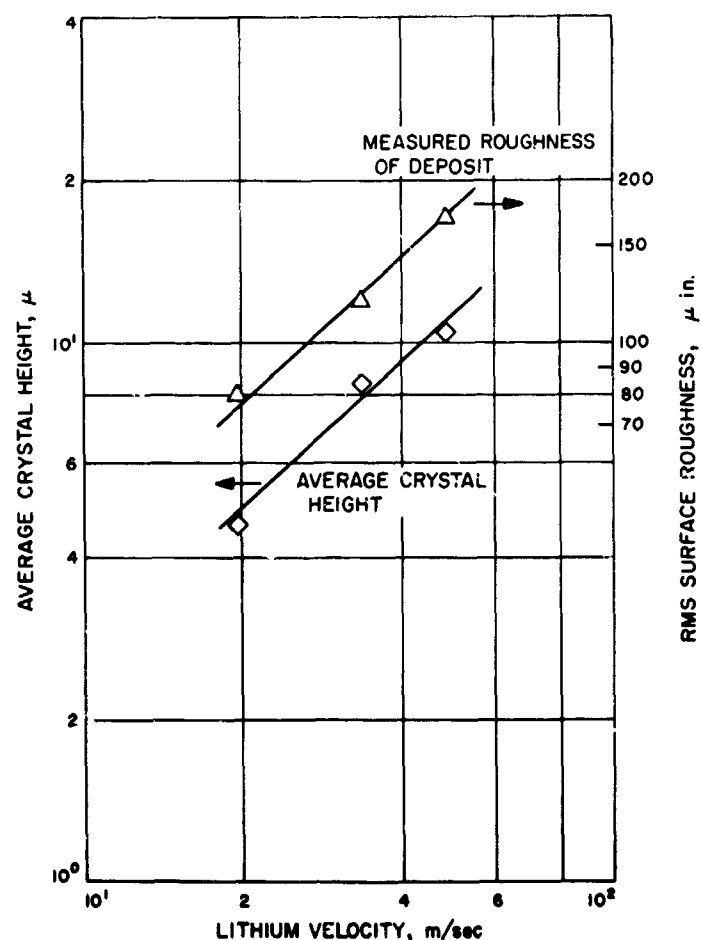


Fig. 16. Roughness and crystal height of yttrium deposit versus lithium velocity



Fig. 17. Test specimen surface in flow channel after 500 hr at 19.6 m/sec (250 ×)
(Photo reduced to 82 %)

flow channel and immediately adjacent to the flow channel (where seepage but little post-test deposition occurred). The most striking characteristic of these photographs is the pronounced grain boundary grooving and the appearance of dislocation pits. The direction of the grooves were completely independent of the direction of flow. The surfaces had no post-test metallurgical preparation, so the grooving and pit formation result from the dissolution of lithium. These surfaces have an appearance typical of over-etched columbium-1% zirconium specimens. A 750 × photomicrograph of the grooving is shown in Fig. 20.

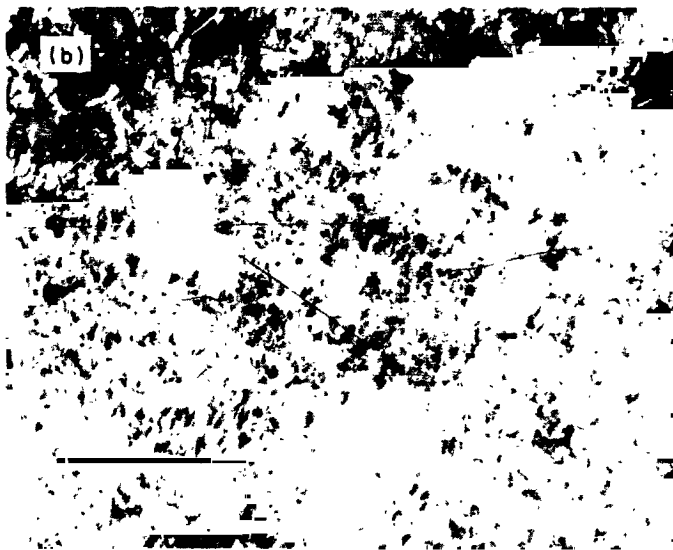
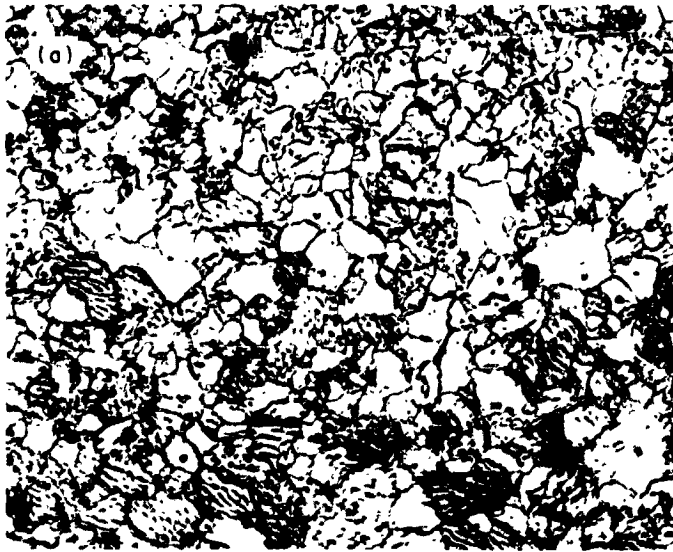


Fig. 18. Test specimen surfaces of 33.6-m/sec flow channel after 500 hr (250 $\times$): (a) adjacent to flow channel; (b) in flow channel (Photo reduced to 82%)

Traverses of the surfaces at $L/d_h = 10$ were made with a microscope with a calibrated vernier focusing mechanism. Divisions on the scale were in 2-micron increments, and repeatability was within about $\frac{1}{2}$ micron with the approach from a consistent direction. The results of measurements on the Cb-1%Zr are shown in Fig. 21 for the three different regions of flow velocity. The arrows define the limits of the flow region.

4. Comparison with Predicted Values

Two extremes of material loss were estimated as illustrated in Fig. 21. One method was measurement of the

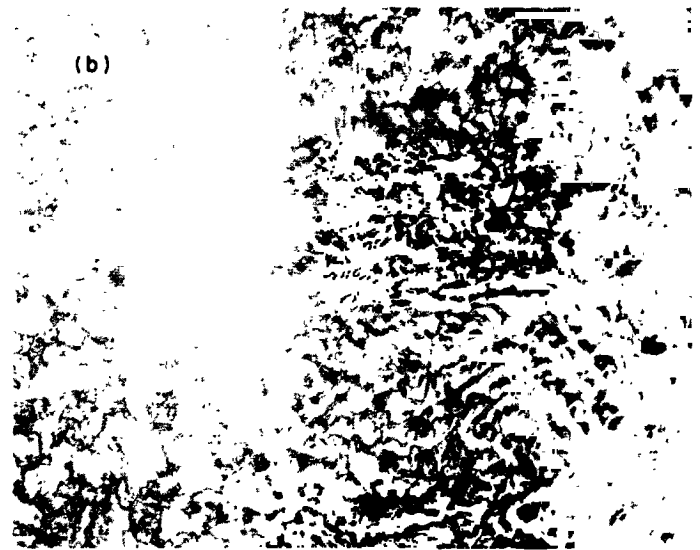
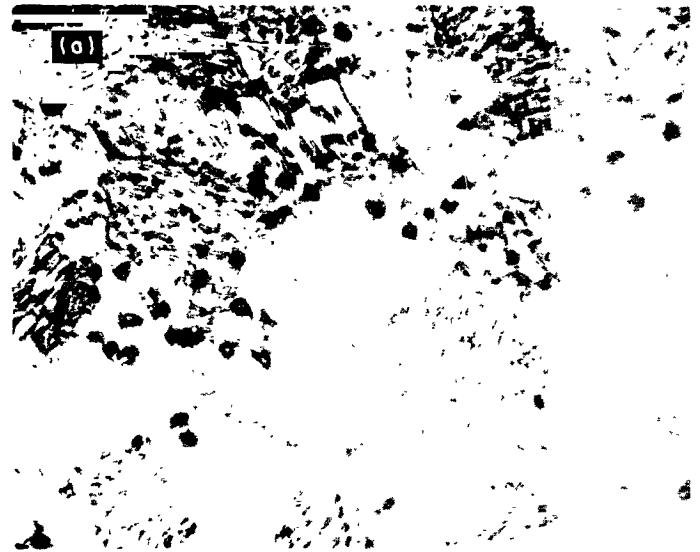


Fig. 19. Test specimen surfaces of 48.5 m/sec flow channel after 500 hr (250 $\times$): (a) adjacent to flow channel; (b) at boundary of flow channel (Photo reduced to 82%)

distance between the peak-to-peak tangent and the lowest valley in the flow channel. The other method was to fair a line through the surfaces outside of the flow region and measure the maximum valley depth below that line. The results of these measurements are plotted on Fig. 22 as a function of velocity. The calculated curve is shown for comparison. The most important finding is that the highest velocity test specimen exhibited a maximum material loss rate of about one-eighth of the predicted value. At 48.5 msec (159 ft/sec) the predicted value is 66.7 microns ($\sim 2\frac{1}{2}$ mil) while the measured estimates range from 5.5 to 8.5 microns. This factor of eight is actually



Fig. 20. Cb-1%Zr dislocation grooving due to lithium dissolution. Test specimen surface adjacent to 48.5-m/sec flow channel (750 X)
(Photo reduced to 82%)

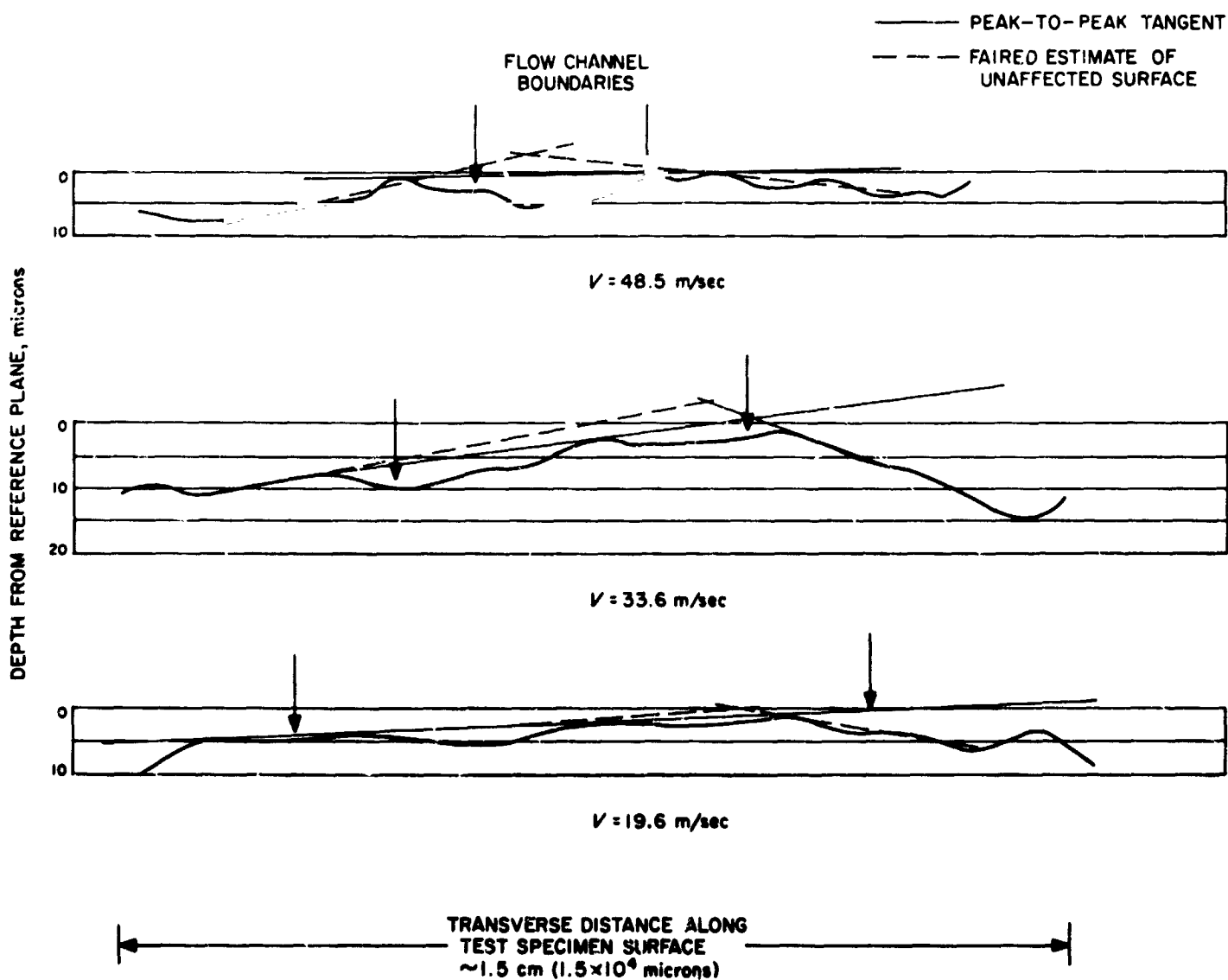


Fig. 21. Test section traverses $L/d_h = 10$

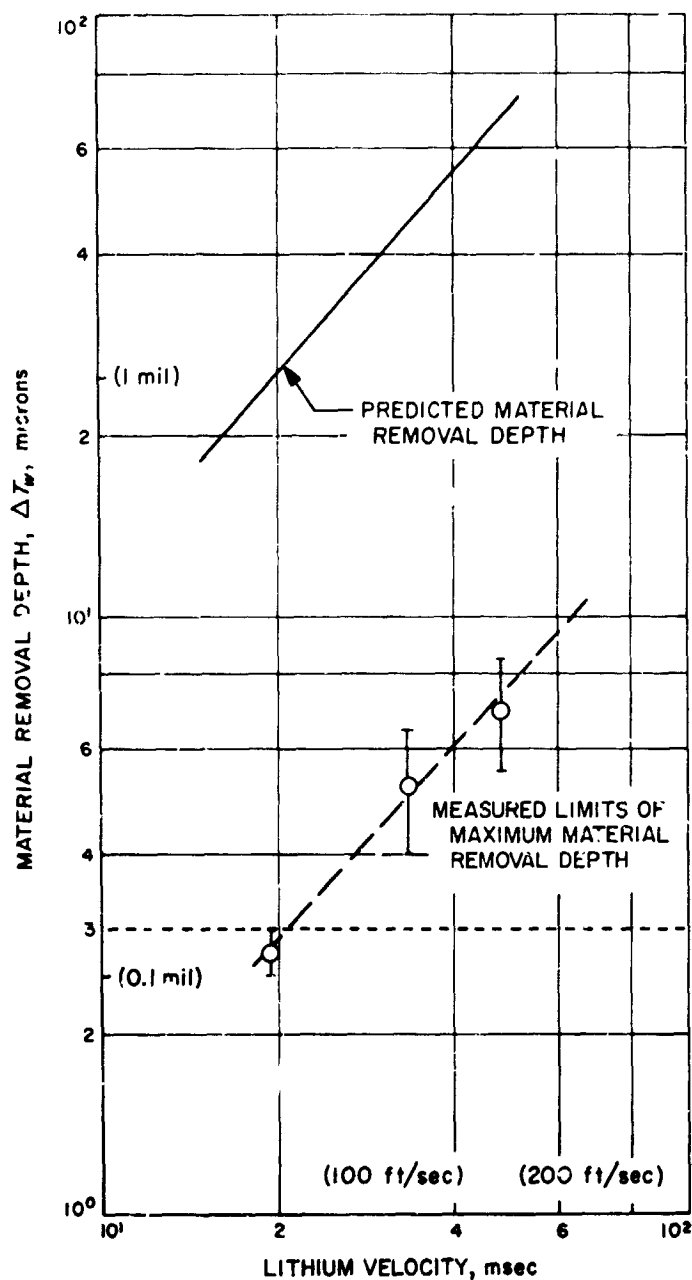


Fig. 22. Comparison of measured and predicted material removal depth versus lithium velocity. 500 hr at 1088°C average lithium temperature, differential temperature $\approx 7.5^\circ\text{C}$

well within the uncertainty of the existing solubility data (a factor of greater than 10 scatter) and the diffusivity calculation (a factor of 2 uncertainty). The slope of the test data is reasonably close to the slope of the predicted values (≈ 0.9) which result from a diffusion-limited process. Therefore, turbulent mass-transfer relations based on simple dissolution may be adequate for conservative estimates of mass transfer in columbium-lithium systems.

For the range 1070 to 1150°C, if the equation for diffusivity is taken as valid, a preliminary empirical relation for mass transfer based on the maxima of the test data is

$$\Delta t_w = 6 \times 10^{-10} k_d \frac{\rho_s}{\rho_w} \tau (T_w - T_c) \quad (10)$$

where k_d is given by Eq. (1).

Further refinement of these calculations, and chemical and metallurgical analyses of the other test loop components, are continuing. When complete, it is anticipated that the constant in Eq. (10) may change by a significant amount due to changes in T_c . In any event these tests have demonstrated that there is no unusual or unexpected corrosion effect resulting from high velocities, and the material loss is significantly less than would be predicted based on the mean values of existing solubility data.

Application of Eq. (1) will be illustrated by consideration of the 10,000-hr operation of a liquid MHD system with a maximum velocity of 150 m/sec (500 ft/sec). The maximum possible temperature difference in the high-velocity region would be on the order of 5°C . Since the flow rate would be approximately 80 kg/sec (130 lb/sec), this temperature difference represents a heat loss and kinetic losses of 1200 kw from the lithium stream, the most likely to be encountered. For this example, the hydraulic diameter at the high-velocity location is 0.38 cm, giving a channel Reynolds number of 1.7×10^6 . The Sherwood number is therefore 4.4×10^3 , and the mass transfer coefficient $k_d = 4.8 \text{ cm/sec}$.

Substitution in Eq. (10) gives $\Delta t_w = 0.027 \text{ cm}$ (0.011 in.). This loss (or deposit thickness) is within acceptable limits, since the wall thickness and minimum channel opening are several times as great. Further studies have been initiated to more accurately determine the temperature and velocity history of the lithium in a 300-kwe liquid MHD system. It is probable that the effective temperature difference will be much less than 5°C , giving a correspondingly smaller mass transfer value.

C. Lithium-Boiling Potassium Test Loop

Operations, Phase II, G. M. Kikin, W. M. Phillips, and M. L. Peelgren

1. Test Loop Operations: November 1966 to January 1967

The period between the end of Phase I and Phase II operations (June to November 1966) was devoted to field

installations and TIG welding into the loop of a new preheater, instrumentation changes, and repairs, addition of three Bourdon gauges to the potassium loop; changes and additions of loop trace heaters and radiator-condenser (R-C) heaters; repairs to valves and manipulators, installation of a screen to stop particulate matter from entering the argon-air heat exchanger; rerouting piping to eliminate argon flow through the radiator-condenser chamber; remagnetization of flowmeter magnets to bring flux back to full strength; repairs to the lithium heater, potassium preheater, and potassium pump power supplies; installation of a backup Dowtherm pump and motor, replacement of catalytic units in the argon purifier; repairs to the lithium pump power supply; and installation of an automatic argon makeup system to maintain tank argon pressure at the desired preset pressure level of about 5 psig.

Initial startup operations included outgassing the tank walls and the loop at 400 to 500°F and flushing the secondary loop with a full charge of potassium. After dumping the flushing charge, the secondary loop was "hard-filled" again with a fresh charge of potassium.

Several smooth starts and stops were made on the lithium pump to ascertain satisfactory pump operation. Heat losses from the boiler, preheater, and lithium heater were next determined under no-flow conditions. These losses were obtained by applying power to the trace heaters on the boiler and directly to the potassium preheater and lithium heater, so as to maintain these components at preselected temperatures. Under steady-state conditions, the power input equals the heat losses at a zero-flow condition. These heat loss tests were run at temperatures from 400 to 1200°F, and the results are plotted in Fig. 23.

Calibration of the potassium and lithium flowmeters was accomplished by comparing the flowmeter signals against the actual potassium and lithium flowrates. These flowrates were calorimetrically determined by measuring the rise in fluid temperature across the lithium heater and the potassium preheater for given power inputs. The calibration factors obtained are: $F_K = 200$ lb potassium/hr-mv and $F_{Li} = 134$ lb lithium/hr-mv.

With the secondary loop completely full of liquid potassium, tests were performed to determine the over-all average heat-transfer coefficient in the boiler for liquid-liquid operation. Variation of the potassium flowrate while maintaining the lithium flow constant will permit determination of the liquid lithium heat transfer

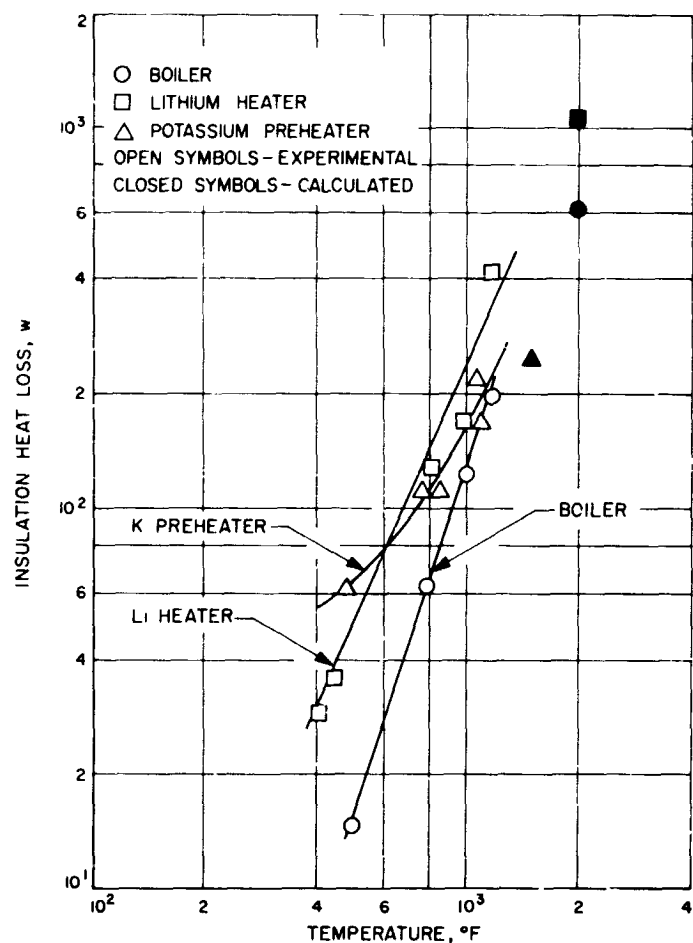


Fig. 23. Lithium heater, potassium preheater, boiler heat losses

coefficient using the "Wilson plot" technique, which is a graphical technique of interpreting heat-transfer coefficients in heat exchange.

Preliminary analysis of liquid-liquid heat-transfer test data indicates the over-all liquid-liquid boiler heat transfer coefficients to be ~ 1500 to 1650 Btu/hr-ft²-°F. This compares very well with the calculated value of U_o (over-all heat transfer coefficient) of 1630 Btu/hr-ft²-°F using the Lubarsky-Kaufman equation for liquid metal heat transfer in annuli.

Following calibration of the flowmeters, the lithium pump flowrate was then determined as a function of the pump speed. This calibration is shown in Fig. 24. The actual flowrate, at the nominal design pump speed of 3500 rpm, is ~ 1000 lb/hr, which is about 17% less than the design flowrate of 1200 lb/hr. During operations the pump was usually operated at ~ 2100 rpm, as previous high-speed operation had resulted in pump seizure. At this lower pump speed the flowrate is ~ 800

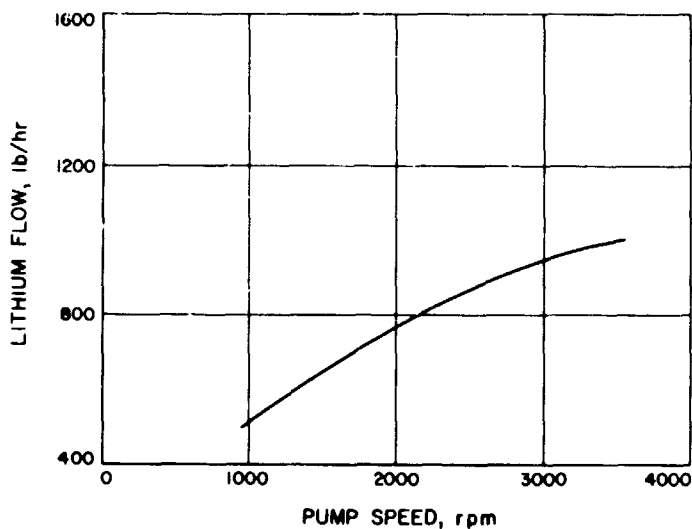


Fig. 24. Lithium flowrate versus pump speed

lb/hr which proved to be satisfactory, as the primary fluid flowrate is not a critical parameter in tests on this two-fluid experimental loop.

After the liquid-liquid tests were completed, boiling was initiated by draining about one-half the potassium out of the secondary loop into the potassium dump tank. Draining the loop of this quantity of potassium produces a liquid level in the radiator-condenser of about half its height. The potassium drain was accomplished in about five individual draining steps wherein a small amount of potassium was drained each time and then the drain valve closed in order to establish loop equilibrium conditions before the next potassium drain. The boiling startup oscillations which started at the time of potassium drain are shown in Fig. 25, which is a Sanborn 8-channel pen-recording showing the following loop

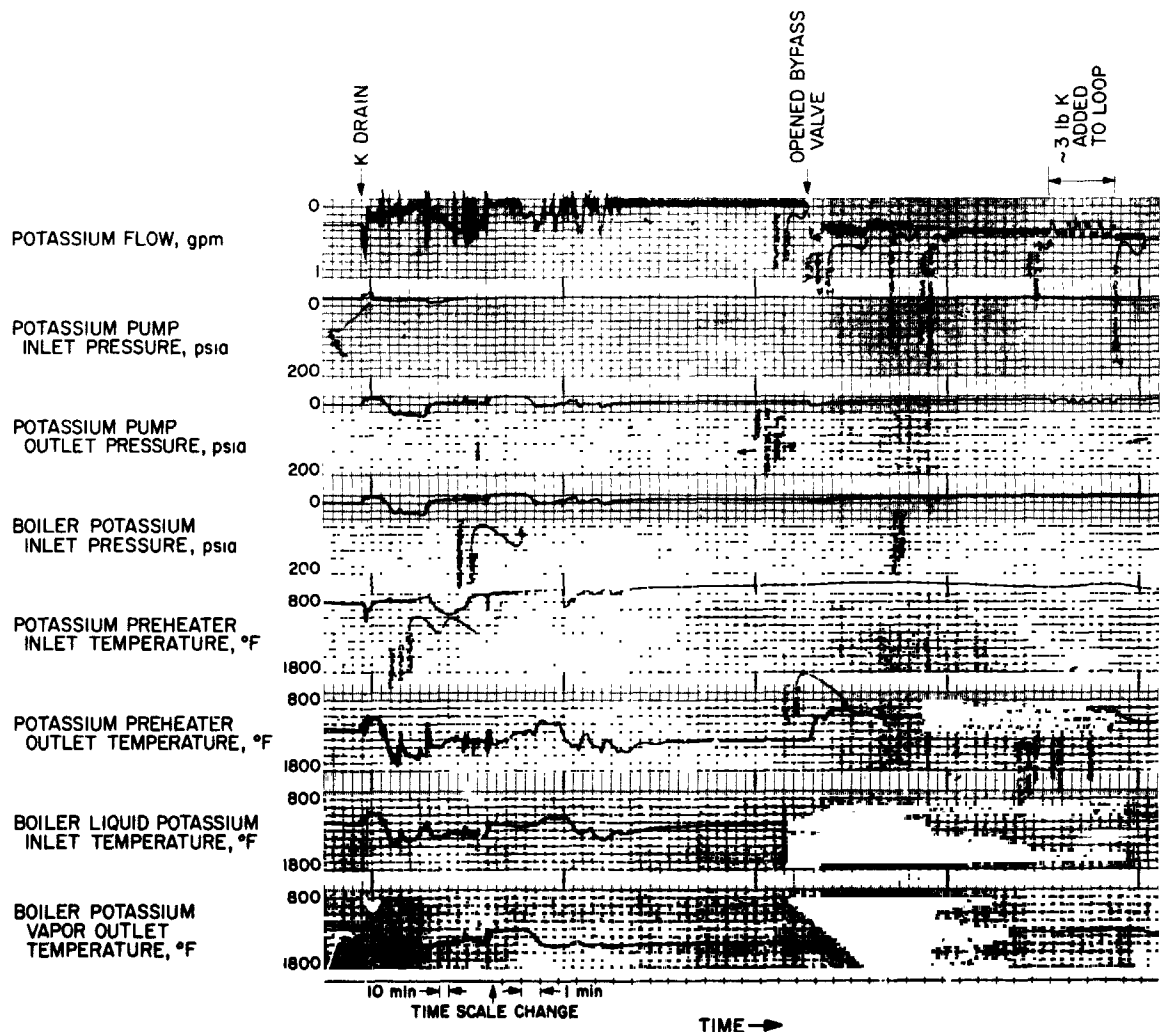


Fig. 25. Sanborn multichannel trace during boiling startup

parameters: potassium flowrate, potassium pump inlet pressure, potassium pump outlet pressure, potassium boiler inlet pressure, potassium preheater inlet temperature, potassium preheater outlet temperature, liquid potassium boiler inlet temperature, and potassium vapor outlet temperature.

Flow oscillations occurred after the turboalternator-bypass valve was closed. These oscillations eventually were damped out and fairly steady boiling occurred, but at a very low potassium flowrate of about 0.07 gpm. The flow oscillations were about ± 0.03 gpm. Opening the bypass valve increased the potassium flow to about 0.4 gpm. However, the flow oscillations increased to about 0.1 gpm (peak-to-peak). Increasing the potassium pump current from 750 to 1000 amp reduced the flow oscillations to about 40% of its former value. In order to determine if even these remaining flow oscillations were due to potassium pump cavitation, potassium was added to the loop from the dump tank, and the potassium level was brought up to two-thirds the height of the radiator-condenser. This potassium addition, plus concurrent heating (using radiator-condenser heaters) of the potassium condensate (thus raising the potassium pump

inlet pressure) resulted in an extremely steady liquid potassium flow. Steady-state boiling at high temperatures (1700 to 1900°F) and high powers (20 to 30 kw) were readily achieved. For this lithium-boiling potassium system, at least, satisfactory nominal operating condition stability can be achieved without resorting to feedback control. Fig. 26 is a typical Sanborn strip chart recording during steady-state boiling operations.

In order to determine the nature of the small potassium flow and pressure oscillations during smooth steady-state boiling, the flow and pressure signals to the Sanborn recorder were amplified $10\times$, and the chart speed was increased to its maximum of 100 mm/sec. Fig. 27 shows the resulting recording. The oscillations of shortest duration are those due to 60 cycle "noise" superimposed on the flowmeter and the thermocouple signals due to instrumentation noise or 60-cycle noise from the loop itself. It can be seen that the peak-to-peak fluctuation of the liquid potassium flowrate was only 0.01 gpm or only about $\pm 1\%$ of the nominal flow (0.45 gpm).

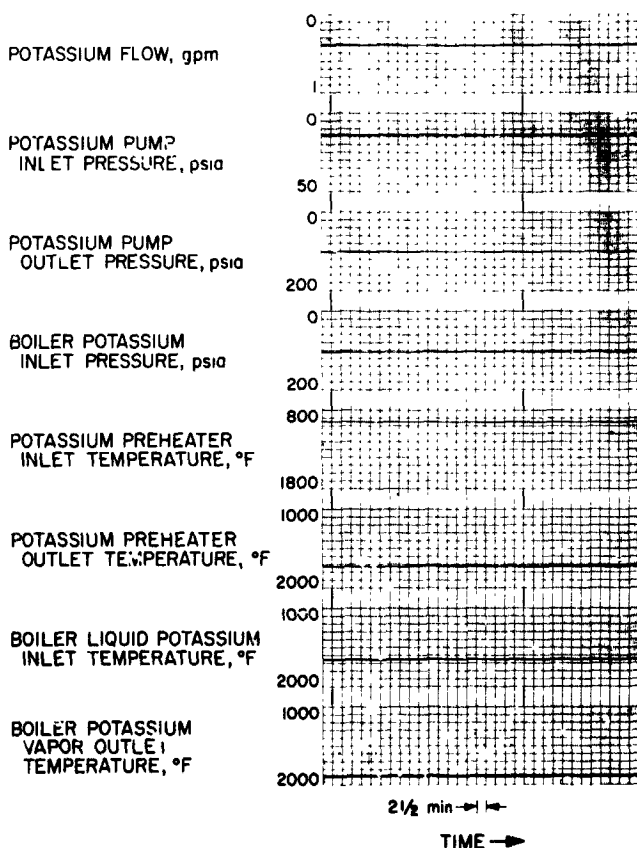


Fig. 26. Sanborn strip-chart trace: steady-state boiling

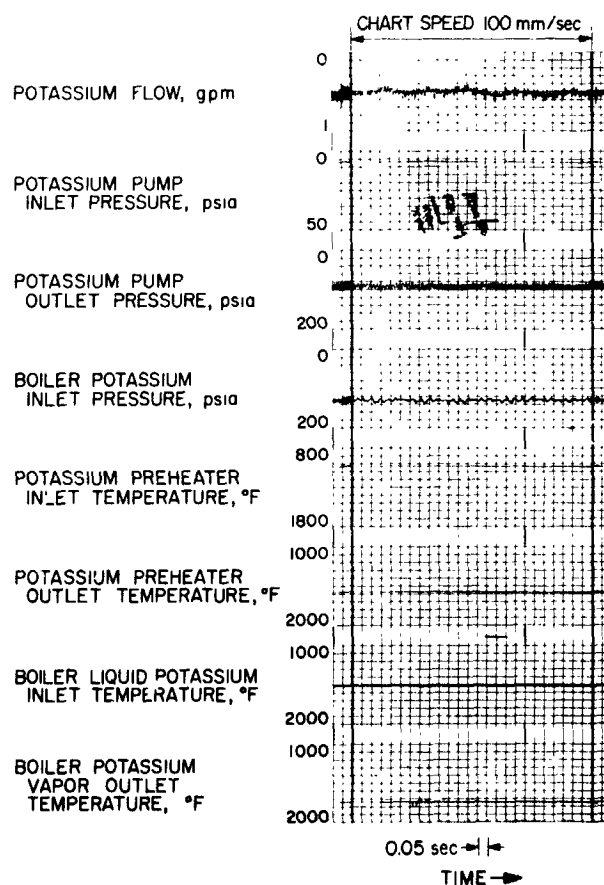


Fig. 27. Sanborn strip chart: steady-state boiling (high-speed and amplified signal)

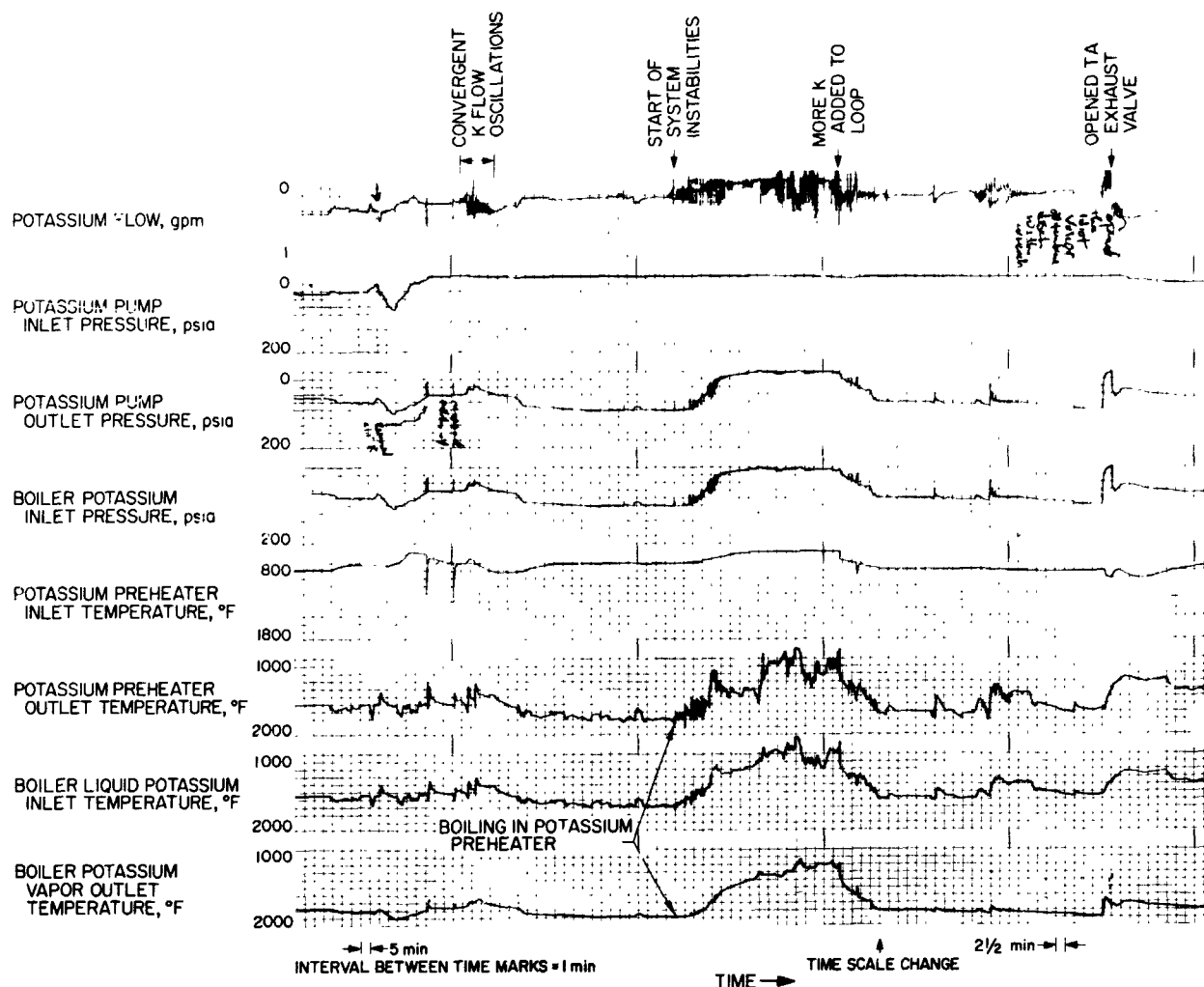


Fig. 28. Sanborn strip-chart recording of system instabilities

Erratic behavior was experienced when all the potassium vapor flow was directed to the turboalternator (T-A), but with the T-A exhaust valve only partially open. This flow restriction at the exhaust valve caused the T-A to back-fill with liquid potassium, causing periodic voiding of the liquid potassium condensate volume in the R-C, since the T-A internal volume is about equivalent to the potassium condensate volume in the R-C.¹ This caused pump cavitation which subsequently would result in flow oscillations and system instability (Fig. 28). Periodically

¹The potassium liquid vapor interface in the radiator condenser is determined by taking a trace of the individual radiator-condenser coil temperatures at any one time. In the condensing region in the top of the coil the temperature will be essentially constant, but will drop rapidly in the lower portion of the R-C which contains the potassium condensate. A typical R-C temperature trace showing the vapor liquid interface is shown in Fig. 29.

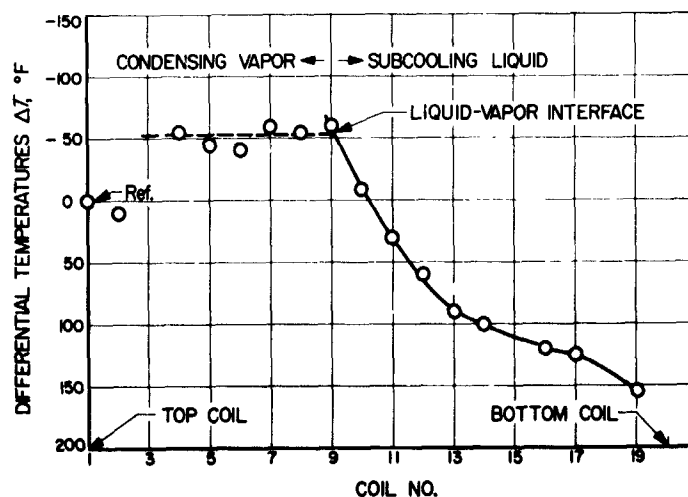
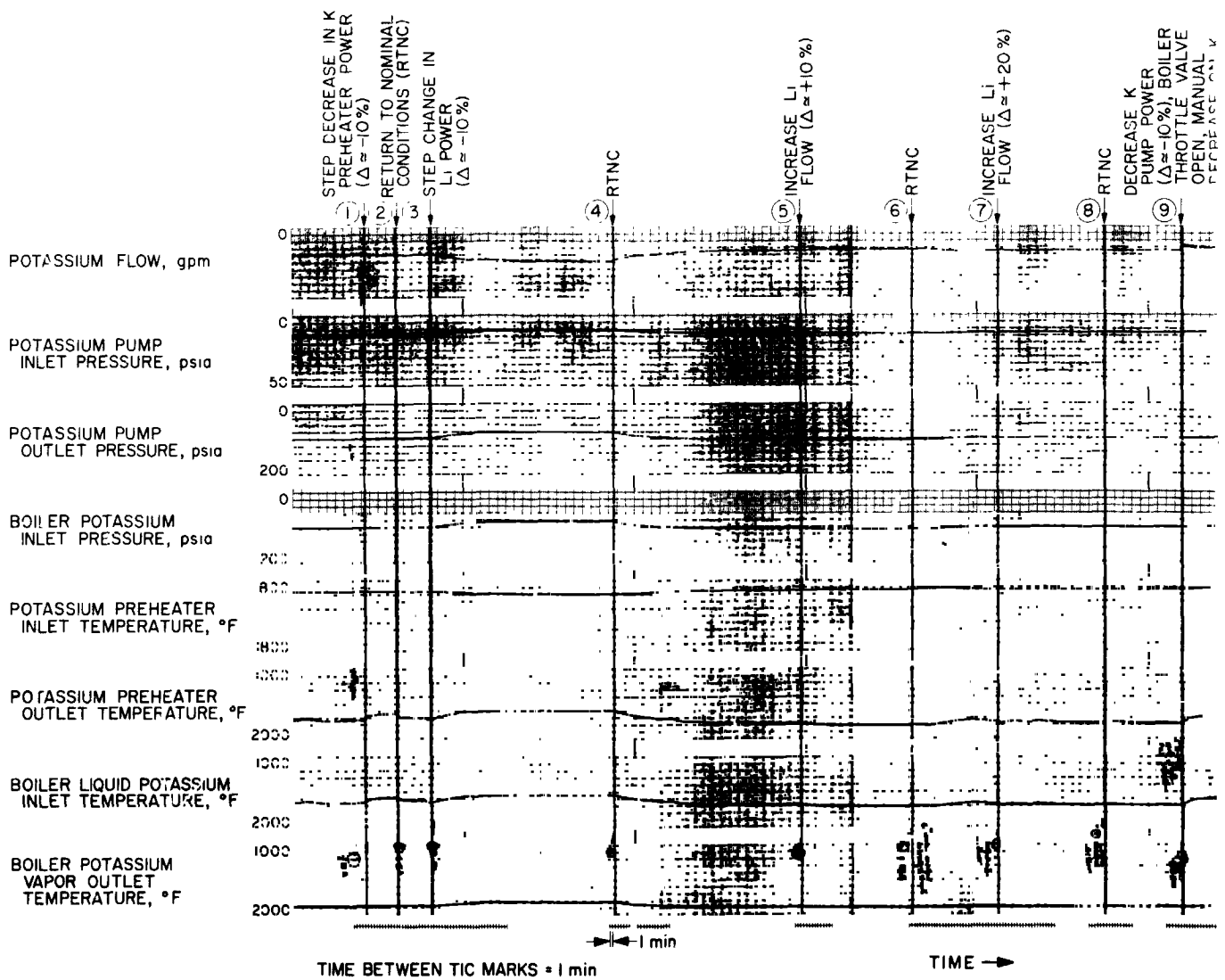


Fig. 29. Plot of radiator-condenser temperature versus axial-coil height



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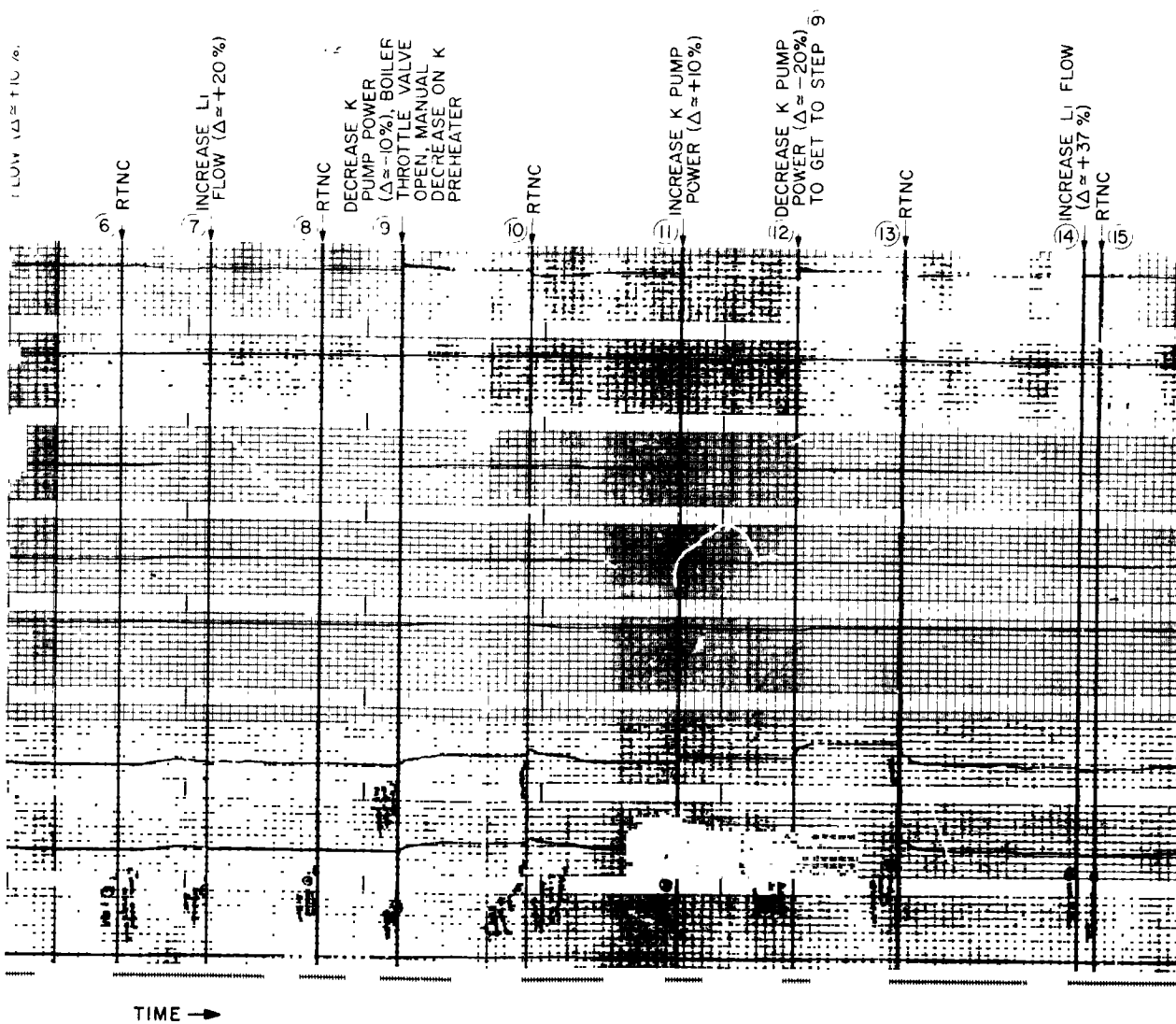


Fig. 30. Sanborn strip-chart recordings: transient tests 1-15

the liquid potassium in the T-A would drain back into the R-C with reestablishment of stable operation. This sequence of events leading to erratic loop behavior was relieved by further opening the T-A exhaust valve; nominal potassium flow then ensued, and system stability was excellent.

A series of tests were performed to determine transient system behavior for various perturbations. The various perturbations (all of which were essentially step changes) fall under the following major classification:

- (1) Step changes in potassium preheater power
- (2) Step changes in lithium heater power
- (3) Step changes in lithium flow

- (4) Step changes in potassium flow
 - (a) Boiler throttle valve fully open
 - (b) Boiler throttle valve partially closed
- (5) System scram—lithium power shutoff

The Sanborn chart recordings of the transient tests (Nos. 1 to 22) are shown in Figs. 30, 31, and 32.

The descriptive title of each of these transient tests are as follows:

- (1) Step decrease in potassium preheater power ($\Delta \approx -10\%$)
- (2) Return to nominal conditions
- (3) Step change in lithium power ($\Delta \approx -10\%$)

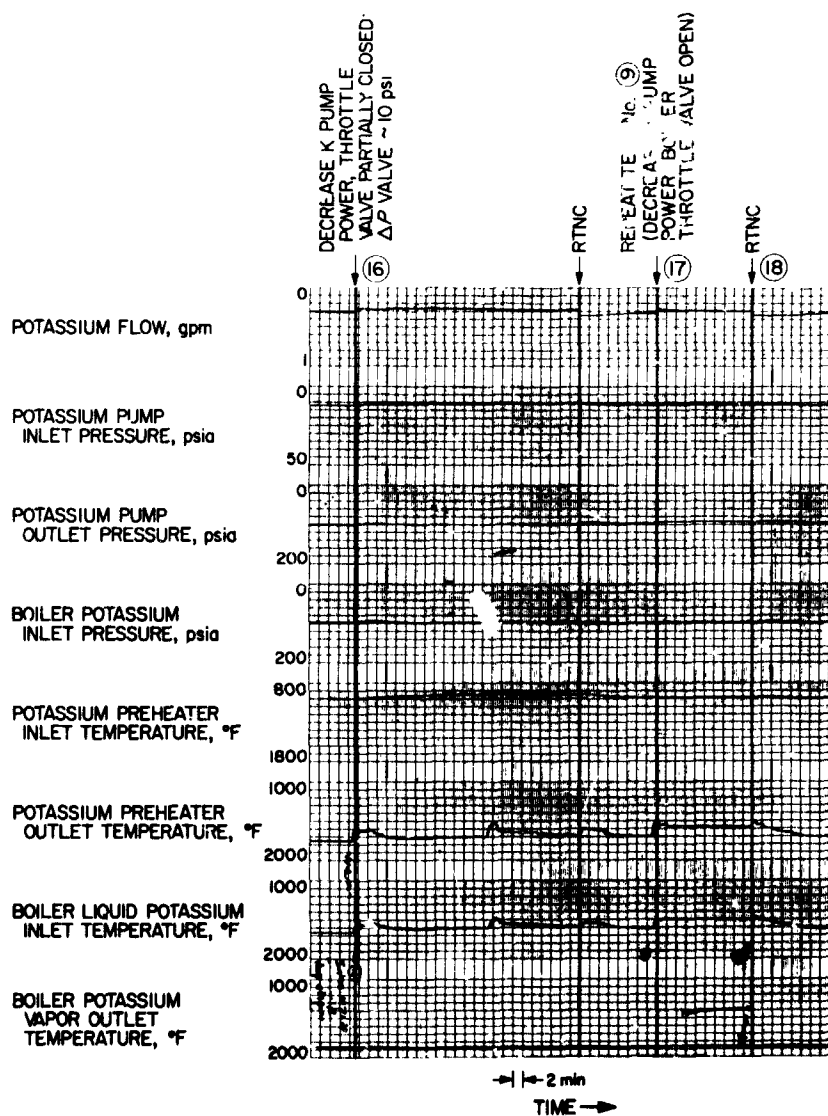


Fig. 31. Sanborn strip-chart recordings: transient tests 15-18

- (4) Return to nominal conditions
- (5) Increase lithium flow ($\Delta \simeq +10\%$)
- (6) Return to nominal conditions
- (7) Increase lithium flow ($\Delta \simeq +20\%$)
- (8) Return to nominal conditions
- (9) Decrease potassium pump power ($\Delta \simeq -10\%$), boiler throttle valve open, manual decrease on K preheater
- (10) Return to nominal conditions
- (11) Increase potassium pump power ($\Delta \simeq +10\%$)
- (12) Decrease potassium pump power ($\Delta \simeq -20\%$) to get to Step 9
- (13) Return to nominal flow
- (14) Increase lithium flow ($\Delta \simeq +37\%$)
- (15) Return to nominal lithium flow
- (16) Decrease potassium pump power as in Step 9 but close throttle valve to give $\Delta P \sim 10$ psi across valve
- (17) Repeat Step 9
- (18) Return to nominal flow
- (19) Decrease potassium pump power as in Step 9 but throttle valve closed more: ΔP valve $\simeq 36$ psi
- (20) Reopen throttle valve
- (21) Scram, lithium power shutoff
- (22) Lithium power on again

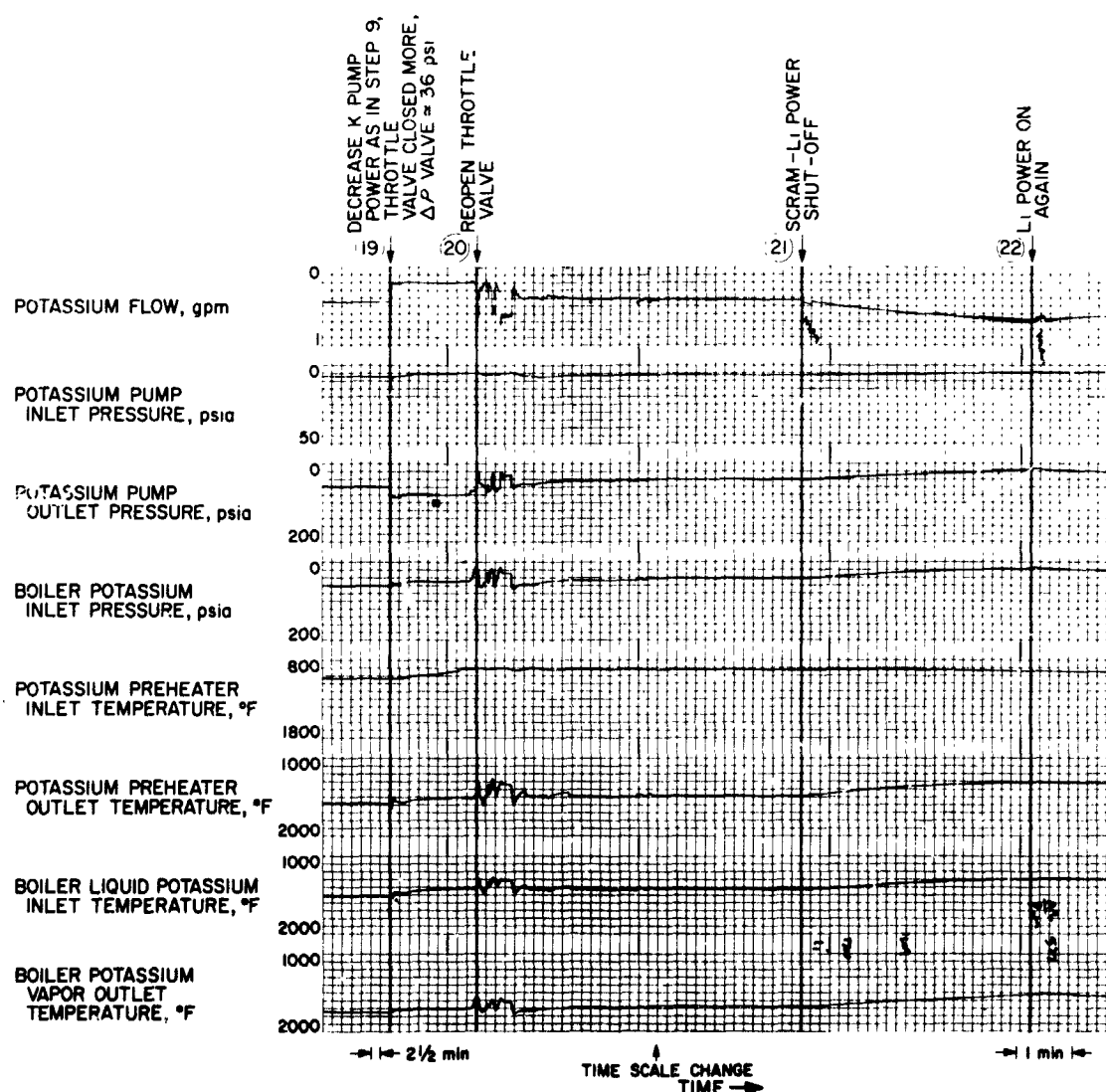


Fig. 32. Sanborn strip-chart recordings: transient tests 19-22

The general conclusion reached following this series of transient loop-behavior tests is that step-change transients of ± 10 to 20% about the operating point in primary flow, secondary flow, power input, and sub-cooling results in smooth transitions to new, stable, operating points.

Boiling test operations were completely shut down in a routine scheduled manner on January 13, 1967, following completion of the initial series of transient tests. Although many attempts were made, operation of the turboalternator was not achieved in spite of the potassium flow, temperature, pressure, and all other pertinent T-A parameters being at their nominal design values. Disassembly and discussion concerning the inoperability of the T-A are presented in Section 3 of this article.

Examination of the loop following the scheduled shutdown revealed small amounts of liquid metal leakage at the bottom of the boiler and at an epoxy-repaired cracked braze-joint on the turboalternator. Analysis of system transient behavior and heat-transfer tests are being carried on and will be reported when completed.

a. Summary of loop operations. Approximately 1000 hr of loop operation were accumulated in this last test period, including ~200 hr of boiling of which 100 hr were at full power (30 kw) and at nominal design temperatures (~1900°F). Approximately 80% of the transient tests have been completed. Those transient tests involving load perturbations were not concluded, as the turbine was inoperable. Over-all loop operation, in general, was very satisfactory. The major conclusions are:

- (1) Drain start is feasible.
- (2) Low-power operation is more unstable than high-power operation due to small magnitude of boiler "pinch-point."²
- (3) High-pressure drop at boiler inlet stabilizes flow but does not necessarily eliminate boiler pressure oscillations.
- (4) Flow-void instabilities (often encountered in natural circulation water boilers) was not observed.
- (5) Flooding-flashing instabilities (not usually encountered in water systems) was the primary instability mode.

²That point in a two-fluid heat exchanger where temperature difference between the two fluids is a minimum.

- (6) High-flow, low-quality boiling is more prone to flooding-flashing instabilities than low-flow, high-quality boiling.
- (7) Boiling "noise" is greater for high-quality flow.
- (8) Response time of Statham and Bourdon pressure transducers is adequately fast.
- (9) Thermocouple locations are critical; 20°F errors can be introduced by small local perturbations.
- (10) Full-power, steady-state operation is extremely stable.
- (11) Step transients $\pm 10\%$ about operating point in primary flow, secondary flow, power input, or sub-cooling result in stable transitions to new operating points.
- (12) Most critical conditions will be startup and low-power operation.
- (13) Programmed start requires intimate knowledge of actual system characteristics (including zero-gravity effects).
- (14) The occurrence of the undesirable phenomenon of boiler flooding-flashing requires the high liquid superheat characteristic of liquid metals. In order to simulate this effect to any degree with water, extremely high purity and very smooth surfaces are required; even then the degree of simulation may not be comparable to liquid metals.
- (15) While not directly concluded from these loop operations, programming potassium flow to heat transferred across the boiler would maintain constant quality or superheat to the turbine which would be highly advantageous to minimize blade erosion.

b. Future goals. The next phase of operations will include: inspection and bench operation of the turboalternator and reinstallation into the test loop; installation of the new cross-flow boiler; and additional full-power loop operations and completion of the transient tests.

2. Loop Disassembly and Cleaning

The lithium-boiling potassium test loop was disassembled following Phase II operations. Both hot traps are to be discarded, a new boiler installed, and the T-A unit is to be opened up for inspection and bench testing. Bolt cutters were used to remove major components from support structures, since new interconnecting piping would have to be fabricated anyway. After this, the loop

components and piping had to be cleaned of residual lithium and potassium.

The liquid metal cleaning operation was accomplished by reacting the liquid metal with Butyl Cellosolve.³ This reactant was chosen for two reasons: primarily, because it is the only reactant known that reacts with a liquid metal in a Cb-I-Zr component without hydriding the columbium; secondly, it appears to be less likely to flash than the reactant normally used on noncolumbium parts (alcohol-kerosene mixture). The reaction rate appears slightly slower than with butyl alcohol. In all component cleaning, the volume of fluid was kept large enough to prevent the bulk temperature from rising appreciably. When cleaning the radiator-condenser coils and the potassium pump, the Butyl Cellosolve was recirculatively pumped through these components to prevent the temperature from rising excessively. After complete reaction between the Butyl Cellosolve and liquid metal had occurred, the parts were then cleaned in alcohol and water rinsed. This method of cleanup of liquid-metal-contaminated components proved to be very effective. Components and portions of the loop to be used again were also pickled to remove any surface contamination.

3. Preliminary Results of Turboalternator Inspection Following Disassembly

At no time during testing were there any indications that the T-A was rotating. Rotation is indicated by alternator output frequency. A decision was made to disassemble the unit for inspection to determine the cause of seizure. After disassembly and cleaning, running clearances of bearings, shaft, wheel and rotor, and the concentricity and parallelism of flanges and mounting faces were determined.

Two items were discovered in the inspection that could give rise to conditions that would prevent rotation of the unit:

- (1) The mounting surfaces for the two bearing assembly flanges were out of parallel by 0.015 in. at about the diameter of 5 in. This is equivalent to an angular misalignment of $0^{\circ}10'$, which is sufficient to prevent rotation of the shaft.
- (2) There was inadequate end clearance between the bore of the alternator case and that portion of

³Glycol-monobutyl-ether. "Butyl Cellosolve," tradename by Union Carbide Corp.

the rotating assembly between the thrust bearing and the rotor. Measurements indicate this clearance to be 0.010 in. at room temperature. This clearance might become an interference at operating temperatures depending on the pessimism of assumptions made regarding temperature gradients. If the clearance was decreased by more than the 0.010 in., the thrust washer would be pressed into intimate contact with the mating portion of the slightly (1°) conical thrust-bearing pads. Photographs of these two surfaces are shown in Fig. 33. The metal transported from one surface to the other indicates that high temperatures and/or pressures were experienced prior to disassembly.

4. Metallurgical Examination of Loop

Metallographic samples have been taken from both loops and are presently undergoing evaluation. Preliminary photos and observations are presented in the following section to indicate the trends observed. A final report on observations will be prepared when metallography and chemical analyses are completed.

Minimal corrosive effects were observed in both loops of the system. A photo of the inner surface of the liquid portion of the potassium loop is presented in Fig. 34. The drawing marks present on the surface are typical of both the as-received tubing as well as almost all of the samples taken from the loop. No evidence of corrosion was detectable in this sample.

The zirconium hot trap used in the liquid portion of the potassium loop is shown in Fig. 35. The inner surface plainly showed the machining marks from fabrication. Discoloration is believed to be a result of the Butyl Cellosolve cleaning agent used to react with the potassium present in the component. No evidence of corrosion was observed during metallographic examination of sections taken from the hot trap.

Changes in the appearance of the surface, however, were detected in the vapor portion of the potassium system. The drawing marks typical of as-received tubing were less prominent, and the surface had a roughened appearance. A section of this potassium turbine vapor by-pass line is shown in Fig. 36. The roughness shown is considerably greater than any detected in any other portion of the loop. Since no preferential attack was observable, this roughening may be a result of erosion by liquid droplets entrained in the potassium vapor.



Fig. 33. Turboalternator thrust bearing pads after disassembly



Fig. 34. Potassium system liquid line, inner surface (14 X)

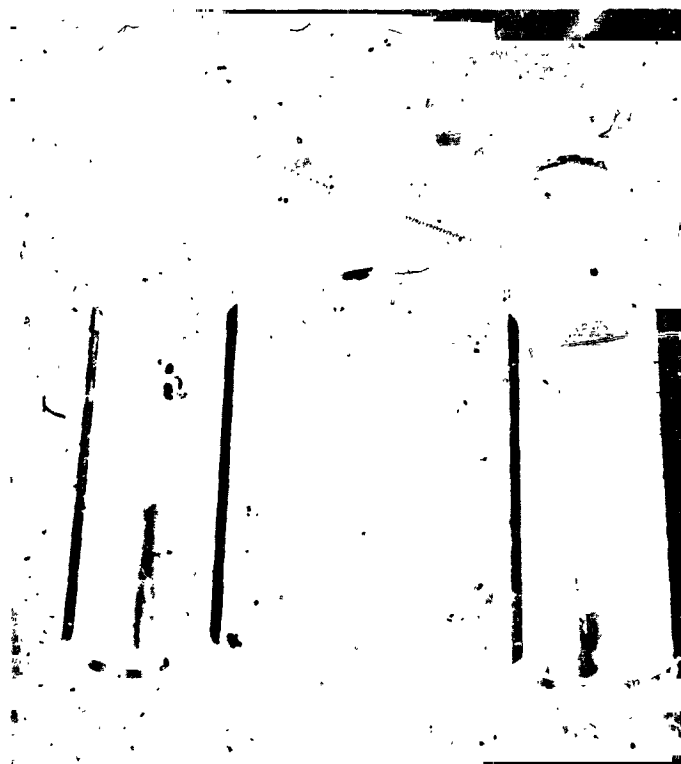


Fig. 35. Potassium hot trap (zirconium getter)

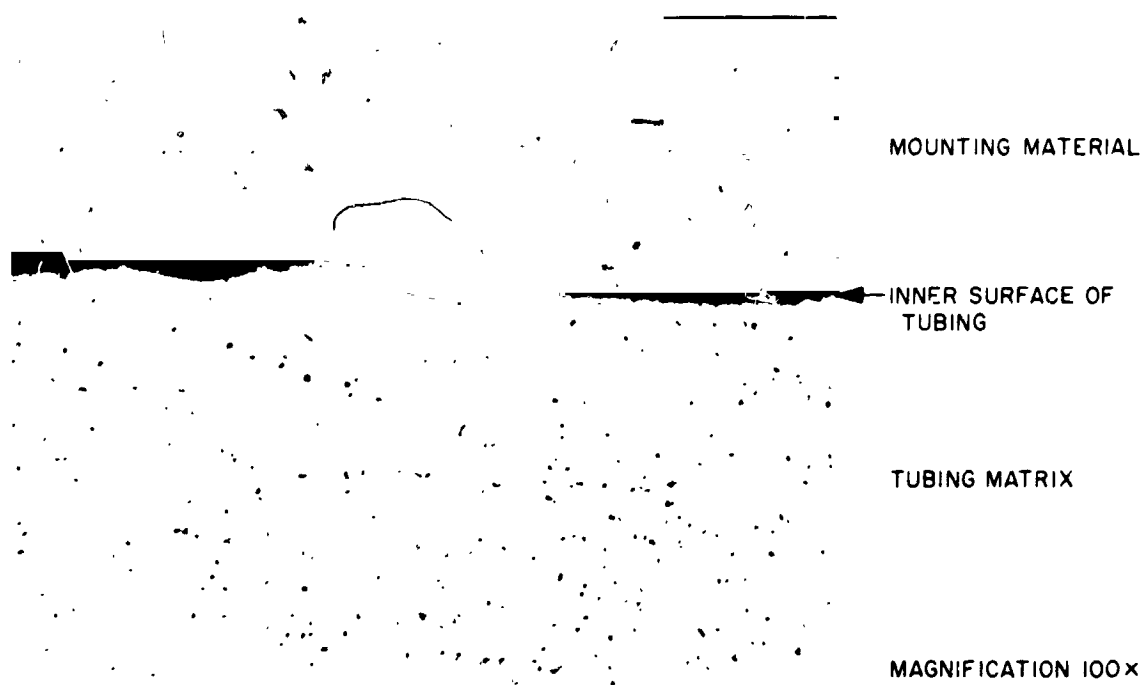


Fig. 36. Inner surface of potassium turbine vapor by-pass line, metallographic section (100 X)

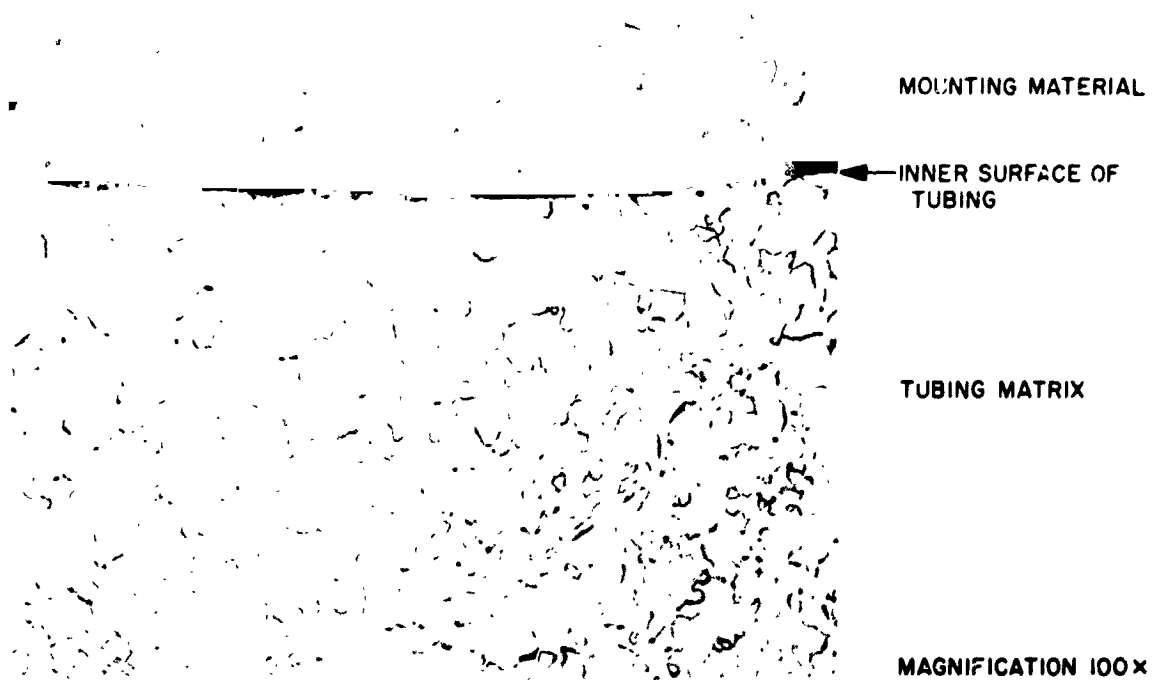


Fig. 37. Inner surface of lithium primary heater coil outlet, metallographic section (100 X)

Tubing sections taken from the boiler display the characteristic drawing marks, as well as light scratches incurred during boiler assembly. Metallographic examination of sections taken from the centerless ground tube, which was the heat-exchange surface between the lithium and the potassium, showed no corrosive effects.

Sections taken from the inlet of the primary lithium heater coil (the coldest spot in the lithium loop) displayed neither corrosive nor mass-transfer effects. The surface of the outlet of the lithium heater coil (hottest part of the loop) retained the characteristic drawing marks; however, some roughening was visible, indicating etching or solution corrosion of this area. This effect was slight, as indicated by the metallographic section (Fig. 37) which displayed very minimal preferential etching of the grain boundaries.

Hot trapping of oxygen in the lithium loop was done with yttrium. The yttrium element was intact after sectioning of the hot trap; however, disintegration took place during the cleaning operation during which the lithium reacted with Butyl-Cellosolve (Fig. 38). Oxygen analyses of the lithium after test indicated 700 ppm, which would allow considerable gettering of oxygen

by the yttrium. Small particles of yttrium were found in one low spot of the loop, indicating chipping of the yttrium getter element, presumably due to embrittlement produced by oxygen pick-up during operation. No evidence of mass transfer of yttrium could be found in any portion of the loop.

a. Chemical analysis results. Sections of both loops were submitted for chemical analysis, as well as samples of the liquid metals. While the results are not yet complete, the available data are presented in Table 2.

Two effects are prominent in these data. The first is depletion of oxygen from the outer surface of the potassium system tubing. This can be attributed to partitioning of oxygen between the tubing and the zirconium foil. The second effect is an increase in hydrogen on the outer surfaces of the potassium system tubing. This is attributed to a small leak of organic coolant which developed during initial loop startup and was subsequently corrected.

An increase is indicated in the carbon concentration at the inner surface of the lithium system tubing. Duplicate samples have been submitted to determine if this is a true effect, and samples from the hot portion of the loop have been submitted for analysis to determine if a carbon depletion exists in that area.



Fig. 38. Lithium hot trap (yttrium getter disintegrated during post-test liquid metal cleaning operation)

Table 2. Analyses of columbium-zirconium alloys

	Element, ppm				
	C	H	O	N	Zr
Typical as-received tubing	50	5	250	50	9000
Potassium vapor tubing					
OD	55	84	50	140	8800
Core	23	17	210	160	9400
ID	23	18	340	160	9000
Lithium system tubing from boiler outlet (cool leg of system)					
OD	46	*	740	540	9000
Core	26	*	690	350	9000
ID	259	*	650	150	10100
Lithium, after operation	—	*	700	—	—
* These samples were cleaned in water to determine extent of hydrogen pick-up. Typical numbers were 500 ppm.					

D. Plasma Studies in the Electron Bombardment Ion Engine—Magnetic Field Effects, T. D. Masek

1. Introduction

This report is part of a continuing study of the plasma in the electron bombardment engine. A previous report (SPS 37-42, Vol. IV, pp. 131-137) discussed the general effects of arc current and voltage on the electron energy distribution function. It was shown that the arc current, by changing the electron-neutral density ratio, significantly affected the high-energy region of the energy distribution. These data were reported for a single magnetic field strength. It is also of interest to study the variation of plasma characteristics with changes in magnetic field magnitude. The present report describes the observed effects of magnetic field strength on the electron energy distribution function for several propellant flowrates and arc conditions.

2. Plasma Measurements

The data presented in this report were taken during the same mercury engine experiments as described above. Thus, the experimental set-up will not be discussed again. As before, the probe positions were 0.65 in. apart axially, beginning 0.18 in. from the screen grid, and were 0.25 in. apart radially, starting 0.50 in. off center. All data are plotted in normalized coordinates.

The axial magnetic field strength is plotted in Fig. 39. This is the nonoperating field and is undoubtedly af-

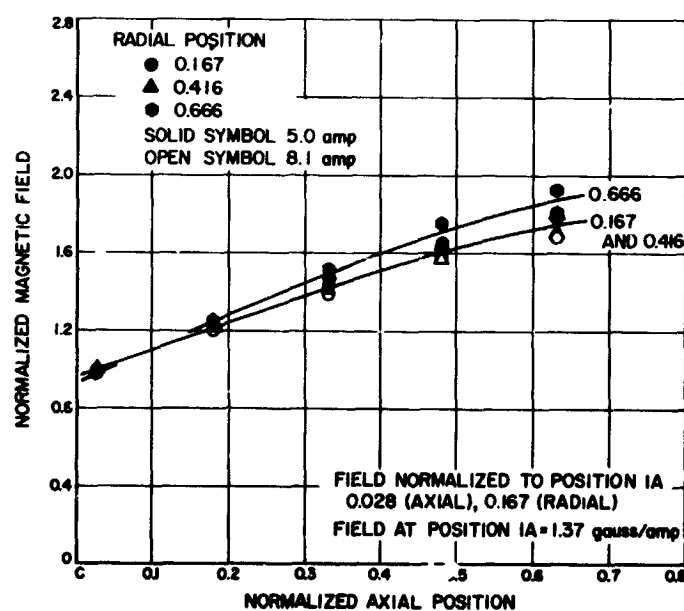


Fig. 39. Axial magnetic field distribution

fected to some extent by the plasma. Although the field lines are somewhat divergent, relative changes in the plasma characteristics can be related to changes in the externally applied field.

Semilogarithmic plots of several Langmuir probe traces are shown in Figs. 40 and 41. This data was obtained at the most central grid position (0.042, 0.167) and with constant arc current. The propellant flowrates, beam currents, and arc voltages are indicated in the figures. The plasma characteristics, ion density, electron temperature and plasma potential, determined from this data are shown in Figs. 42 to 44. The additional data points shown in Figs. 42 to 44 were obtained from plots similar to Figs. 40 and 41.

It is important to note that although many of the semilogarithmic curves are straight lines to the left of the "knee" for less than an order of magnitude in probe current, all data has been reduced on the basis of these straight lines. This should be a reasonable procedure in light of the discussion in SPS 37-42, Vol. IV. (That is, the electron energy distribution for high arc currents, 10 to 14 amp, appears to be approximately a Maxwellian distribution disturbed by high-energy electron gain or loss.) The reasoning behind the selection of these straight lines and the mechanisms causing the shifts in distribution will be presented in the following discussion.

a. Distribution function variations. In studying Figs. 40 and 41, it is readily apparent that the electron energy distribution significantly changes its nature between magnet currents of 5 and 9 amp. The distribution is one of excess high-energy electrons at low magnet currents but gradually changes to one lacking high-energy electrons at higher currents. This is indicated by the fact that the data falls above the Maxwellian line for negative probe voltages at low magnet currents but lies below the Maxwellian line at higher currents. This trend persists throughout the data for all flowrates. At lower arc currents the data tend toward the excess electron nature indicated in SPS 37-42, Vol. IV.

The mechanism producing the change in the distribution function can be explained as follows. First, it is necessary to recognize that the axially applied magnetic field traps radially directed electrons in circular orbits. This trapping in general restricts the radial flow of electrons. Thus, radial electron flow depends on collisions of electrons with other particles. However, electrons can move freely axially. This axial freedom allows most electrons to reach one or both ends of the engine housing.

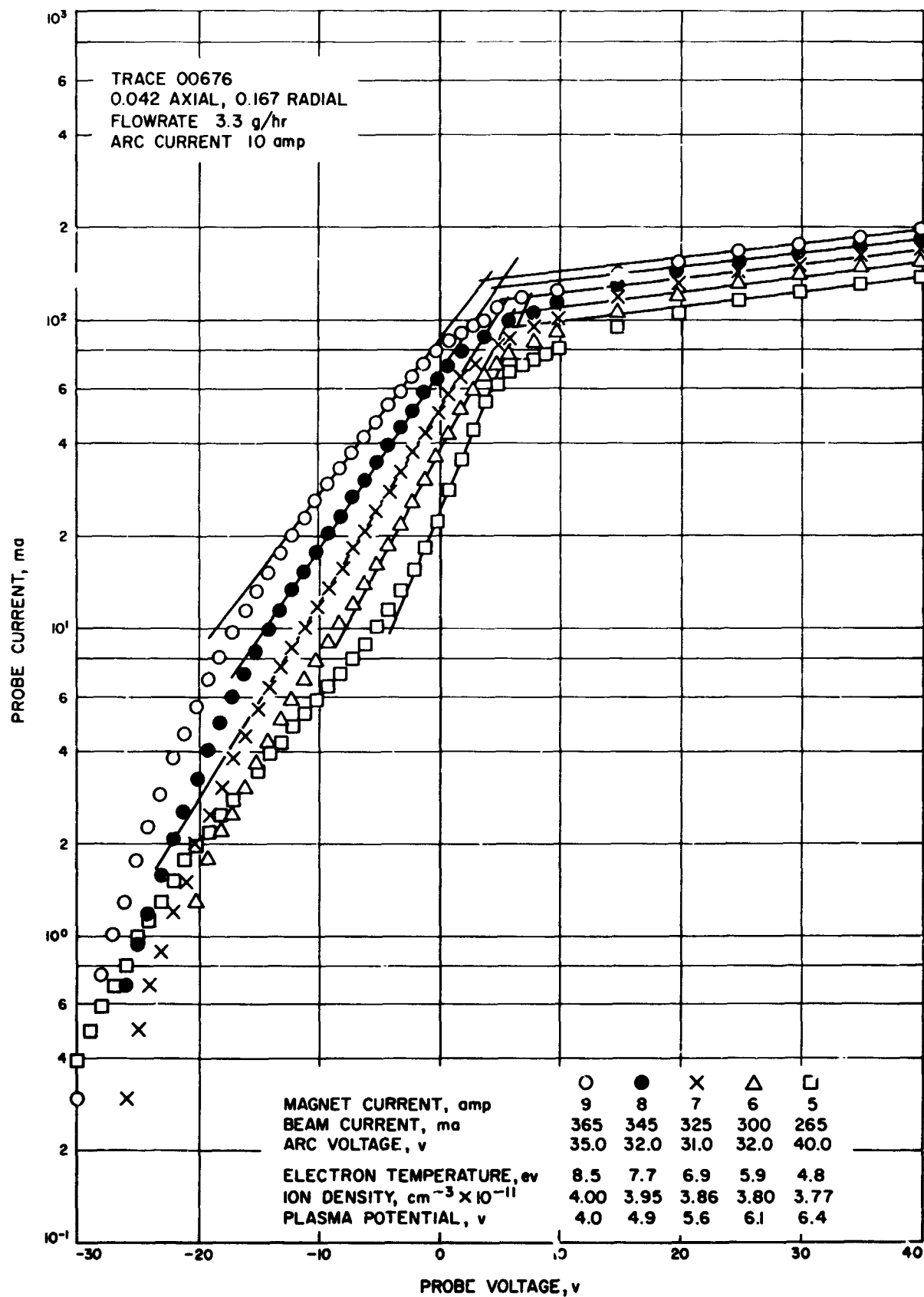


Fig. 40. Semilogarithmic plot of Langmuir probe data with variable magnetic field, 3.3 g/hr

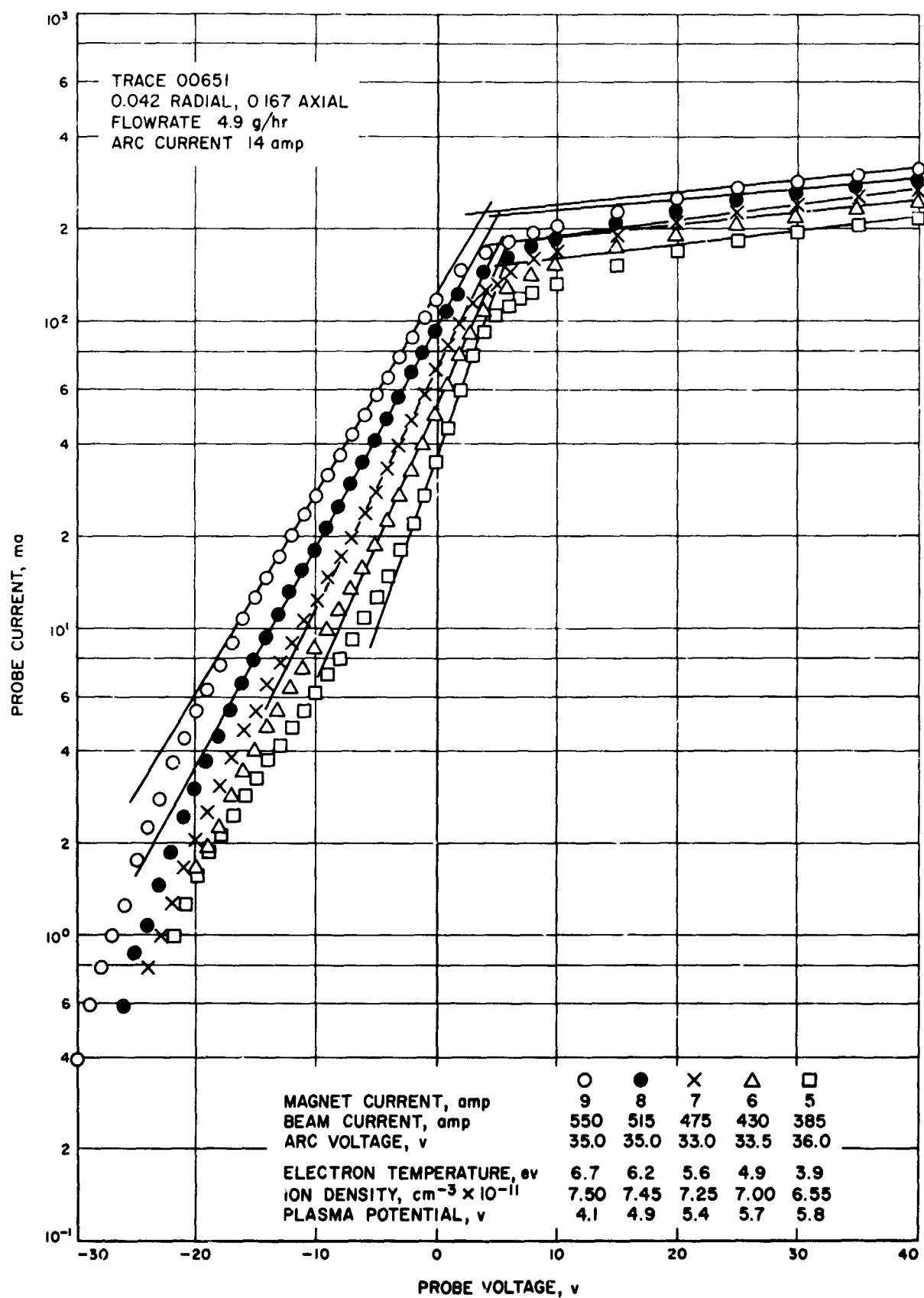


Fig. 41. Semilogarithmic plot of Langmuir probe data with variable magnetic field, 4.9 g/hr

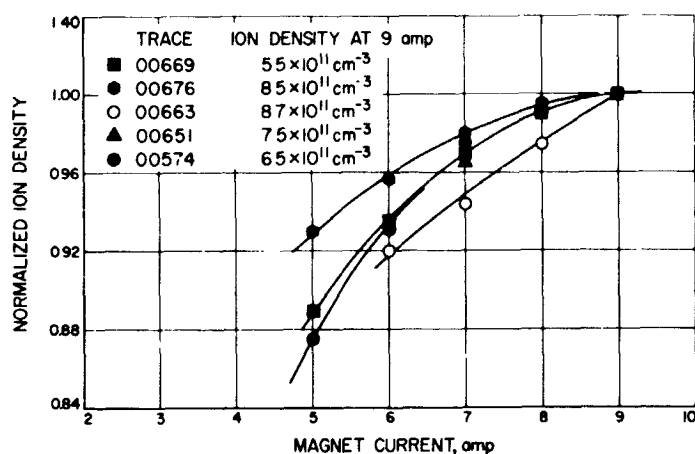


Fig. 42. Normalized ion density variation with magnetic field strength

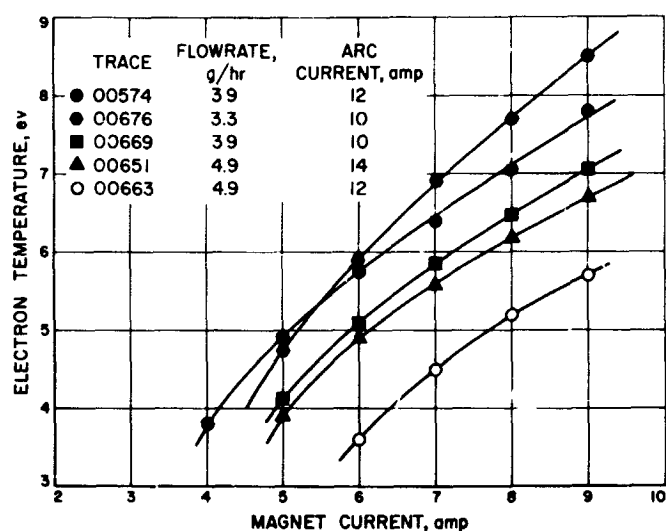


Fig. 43. Maxwellian electron temperature variation with magnetic field strength

Once trapped by the magnetic field, electrons can travel only radially by colliding with another particle (an electron, ion, or neutral atom). The rate of electron flow across the chamber, cathode to anode, is thus determined by the highest collision frequency and by the magnetic field strength. Since the electrons move approximately one cyclotron radius per collision, the higher the axial magnetic field the more collisions required to reach the anode. This indicates that the degree to which the electrons approximate a Maxwellian distribution (determined by electron-electron collisions) and the degree of ionization (determined by electron-neutral collisions) depend on the axial magnetic field strength.

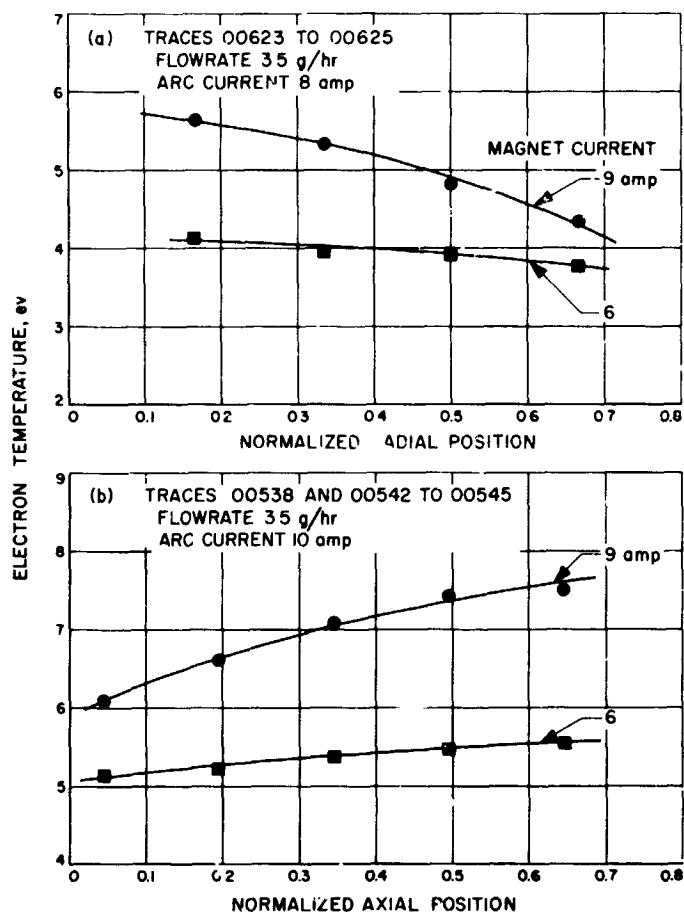


Fig. 44. Maxwellian electron temperature at two magnetic field strengths (a) radial variation; (b) axial variation

Of most significance in discussing the electrons is a distinction between two possible electron groups. Electrons emitted from the cathode, termed primary electrons, initially have energies equal to the sum of the arc voltage and the plasma potential (on the order of 40 eV in the present case). The second electron group is of lower energy, 4 to 10 eV, and is formed by electrons produced in ionization and in primary electron inelastic collisions. The rate at which primary electrons transfer their energy to neutrals, ions, and other electrons determines the total energy distribution among all the electrons. Therefore, the relaxation time for primary electrons is important and must be compared with the time required for high-energy electrons to traverse the chamber radius.

The various collision times and the primary electron relaxation time were summarized in Ref. 9. The time required for 40 eV electrons to relax into a Maxwellian distribution of about 7 eV with an electron density of $5 \times 10^{11} \text{ cm}^{-3}$ is shown to be approximately $1.8 \times 10^{-8} \text{ sec}$.

The primary electron-ion or primary electron-Maxwell electron collision time is about 6×10^{-6} sec. The primary-neutral times are about 3×10^{-6} for elastic collisions and 8×10^{-6} for inelastic collisions for a neutral density of $5 \times 10^{11} \text{ cm}^{-3}$. Since the ion-neutral ratio changes across the chamber, and since a neutral density of $5 \times 10^{11} \text{ cm}^{-3}$ is probably a good average, 3×10^{-6} sec will be used as the average scattering collision time.

With an average magnetic field of 20 gauss, the cyclotron radius of 40 ev electrons is about 1.1 cm. The chamber radius is about 7.5 cm, so that a 40-ev electron requires about seven collisions to reach the anode. These seven collisions require, using the previous collision time, approximately 2×10^{-5} sec, or about the same time as for primary relaxation.

However, as the magnetic field is reduced, two effects occur. First, the cyclotron radius increases, thereby reducing the number of collisions required to reach the anode. Second, less energy is transferred to the lower energy electron group, reducing the group energy. Using a Maxwellian energy of 5 ev and a field of 11 gauss (Fig. 41), the relaxation time is about 2.2×10^{-5} sec, and the time required for 40 ev electrons to traverse the chamber is about 1.2×10^{-5} sec. Thus, these electrons escape in about half the time required for relaxation.

The previous collision time comparisons, in part, provide the explanation for the distribution function changes with magnetic field changes found experimentally. However, an additional consideration, relating to high-energy electron loss previously discussed, is also required to complete the explanation. At high magnetic fields, and with high arc currents (SPS 37-42, Vol. IV), the distribution lacks high-energy electrons relative to a Maxwellian distribution as seen in Figs. 40 and 41. Electrons with energies greater than the cathode sheath potential (40 ev in the present example) can overcome this barrier at the housing and escape to neutralize ions which also diffuse to the housing. Although only the portion of electrons with energy greater than this sheath potential can escape to the housing, some electrons from the lower energies will increase in energy through collisions to fill in the energy gap. This causes the data to decrease continuously and to begin decreasing at energies below 40 ev.

The mechanisms for changing the distribution function with magnetic field changes can now be clarified. At high fields, most of the primary electrons relax into a basically Maxwellian distribution before reaching the anode because of the number of collisions required to traverse the

chamber. The loss mechanism discussed in the last paragraph for high-energy electrons causes the distribution to lack these electrons relative to a normal Maxwellian distribution. As the field is reduced, the number of collisions required for electrons to reach the anode is reduced. This reduces the number joining the Maxwellian group and leaves an excess of high-energy primary electrons relative to the Maxwellian distribution indicated by the straight lines through the data. It is expected that the primary group is not monoenergetic, because the electrons have at least a few collisions before reaching the anode. The exact form of this high-energy distribution will be discussed later in this series of reports.

b. Plasma characteristics variations. With the previous discussion for reference, the electron temperature, ion density, and plasma potential variations with magnetic field can be considered. These characteristics will also be discussed in terms of relative field changes. It is clear from Figs. 42, 43, and 45 that electron temperature and ion density are directly proportional to a function of the magnetic field while the plasma potential varies inversely with field.

The electron temperature curves shift upward consistently with reducing flowrate and increasing arc current. This corresponds to increasing propellant utilization efficiency. A similar upward shift occurs in the normalized ion density curves, although not quite so consistently. The plasma potential curves do not shift consistently. This can probably be attributed to the differences in arc voltage and the data reduction technique

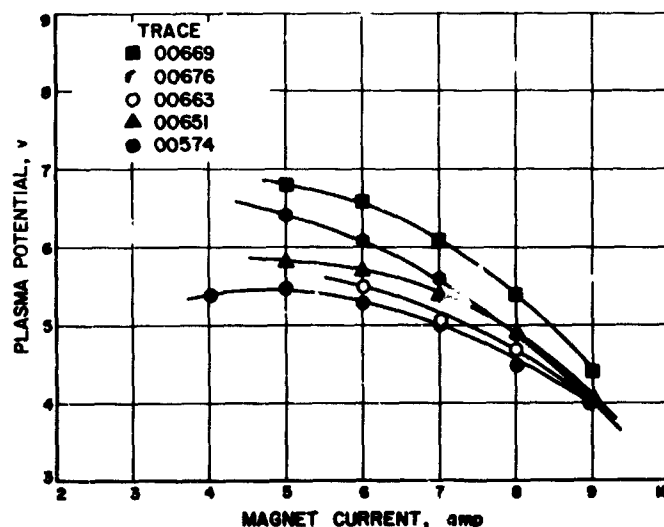


Fig. 45. Plasma potential temperature variation with magnetic field strength

utilizing the low-energy Maxwellian lines. However, the variation of plasma potential with field is quite consistent.

Considering any initial magnet current, a reduction in this current reduces the Maxwellian electron temperature because of the reduced energy transfer from the primary electron group. This decrease is apparent in Fig. 43. An additional effect to be expected in this regard is an improvement in electron temperature uniformity spacially with reducing magnetic field. An example of this improvement is shown in Fig. 44. The semilogarithmic plot of the data for these is not included for brevity. The figures show that high-energy electrons distribute their energy more uniformly at lower magnetic fields, since the trapping effect is reduced.

The plasma potential represents the ability of electrons to diffuse away from the ions and form a net space charge. Although the electron energy drops significantly with magnetic field, the velocity drops as the square root of the energy. At the same time, the distance electrons can move radially changes inversely with the field. The high magnet current portions of Fig. 43 indicate curve slopes of about 0.5 to 0.8 eV/amp, and the corresponding curves in Fig. 45 have slopes of about 1 v/amp. At low currents these slopes are about 1.0 to 1.5 eV/amp and

0.1 to 0.3 v/amp. This indicates that a lower magnetic field allows electrons to diffuse more rapidly, causing the plasma potential to increase. But the electron energy decrease partially compensates for this freedom and tends to slow the plasma potential increase. The data from trace 00574 in Fig. 44(a) actually begins to decrease, reflecting the compensating effect. In addition, the primary electron flow increase at the anode caused by the reduced field reduces the number of Maxwell electrons reaching the anode for a given arc current. This tends to increase the anode sheath potential since the sheath changes to reduce the flow of lower energy electrons. This sheath increase also tends to shift the plasma potential.

3. Conclusions

At the present time, a method for predicting the magnitude or variations of the previously discussed characteristics is not available. It is believed that the insight gained in basic studies described here and in SPS 37-42, Vol. IV will aid in formulating such a method.

Additional data must be analyzed to verify the trends reported here. Data obtained in a conical engine operated with both mercury and cesium will be used for this purpose. The basic information gained in these studies should greatly aid in engine design and optimization.

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XIII. Liquid Propulsion

PROPULSION DIVISION

N67-29154

A. Reaction Control Gas Supply System,

T. A. Grouble

1. Introduction

Experiments relative to the use of neat hydrazine as a monopropellant fuel to generate gases for an attitude control system for unmanned space missions (warm gas attitude control system) were discussed in SPS 37-36, Vol. IV, p. 50, and SPS 37-37, Vol. IV, p. 159. Now, an additional series of tests have been performed to define the problems of large numbers of pulses (i.e., several days of intermittent pulsing), and to investigate filter contamination from generator by-products.

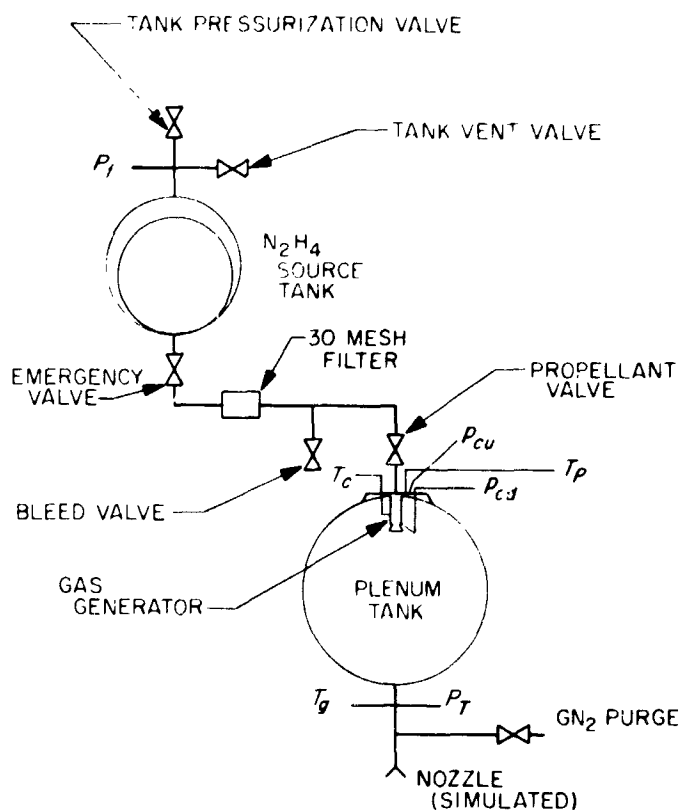
2. Pulse Mode Test

The method used for testing the 0.5 lb_r equivalent thrust gas generator system is shown in Fig. 1. Several firing tests were performed with this configuration to establish such basic performance parameters as chamber pressure and chamber temperature. The gas generator was essentially off-the-shelf hardware, and, more importantly, it was originally designed as a thruster using hydrazine-hydrazine nitrate as the propellant. The per-

formance of this generator with neat hydrazine, however, was adequate for the purposes of this preliminary system. The chamber pressure (p_c) was nominally 90 to 110 psig, the flow rate ($\dot{w}$) was 0.0041 lb_m/sec, and the chamber temperature (T_c) was 1545°F. These measurements were at steady-state conditions (~30 sec firing time), and were degraded considerably during pulse mode operation ($p_c = 50$ to 80 psig; $T_c = 1000$ to 1500°F; $\dot{w} = 0.003$ lb_m/sec).

The gas generator system was connected to a Ranger-type attitude control thruster system (Fig. 2) through a 9- μ mechanical labyrinth filter of JPL design, and a 2- μ Millipore¹ mechanical filter. A firing of this combined system was performed for 30 continuous pulses, with an average pulse length of 30 msec. Two attitude control system (ACS) thrusters were cycled such that one thruster was continuously on, and the other operated at random intervals. The warm gas system (WGS) maintained a pressure of 15 $\pm$ 2 psig during the test, and met the demand of the ACS.

¹Millipore Corp., Bedford, Massachusetts.



- T_p MOUNTING PLATE TEMPERATURE
 P_1 FUEL TANK PRESSURE
 T_c GENERATOR CHAMBER TEMPERATURE
 T_g GAS TEMPERATURE
 P_{cd} GENERATOR CHAMBER PRESSURE (DOWNSTREAM)
 P_{cu} GENERATOR CHAMBER PRESSURE (UPSTREAM)
 P_T PLENUM TANK PRESSURE

Fig. 1. Warm gas system schematic

An examination of the filters was made at the end of this test firing. The 9- μ filter was quite dirty with particles from 8 μ to 250 μ in diameter, which appeared to be catalyst fines from the gas generator packed bed. The 2- μ filter was very clean with only a few 2- μ to 3- μ particles.

The 2- μ filter was recleaned; a new 5- μ mechanical filter² was installed in place of the 9- μ filter while the latter was being cleaned. A WGS firing of 50 pulses (pulse duration_(avg.) = 100 msec) with 3 ACS valves open was performed. The combined system worked very well, with the WGS meeting the ACS demand on the plenum tank. Table 1 describes the recorded performance of the WGS parameters. At the termination of this test, the

²Permanent Filter Corp., Compton, California.

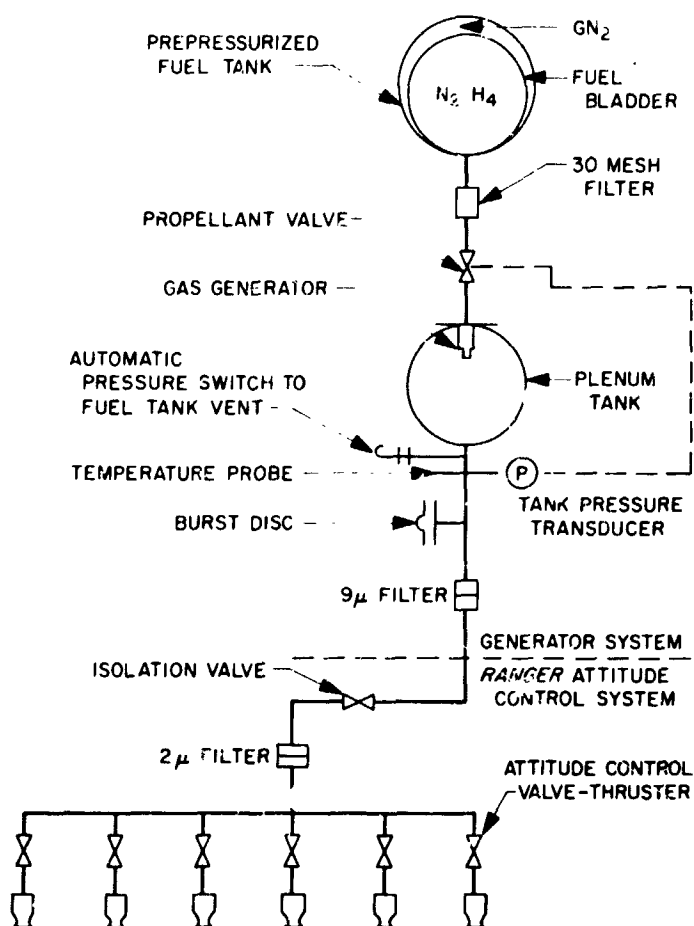


Fig. 2. Warm gas attitude control system schematic

filters were again examined. The 5- μ filter had trapped several metallic and non-metallic particles from 1 to 100 μ , and one 520- μ particle. A green residue of crystalline structure was also found in the filter; a chemical analysis indicated that the green substance was inorganic. Logically, it was formed as a result of an electrolytic couple of aluminum and stainless steel in the system. These ingredients, in the presence of moisture, ammonia, and traces of hydrazine, form a good battery. The 2- μ Millipore filter was not discolored, and only a few 5- μ particles were present. This is interesting because the Millipore-type filter was expected to produce more of the "green" contamination.

The test was repeated using a clean 2- μ filter and a Vacco³ 5- μ filter in place of the Permanent filter. Post-test examination revealed that the Millipore filter was slightly dirty (4- μ to 2- μ particles) but had no "green"

³Vacco Valve Co., El Monte, California.

Table 1. Results of filter test firing conditions: pulse mode (including pulse no. 1)

Measurement	Value
P_{td}	87.0 psig (avg.)
P_f	214.5 psig
P_r	13.1 to 15.0 psig
P_{ru}	129.0 psig (avg.)
T_c	1547°F (avg.)
T_p	96°F (avg.); 120°F (max.)
T_o	97°F (avg.); 104°F (max.)

contamination. The Vacco filter had both non-metallic and metallic particles 1 μ to 60 μ . No "green" contamination was present.

3. Long-Duration Test

It appeared at this time that the contaminants generated by the WGS could be filtered out; it would be desirable, therefore, to run the combined warm gas attitude control system (WGACS) for a cycling test of long duration, based on a *Voyager* condensed mission profile. In preparation for this test, both the WGS and ACS were disassembled and inspected. The WGS had an external green deposit on the generator chamber flange, and the bottom of the plenum tank near the outlet had a uniform coat of loose catalyst dust on the surface. The pressure transducers were removed, and the $\frac{1}{8}$ in. pressure tube on the upstream chamber pressure (p_{ru}) was found partially plugged. Small orifices throughout the system are particularly vulnerable to obstruction.

Examination of the ACS revealed a deposit on the thruster valve seats and pintle. A chemical analysis indicated that the deposit was organic. A possible source of the organic material was found on the WGS: a circle of black paint applied to the bottom of the generator mounting plate prior to assembly had not been found in pre-assembly cleaning.

Under operating conditions, a new Eckel⁴ propellant valve was cycled with water to provide an indication of how the valve would withstand 4000 cycles of operation. At the end of this test, there were no leaks in the valve and it appeared to operate properly.

Because part of the WGS concept was a "blowdown" method of fuel feed, the level of fuel tank pressurization was calculated to be approximately 1000 psig. However, because a suitable tank could not be readily obtained, a

⁴Eckel Valve Co., San Fernando, California.

Table 2. Long-duration test parameters

Measurements	
T_c	Gas generator wall temperature
T_o	Gas temperature
P_f	Fuel tank pressure
p_{td}	Generator chamber pressure (downstream)
p_{ru}	Generator chamber pressure (upstream)
P_i	Plenum tank pressure
Functions (remote operation)	
ACS thruster valve cycling	
Fuel vent valve	
Fuel line emergency valve	
Plenum tank vent valve	
Generator propellant valve	
Stop/start over-ride switch	

modified *Mariner* Mars 1964 fuel tank was used with a propellant bladder installed. This tank was not designed for pressures greater than 400 psig, and required remote replenishment at selected intervals.

The entire system was reassembled and a test firing was conducted to check the total system and to make a final evaluation of the 9- μ labyrinth and 2- μ mechanical filters. This firing was of 30 min total duration at a pulse rate of 600 msec on, and 2.0 sec off. After this test, the 2- μ filter was examined and found to have several small particles (~ 3 to 5μ) and the 9- μ filter had fifty to sixty 10- to 30- μ particles. The particles in both filters were of metallic and non-metallic origin. The 2- μ filter element also had a colorless stain and appeared wet. This level of filter contamination was judged acceptable.

The first long-duration mission simulation operation on the WGACS was so conducted that the 21.5 lb_m load of hydrazine in the fuel tank was initially pressurized to 400 psig, and permitted to blow down to 200 psig before repressurization. The system was installed in a protective barricade, and parameter measurements were brought out through an access port. Table 2 lists the parameters monitored. These measurements were recorded on strip charts running at slow speed, and cycled 1 min on every 5 min.

The ACS thruster cycles began according to the sequence in Table 3. The propellant valve was armed, and control was given to the automatic pressure switch (modified pressure transducer). The system operated normally for 10 hr (368 generator pulses) before an electro-mechanical relay in the pressure switch amplifier failed, causing the gas generator to remain on, and thus creating an overpressure in the plenum tank. At this time both

Table 3. ACS thruster sequence chart

Simulated operation	Thruster response
Separation rate removal, and solar reference acquisition	Open 3 valves for 20 min continuously
Cruise mode	Open 1 valve every 3.3 sec for 60 msec for 8 days
Commanded turn	Open 3 valves for 5 min
Cruise mode	Open 1 valve every 3.3 sec for 60 msec for 8 days
Commanded turn	Open 3 valves for 5 min
Cruise	Open 1 valve every 3.3 sec for 60 msec for 8 days
Commanded turn	Open 3 valves for 5 min
Cruise	Open 1 valve every 3.3 sec for 60 msec for 8 days
Orbital insertion maneuver	Open 6 valves for 165 min
Cruise	Open 1 valve every 3.3 sec for 60 msec for 8 days

safety devices⁵ in the system were activated and the test was terminated.

After repairs to the amplifier circuit and replacement of the burst disk, a second long-duration mission simulation operation was initiated using the same ACS sequence of events. This time, the system operated for 15 hr before automatic shutoff terminated the test; no obvious mechanical failure could be found for this overpressure in the plenum tank.

Subsequently, the expected thrust levels in the ACS thruster changed, creating a need for a larger gas generator to meet the demand. As a result, an off-the-shelf 5.0 lb_r equivalent thrust gas generator was modified to incorporate a "shower head" injector for use with the Shell 405 catalyst⁶. This generator was tested independently and modified until its performance was acceptable for this system. It was not, however, optimized for use as a gas generator (i.e., NH₃ dissociation was still ~50%). The plenum tank was rebuilt to permit installation of this larger generator, and several test-stand firings were made. This generator had been originally designed for neat hydrazine, and its steady-state performance was much smoother than the original, smaller unit. The cold-bed

⁵The burst pressure of a burst disk on the plenum tank was set at 30 psig; a pressure switch on the plenum tank was set to vent the fuel tank when pressure exceeded 27 psig.

⁶Shell Development Corp., Emeryville, California.

ignition delay time⁷ was considerably better as well: 89 msec vs 150 msec for the smaller unit.

The WGACS was reassembled using the larger gas generator and the modified pressure transducer was replaced with a pressure switch⁸ to control the pressure in the plenum tank. A new test plan was devised utilizing the same sequence of ACS operation listed in Table 3, except that the cruise mode durations were reduced to provide a 20-day continuous test.

In the third WGACS long-duration test, the system operated continuously for 8 days without disturbance until the burst disk in the plenum tank failed, whereupon the generator operated until all the fuel was depleted. The failed burst disk was examined and found to have been chemically attacked (etched), and, being a thin, aluminum membrane originally (~0.005 in.), the disk had weakened to the point of failure. The electrolytic action that occurred earlier in the filters apparently had appeared again, this time on the disk. The performance of the WGACS from this test is presented in Table 4.

The WGACS was disassembled and examined. The filters had a very large quantity of particulate matter in each element, and both had been severely attacked chemically. Otherwise, the WGS was normal and appeared unaffected by the test. The internal surfaces of the ACS thruster manifolds, however, were heavily spotted with a light brown stain, and the thruster valve seats and ball plunger were coated with a light brown crystalline residue. Corrosion was apparent throughout the ACS components.

A post-test firing was made to provide a gas sample from the plenum tank. The gas was collected by an

⁷Delay time is measured from propellant valve open to *p*_r rise.

⁸Custom Component Switches Inc., Burbank, California.

Table 4. Results of 8-day test

Measurement	Value
P_r	{ 17.8 psig (max.) 13.6 psig (min.) (Nominal = 15.0 ± 1.5 psig) 15.8 psig (ave.)
P_f	2.5 psig/min (ave.) decay
P_{cu}	80 psig (ave.)
P_{cd}	60 psig (ave.)
Total generator cycles	1695
T_g	75°F (ave.)
T_c	560°F (ave.)

evacuated glass ampul with a remote valve on the end, which was connected to the tee of the plenum tank outlet. One second after ignition, the remote valve was opened for 2 sec and then closed. The ampul was removed and the gas sample analyzed. The general equation for the decomposition of hydrazine is



where x = the mole fraction of NH_3 dissociated. Assuming 50% NH_3 dissociation, the following results, expressed in mol%, would be expected:

$$\text{NH}_3 = 28.6\%$$

$$\text{N}_2 = 28.6\%$$

$$\text{H}_2 = 42.8\%$$

The results of the analysis, expressed in mol%, follow:

H_2	Ar	O_2	N_2	NH_3	H_2O	C.	Iso-propyl alcohol
37.16	0.32	0.35	37.03	23.62	1.07	0.41	0.04

Argon is expected in the analysis because an argon back-fill is used to isolate nitrogen in the analysis of the products. Isopropyl alcohol was present because most of the WGS components were cleaned with this liquid. It appears that a large amount of water formed with some hydrazine in solution. This would indicate inefficient operation of the generator during such short pulses. Most likely, the water is the result of initially-admitted trace amounts of fuel that passed through the catalyst bed without reacting; the fuel normally contains from 1 to 2% water. After several hundred cycles, these trace amounts eventually accumulated into a sizable quantity.

4. Conclusion

The feasibility of the hydrazine plenum warm gas attitude control system was demonstrated by the use of breadboard-type hardware. The test program was hampered by support equipment failures. Several important system problem areas were identified, primarily in the areas of particulate and liquid contamination of the gas, and materials compatibility. No serious or unsolvable problems were encountered. A careful design, with appropriate provision for filtering the gases, should prove highly successful.

B. Liquid Propulsion Systems, L. R. Toth and O. F. Keller

1. Introduction, L. R. Toth

The Advanced Liquid Propulsion Systems program is investigating selected problems generated by spacecraft operational requirements for propulsion systems capable of high inherent reliability, long-term storage in the space environment, multiple start in free fall (zero gravity), and engine throttling. The proposed solutions to these problems are coordinated for integration into a system.

Beginning with SPS 37-8, Vol. IV, periodic reports describe the progress of work on the various parts of a specific system. Recent advances in material compatibility and bladder development include:

- (1) Material compatibility with hydrazine and hydrazine/hydrazine nitrate mix propellants.
- (2) Heat sterilization of ethylene-propylene elastomeric material with hydrazine.

2. Material Compatibility, L. R. Toth

A number of spacecraft construction materials are subjected to long term compatibility tests in hydrazine (N_2H_4), hydrazine/hydrazine nitrate mix (75% N_2H_4 , 24% $\text{N}_2\text{H}_5\text{NO}_3$, 1% H_2O), and nitrogen tetroxide (N_2O_4) propellants.

Each material sample is sealed in a glass ampul; a lox-cleaned pressure gage is attached to permit termination of the test if the internal pressure buildup from decomposition reaches 40 psia. This predetermined cut-off point would preclude failure of the glass ampul. Prior to use, the sealed ampuls are stored at $110 \pm 5^\circ\text{F}$ in the material compatibility test facility located at the Edwards Test Station.

Metal specimens are cylinders $\frac{1}{4}$ -in. in diameter and 3 in. long; they are half-immersed in 20 cm^3 of liquid propellant. The elastomeric specimens are typically thin sheets $\frac{1}{2}$ in. wide, $\frac{1}{4}$ in. long and 0.03 to 0.06 thick; these are fully immersed in 40 cm^3 of propellant.

All test and specimen data have been fully documented for comparison with post-test analyses. This includes such pertinent controlling information as fixture internal geometry and volume, coding, material physical measurements and properties, propellant composition and volume, and the cleaning procedures involved during preparation. The evaluation of results includes analysis for propellant decomposition, metallurgical analysis to determine the

effects of exposure on the basic material at the liquid interface, and the significance of pitting that results from material constituents. Permeation rates are measured for elastomeric specimens. Complete information will be included in a formal report to be released at the conclusion of this Phase I test program.

The following summarizes the results of tests in progress, and tests that were terminated, but not reported previously.

- (1) The status and results for the hydrazine propellant test are summarized in Table 5.

- (2) The status and results for the hydrazine/hydrazine nitrate mix propellant test are summarized in Table 6.

- (3) The nitrogen tetroxide propellant Phase I testing was concluded on February 1, 1967. A total of forty-four test capsules and specimens of the overall program remained at this point. The complete lot is being subjected to post-test analysis and evaluation, and will be reported at a later date. It is noted that throughout this nitrogen tetroxide Phase I testing, there was no indication of an internal pressure build up.

Table 5. Results of compatibility tests with hydrazine

Material	Sample No.	Description	Days to reach		Pressure at 200 days, psia	Total number of days in test	Pressure attained, psia	Test in process as of 3/1/67
			20 psia	35 psia				
Stainless steel	20	19-9 DL	405	1064	12	1154	37	
	19	430	172	374	22	408	39	
	22	440c	191	313	21	461	39	
	16	410	171	311	24	367	40	
	137	310	126	257	26	278	40	
	136	304L	84	205	34	239	40	
	17	416	56	103	—	119	40	
	135	304	54	97	—	110	40	
	18	420	42	91	—	108	38.5	
	134	303	35	63	—	86	41	
	183	A286	40	61	—	67	40	
Aluminum	244	356-T6	— ^a	—	0	692	3.5	Yes
	243	356-T6	— ^b	—	2	692	8.5	Yes
	184	7178	457	773	9	835	39.5	
	185	7178	335	626	12	711	41.5	
	24	6061-T6	201	421	19	506	41.5	
	186	7075-T6	89	275	30	297	42	
	187	7075-T6	276	453	—	635	43	
Miscellaneous	94	Platinum foil	— ^c	—	2	1398	16.5	Yes
	163	Teflon-aluminum laminate	663	1044	6	1198	42	
	23	1018 steel	133	210	30	286	39	
	90	Gold	78	119	—	166	40	
	133	6061-T6 Al (brazed)	— ^d	—	2	1317	19.5	
	49	6061-T6 Al and 75-A Ti (brazed)	130	288	29	343	39	
	47	6061-T6 Al and 303 SS (brazed)	148	169	—	180	31	
	249	Vicalloy	43	85	—	100	42	
Lubricants	108	6061-T6 Al with Apiezon-L	— ^e	—	2	1314	15.5	Yes
	105	6061 T6 Al with Rulon	1182	— ^f	1	1314	24	Yes
	101	6061-T6 Al with Rulon	25	49	—	77	48	
	107	6 AL-4V Ti with Apiezon-L	1096	— ^g	2	1314	26	
	103	6 AL-4V Ti with Rulon	254	361	14	431	42	
	106	347 SS with Apiezon-L	126	247	25	273	40	
	102	347 SS with Rulon	57	95	—	121	40	
^a 3.5 psia at 677 days. ^d 19 psia at 1304 days. ^g 25.5 psia at 1300 days. ^b 8 psia at 676 days. ^e 15 psia at 1300 days. ^c 16 psia at 1385 days. ^f 23.5 psia at 1294 days.								

Table 5 (contd)

Material	Sample No.	Description	Days to reach		Pressure at 200 days, psia	Total number of days in test	Pressure attained, psia	Test in process as of 3/1/67
			20 psia	35 psia				
Ceramics and lubes	40	3/8 in. ceramic ball (Al ₂ O ₃)	482	895	9	983	40	
	49	3/8 in. ceramic ball (Al ₂ O ₃)	326	483	17	571	40	
	51	3/8 in. ruby ball	182	397	22	449	40	
	104	Ceramic ball and Rulon	646	1059	2	1134	41.5	
	146	1/4 in. ceramic ball and moly coat	379	1030	10	1066	39	
	109	Ceramic ball and Apiezon L	259	410	13	457	40	
Butyl elastomers	194	Parker B 496-7	64	118	—	126	41	
	193	Parker B 496-7	61	118	—	126	40.5	
	195	Parker B 496-7	61	117	—	126	41.5	
	197	Parker B 480-7	47	85	—	109	40	
	198	Parker B 480-7	42	83	—	98	42.3	
	196	Parker B 480-7	41	82	—	98	42	
	122	Hadbar #XB-800-71	38	79	—	97	39	
	53	Stillman SR 634-70	44	73	—	77	37	
	149	Stillman SR 634-70	27	47	—	56	40	
	129	Stillman SR 634-70	2	9	—	16	50	
	148	Stillman SR 613-75	17	24	—	36	58	
	61	C.H.R. #4131	35	67	—	72	38	
	57	C.H.R. #3609	32	60	—	68	40	
	147	Fargo FR 6-60-26	32	61	—	74	40	
	125	Fargo FR 6-60-26	<1	10	—	26	48	
	55	Fargo FR 6-60-27	19	44	—	49	38	
EPR elastomers	155	Parker E 515-8	— ^h	—	4	231	17	Yes
	15	Parker E 515-8	— ⁱ	—	—	89	12	
	200	Parker E 529-6	367	689	12	842	43	
	201	Parker E 529-6	325	650	12	742	41	
	199	Parker E 529-6	262	528	15	613	42	
	162	Stillman SR EX 983S-75	198	389	20	476	40	
	150	Stillman SR EX 983S-75	119	297	24	317	37	
	164	Stillman SR 722-70	112	165	—	175	40	
	165	Stillman SR 722-70	107	161	—	175	40	
	168	Stillman SR 722-70	31	53	—	71	46	
	63	Stillman SR EX 1004-70	16	26	—	30	41	
	11	Thiokol 132/54	5	8	—	— ^f	40	
SBR elastomers	58	C.H.R. #3818	90	200	35	309	40	
	123	C.H.R. #4610	54	198	35	239	40	
	59	C.H.R. #3906	85	—	—	492	16	
	60	C.H.R. #3954	64	114	—	124	40	
	124	C.H.R. #4762	36	67	—	97	42	
Miscellaneous elastomers	157	G.E. RTV-60 (Silicon)	189	296	21	359	40	
	62	C.H.R. #4706 (PBR)	21	42	—	49	39	
	156	DuPont 5109D-390 (EPTR)	3	7	—	8	42.5	
Butyl bonded to 6-4 titanium strips	120	Stillman SR 634-70	68	208	33	240	40	
	121	Stillman SR 634-70	52	133	—	180	31	
	119	Stillman SR 634-70	52	121	—	180	40	
	118	Stillman SR 634-70	52	121	—	155	40	
	171	Hadbar XB-800-71	45	74	—	86	40	
	170	Hadbar XB-800-71	45	74	—	86	40	
	169	Hadbar XB-800-71	42	67	—	86	44	

^h16.5 psia at 1250 days.ⁱ10 psia at 189 days.

Table 5 (contd)

Material	Sample No	Description	Days to reach		Pressure at 200 days, psia	Total number of days in test	Pressure attained, psia	Test in process as of 3/1/67
			20 psia	35 psia				
Bonded to 4 titanium strips	203	Parker E-529-6	274	531	15	608	41	
	204	Parker E-529-6	200	377	20	541	43	
	182	Parker E-515-8	211	528	19	770	41	
	175	Stillman SR 722-70	93	259	32	322	40	
	177	Stillman SR 722-70	91	239	33	339	40	
	176	Stillman SR 722-70	84	182	38	283	43	
	174	Hadbar EPR 500-2	50	88	—	115	42	
	172	Hadbar EPR 500-2	47	88	—	108	42	
	173	Hadbar EPR 500-2	43	78	—	97	42	
N ₂ H ₄ only	245	Control sample — blank	— ^j	—	2	692	7	Yes Yes
	179	Control sample — blank	— ^k	—	4	800	8.5	
	138	Control sample — blank	1188	— ^l	3	1317	22.5	
	206	Control sample — blank	597	786	5	854	39.5	
	145	Control sample — blank	532	1101	9	1208	42	
	158	Control sample — blank	590	1012	6	1137	42	
	205	Control sample — blank	408	677	9	741	41	
	178	Control sample — blank	326	566	15	642	41	
	189	Control sample — blank	172	337	22	433	45	
	207	Control sample — blank	86	139	—	180	40	
	130	Control sample — blank	66	220	34	277	40	
	126	Control sample — blank	44	84	—	86	40	

^j 17 psia at 676 days.
^k 8.5 psia at 800 days.
^l 22 psia at 1303 days.

Table 6. Results of compatibility tests with hydrazine/hydrazine nitrate mix (75 % N₂H₄, 24 % N₂H₅NO₃, 1 % H₂O)

Material	Sample No.	Description	Days to reach		Pressure at 200 days, psia	Total number of days in test	Pressure attained, psia	Test in process as of 3/1/67
			20 psia	35 psia				
Stainless steel	143	310	301	686	13	809	40	
	231	310	250	568	16	666	45	
	232	310	200	539	20	606	40	
	142	304L	194	322	20	350	39.5	
	236	304L	127	251	28	319	43	
	235	304L	64	149	—	191	43	
	233	347	51	120	—	129	40	
	234	347	46	108	—	134	44	
	95	347	48	104	—	118	40	
	45	347	26	49	—	55	40	
	141	304	264	423	15	541	40	
	230	304	41	57	—	70	40	
	97	304	31	45	—	48	38	
	229	304	14	40	—	42	40	
	96	303	29	46	—	49	37	
	227	303	10	16	—	17	45	
	228	303	10	13	—	17	44	
	41	416	12	18	—	29	54	

Table 6 (contd)

Material	Sample No.	Description	Days to reach		Pressure at 200 days, psia	Total number of days in test	Pressure attained, psia	Test in process as of 3/1/67
			20 psia	35 psia				
Titanium	212	6Al-4V Ti	— ^a	—	4	832	13	Yes
	213	6Al-4V Ti with 6061-T6 Al	— ^b	—	3	832	9	Yes
	214	6Al-4V Ti with 321 SS	194	414	16	594	40	
	209	6Al-4V-ELI Ti	— ^c	—	3	832	8	Yes
	210	6Al-4V-ELI Ti with 6061-T6 Al	— ^d	—	3	832	8.5	Yes
	211	6Al-4V-ELI Ti with 321 SS	237	561	21	572	44	
	215	5Al-2.5 Sn Ti	761	— ^k	5	832	23	Yes
	216	5Al-2.5 Sn Ti with 6061-T6 Al	594	— ^l	7	832	31	Yes
	217	5Al-2.5 Sn Ti with 321 SS	213	531	19	596	41.5	
	218	5Al-2.5 Sn-ELI Ti	— ^e	—	2	832	7.5	Yes
	219	5Al-2.5 Sn-ELI Ti with 6061-T6 Al	— ^f	—	3	832	15	Yes
	220	5Al-2.5 Sn-ELI Ti with 321 SS	215	485	19	586	40	
	221	6Al-6V-2 Sn Ti (annealed)	— ^g	—	3	832	9	
	222	6Al-6V-2 Sn Ti with 6061-T6 Al	— ^h	—	2	832	5	Yes
	223	6Al-6V-2 Sn Ti with 321 SS	256	589	17	657	44	Yes
	224	6Al-6V-2 Sn Ti (heat treated)	— ⁱ	—	1	832	9	Yes
	225	6Al-6V-2 Sn Ti with 6061-T6 Al	— ^j	—	3	832	11	Yes
	226	6Al-6V-2 Sn Ti with 321 SS	219	495	19	586	40	
	46	6Al-4V Ti	91	138	—	158	41	
Aluminum	43	6061 T6 Al	— ^m	—	5	1413	19	Yes
	237	6061-T6 Al	734	— ⁿ	6	779	21	Yes
	42	6061-T6 Al	783	1437	9	1413	35	Yes
	238	6061-T6 Al	116	163	—	191	44	
	139	6061-T6 Al (brazed)	— ^o	—	3	1317	17.5	Yes
	100	6061-T6 Al (sprayed with Rulon)	405	843	10	1030	40	
Miscellaneous metals	99	6061-T6 Al brazed to 304 SS	20	33	—	34	37	
	98	6061-T6 Al brazed to 303 SS	14	25	—	28	40	
	250	Vicalloy	8	11	—	12	42	
	52	3/8 in. ceramic ball, Al ₂ O ₃	91	173	—	190	38.5	
	44	Ni—Inconel—C	12	20	—	29	52	
	50	3/8 in. sapphire ball	138	340	27.5	383	41	
Butyl elastomers	65	FR 6-60-26 (Fargo)	23	267	30	330	40	
	127	FR 6-60-26 (Fargo)	14	34	—	39	40	
	64	FR 6-50-3 (Fargo)	68	119	—	151	45	
	131	SR 634-70 (Stillman)	12	26	—	31	43	
Fuel mix only	151	Control sample — blank	— ^o	—	2	1265	7.5	Yes
	152	Control sample — blank	— ^p	—	2	1265	5.5	Yes
	153	Control sample — blank	— ^q	—	2	1265	12.5	Yes
	154	Control sample — blank	— ^r	—	2	1265	5.5	Yes
	239	Control sample — blank	— ^s	—	4	779	15	Yes
	144	Control sample — blank	766	1255	8	1317	37.5	Yes
	241	Control sample — blank	520	— ^u	11	779	35	Yes
	240	Control sample — blank	408	606	11	668	40	
	128	Control sample — blank	31	54	—	86	48	
	132	Control sample — blank	20	54	—	86	44	
^a 13 psia at 828 days.			^k 5 psia at 821 days.			^o 7.5 psia at 1250 days.		
^b 9 psia at 821 days.			^l 9 psia at 820 days.			^p 5 psia at 1250 days.		
^c 7.5 psia at 820 days.			^m 11 psia at 820 days.			^q 12 psia at 1250 days.		
^d 8 psia at 820 days.			ⁿ 22 psia at 820 days.			^r 5.5 psia at 1250 days.		
^e 7 psia at 820 days.			^o 30.5 psia at 820 days.			^s 15 psia at 769 days.		
^f 14.5 psia at 820 days.			^p 18.5 psia at 1436 days.			^t 21 psia at 769 days.		
^g 9 psia at 820 days.			^q 17 psia at 1304 days.			^u 34 psia at 769 days.		

3. Bladder Development, O. F. Keller

a. Heat sterilization compatibility of EPR in hydrazine.

A description of the heat sterilization compatibility test set-up, test equipment, and some preliminary heat-sterilization compatibility data are included in Ref. 1.

The first, or exploratory, phase of the heat-sterilization compatibility program now has been completed. Table 7 is a revised compilation of heat sterilization test histories. Container no. 1 was removed from the environmental test chamber after Op. 2 (operation 2), container no. 2 after Op. 4, container 3 after Op. 5, container 4 after Op. 6, container 5 after Op. 7, and container 6 after Op. 9. The durations of heat soak shown in Table 7 are accumulative (i.e., samples removed after a given test operation have also been exposed to the heat soak durations listed for all previous test operations). Table 8 provides a more complete compilation of heat-sterilization test data.

Fig. 3 represents the container pressure ranges for Op. 1 through Op. 9. Fig. 4 shows the sensitivity of the pressure generation rate to changes in test temperature.

Upon completion of Op. 9 and removal of container 6 from the environmental test chamber, the original amount of hydrazine placed in container 3 had been reduced by a factor of one-half. It is likely that a relatively large percentage of the missing hydrazine was vented between Op. 8 and Op. 9 when an attempt was made to "zero" the pressure transducer. Moreover, the relatively small changes in weight and shore hardness of the patch-test samples indicate that decomposition of the hydrazine rather than degradation of the EPR patch-test samples account for the greatest loss. Decomposition is further

Table 7. Ethylene propylene patch-test samples* heat-sterilization test histories

Test operation	Duration of heat-soak, hr	Heat-soak temperature, °F	Initial container pressure, psig	Final container pressure, psig	dP/dt (psi/hr)
Op. 1	1.33	300	50	72	16.5
Op. 2	60	170	26	31	0.083
Op. 3	24	235	40	58	0.750
Op. 4	71	235	37	94	0.803
Op. 5	9	300	50	142	10.2
Op. 6	28.50	300	47	302	8.95
Op. 7	88.25	300	55	708	7.41
Op. 8	164.50	300	40	1382	8.16
Op. 9	45	300	1252	1604	7.82

*Stillman Compound No. SR 722-70.

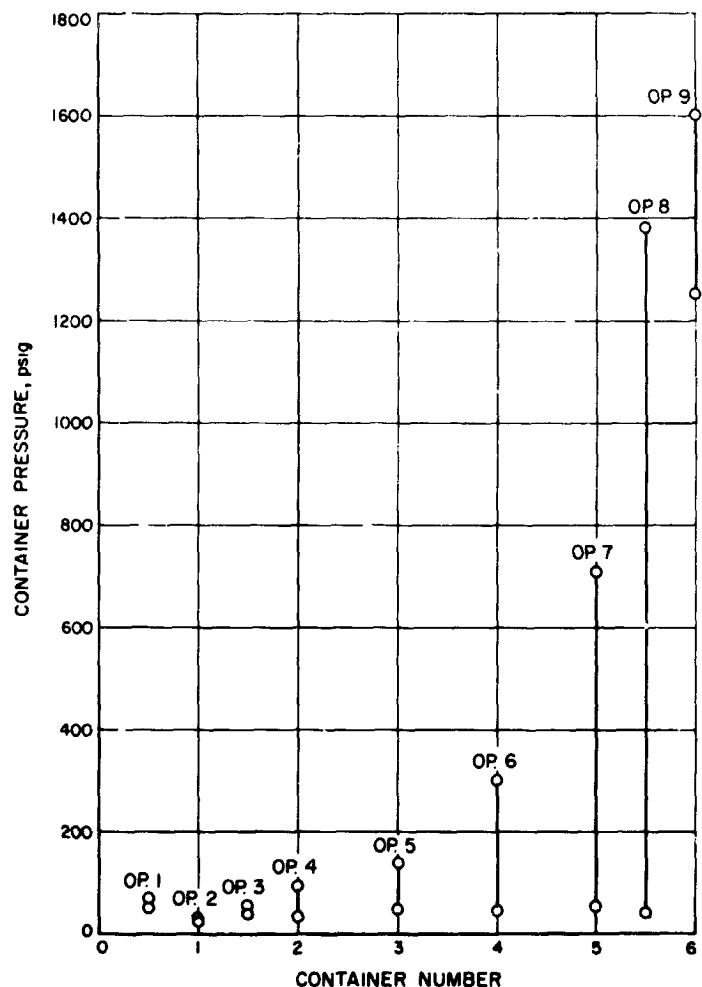


Fig. 3. Operating pressure ranges in test containers

indicated by Fig. 5, which shows changes of hydrazine concentration in the various containers after removal of each container from the environmental test chamber.

At the start of the test, the hydrazine had been colorless. When container 1 was removed from the environmental test chamber following Op. 2, the hydrazine was light brown in color, but still translucent.

The permeability data shown in Table 8 indicate inconsistency for patch-test samples removed from containers 2 and 5. Permeability tests of these samples now have been re-run. The revised permeability data are indicated in a separate column of Table 8.

Patch-test samples removed from container 6 after Op. 9 were changed very little in physical appearance, an indication that this material has excellent potential for use as an effective hydrazine bladder.

Table 8. Ethylene propylene patch-test samples heat-sterilization test data

Item	Sample No.	Thickness, in.	Initial weight, g	Final weight, g	Net increase (—decrease)	% increase (—decrease)	Permeation, Mg N ₂ H ₄ per hr per in. ²	Permeation, Mg N ₂ H ₄ per in. ² (re-run)	Initial shore hardness A ± 2.0	Final shore hardness A ± 2.0	N ₂ H ₄ color	N ₂ H ₄ concentration, %	Samples re-moved following	Compound No.	Container No.
1	66 X 09200-1-a ₁	0.037/0.038	1.3635	1.3662	0.0027	0.20	0.030	—	70.0	69.5	None	98.8	Op. 2	SR722-70	1
2	-1-a ₂	0.035/0.036	1.2930	1.2975	0.0045	0.35	0.030	—	70.0	69.5	—	—	—	—	—
3	-1-b ₁	0.037/0.041	3.4570	3.4798	0.0228	0.66	0.020	—	69.5	69.5	—	—	—	—	—
4	-1-b ₂	0.039/0.042	3.1698	3.1822	0.0124	0.39	0.040	—	71.5	69.5	—	—	—	—	—
5	-2-a ₁	0.035/0.038	1.3550	1.3608	0.0058	0.43	0.002	0.040	72.0	69.0	Light brown	98.6	Op. 4	—	2
6	-2-a ₂	0.036/0.038	1.3084	1.3170	0.0086	0.66	0.001	0.039	70.5	69.0	—	—	—	—	—
7	-2-b ₁	0.037/0.039	3.2670	3.2820	0.0150	0.46	0.001	0.045	67.5	69.0	—	—	—	—	—
8	-2-b ₂	0.040/0.043	3.4814	3.5037	0.0223	0.64	0.002	0.033	65.5	69.0	—	—	—	—	—
9	-3-a ₁	0.037/0.038	1.3420	1.3582	0.0162	1.21	0.030	—	71.5	67.0	—	98.4	Op. 5	—	3
10	-3-a ₂	0.033/0.034	1.2184	1.2338	0.0154	1.27	0.010	—	72.0	70.0	—	—	—	—	—
11	-3-b ₁	0.040/0.043	3.4260	3.4638	0.0378	1.10	0.020	—	71.5	68.0	—	—	—	—	—
12	-3-b ₂	0.040/0.042	3.4193	3.4590	0.0397	1.16	0.002	—	65.5	69.0	—	—	—	—	—
13	-4-a ₁	0.035/0.037	1.3033	1.3007	(—0.0026)	(—0.20)	0.034	—	72.5	71.0	—	97.6	Op. 6	—	—
14	-4-a ₂	0.034/0.036	1.2904	1.2876	(—0.0028)	(—0.22)	0.035	—	73.0	70.0	—	—	—	—	—
15	-4-b ₁	0.035/0.038	3.2336	3.2332	(—0.0004)	(—0.01)	—	—	66.5	72.0	—	—	—	—	—
16	-4-b ₂	0.038/0.041	3.2397	3.2391	(—0.0006)	(—0.02)	—	—	67.0	68.0	—	—	—	—	—
17	-5-a ₁	0.037/0.039	1.3396	1.3529	0.0133	0.99	0.130	0.046	71.5	69.0	—	96.7	Op. 7	—	5
18	-5-a ₂	0.032/0.035	1.2083	1.2183	0.0100	0.83	0.160	0.042	72.5	69.0	—	—	—	—	—
19	-5-b ₁	0.041/0.044	3.5659	3.6000	0.0341	0.96	—	—	69.5	68.0	—	—	—	—	—
20	-5-b ₂	0.039/0.043	3.4633	3.4924	0.0291	0.84	—	—	69.5	68.0	—	—	—	—	—
21	-6-a ₁	0.033/0.035	1.2372	1.2361	(—0.0011)	(—0.09)	0.036	—	71.5	73.0	—	95.1	Op. 9	—	6
22	-6-a ₂	0.037/0.040	1.4035	1.4061	(—0.0024)	(—0.17)	0.026	—	71.0	70.0	—	—	—	—	—
23	-6-b ₁	0.035/0.039	3.2000	3.1956	(—0.0044)	(—0.14)	—	—	72.5	74.0	—	—	—	—	—
24	-6-b ₂	0.042/0.045	3.4977	3.4944	(—0.0033)	(—0.09)	—	—	73.0	70.0	—	93.7	—	—	—

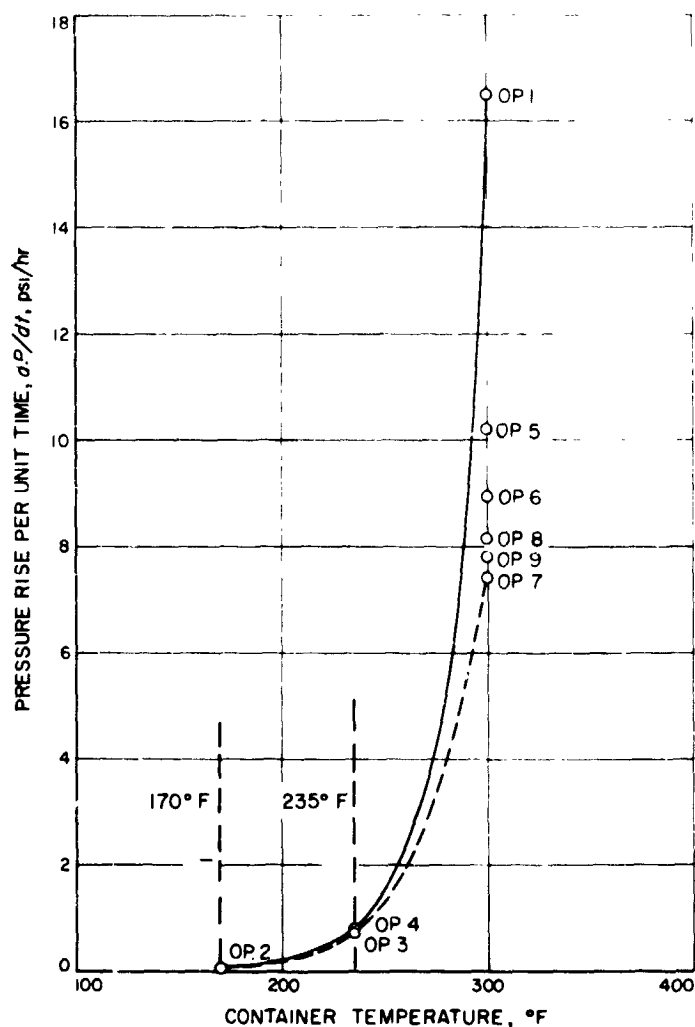


Fig. 4. Temperature sensitivity of EPR patch-test samples in N_2H_4 .

b. Stability of hydrazine. During previous glass vial testing with elastomers in hydrazine (Ref. 3), large quantities of NH_3 , nitrogen, and hydrogen were present. The equation,



might reasonably approximate the chemical breakdown of hydrazine. Substituted hydrazines such as MMH (monomethylhydrazine) are more stable than N_2H_4 .

C. Injector Development: Impinging Sheet Injectors, R. W. Riebling

Introduction

A jet of liquid from a round orifice directed tangentially against a solid, concave deflector surface spreads to form

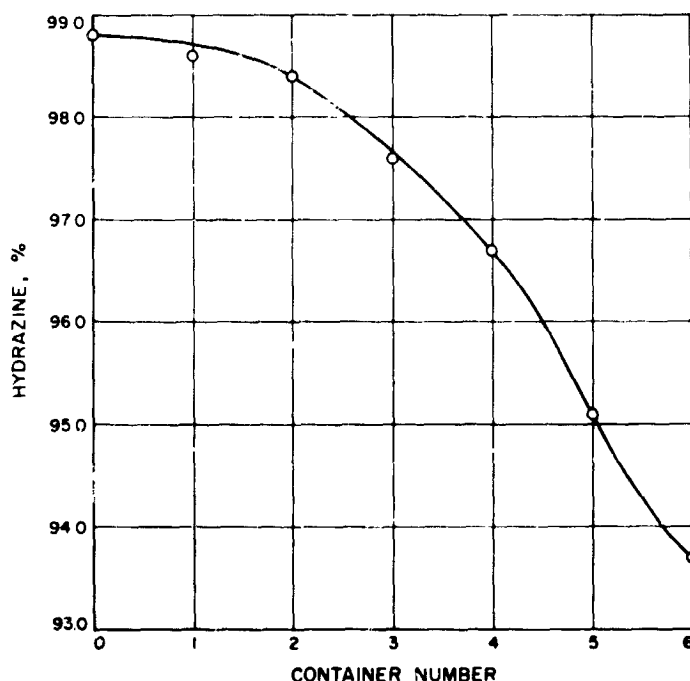


Fig. 5. Final hydrazine concentration in test containers

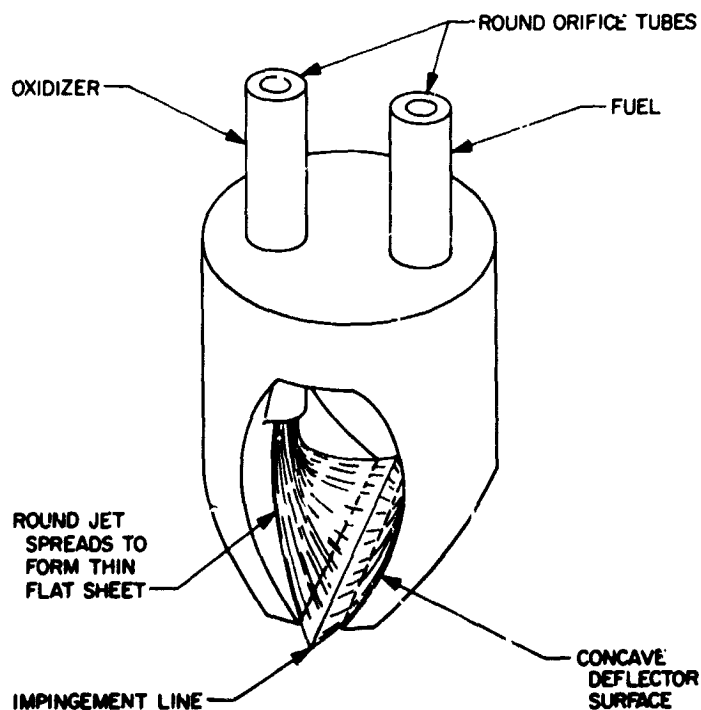


Fig. 6. Typical impinging-sheet injector element

a planar (typically, 0.005-0.010 in. thick) flowing sheet. A number of novel injector elements can be built around this sheet formation process; one of these, known as the impinging-sheet element, is shown in Fig. 6. In this

element, flat sheets of fuel and oxidizer are formed separately, then brought together along an impingement line. Impinging-sheet injectors have been under development at the Jet Propulsion Laboratory for some time. Early work by Evans (SPS 37-34, Vol. IV, p. 174, and SPS 37-35, Vol. IV, p. 152), produced design criteria for single impinging-sheet elements with the nitrogen tetroxide (N_2O_4) hydrazine (N_2H_4) propellant combination at a mixture ratio of 1.2. The work was done at the 25-lb_r-thrust-per-element level. These criteria were applied directly to the design of a multiple-element, 2000-lb_r-thrust, impinging-sheet injector that was operated at the same mixture ratio, the same level of thrust-per-element, and with the same propellant combination. The results demonstrated that impinging-sheet injectors exhibit a number of distinct advantages over the more conventional types. Some of these advantages are:

- (1) Relative insensitivity of performance to manufacturing tolerances.
- (2) Superior performance levels (measured as characteristic exhaust velocity c^* in ft/sec).
- (3) Wide throttling capability using several different throttling techniques without appreciable degradation in characteristic velocity efficiency, η_{cv} .
- (4) Extremely smooth and stable combustion.

Because of these inherent advantages, the impinging-sheet injection concept seems particularly well suited for a wide variety of other propellant combinations (especially the high-energy, space-storables) over a broad spectrum of space mission profiles. The principles of sheet formation, and the relationships governing the combination of sheets into injection elements are discussed in Refs. 4 and 5, respectively.

To extend this promising new injection technique to other applications, some additional design criteria are required. One area of particular interest is the protection of impinging-sheet elements during throttled operation when they are shut off and exposed to the extreme convective heat transfer environment without cooling by the normally-present flow of propellants along the deflectors. The multiple-element, 2000-lb_r-thrust injector mentioned previously had elements mounted externally to the injector face; some of these elements were severely eroded when not cooled by propellant flow (Fig. 7). One possible remedy is to recess the elements within the injector face to reduce convective heat transfer and provide greater opportunity for conduction of heat away from shut-off elements and into the body of the injector.

Another area of interest concerns the feasibility of scaling up the present designs to higher levels of thrust-per-element. Both the mixing and the atomization of a pair of impinging *jets* of unlike propellants are severely impaired as the jet diameters are increased (SPS 37-36, Vol. IV, p. 174), which results in degraded performance. It must be learned whether unlike impinging *sheets* are subject to the same critical limitations.

To investigate both areas simultaneously, a 100-lb_r thrust unlike impinging-sheet single element was designed, fabricated, and test-fired with N_2O_4 , N_2H_4 . Auxiliary attachments permitted controlled variation of the element's external geometry to include the simulation of a fully recessed installation. The effects of several different external configurations on performance were measured, and a way was found to bury the impinging-sheet elements within the face of an injector without an appreciable decrease in performance below that obtained when elements are mounted in a fully-external position. However, the c^* efficiency level of the 100-lb_r-thrust, single element was only about 85% (in both the external and the recessed positions). Because the earlier 25-lb_r elements were about 95% efficient, these results indicated that the performance of impinging-sheet elements is degraded as their physical size is increased. A series of experiments was then conducted in an attempt to understand more fully the nature of this phenomenon. This report will present and discuss the results of this investigation.

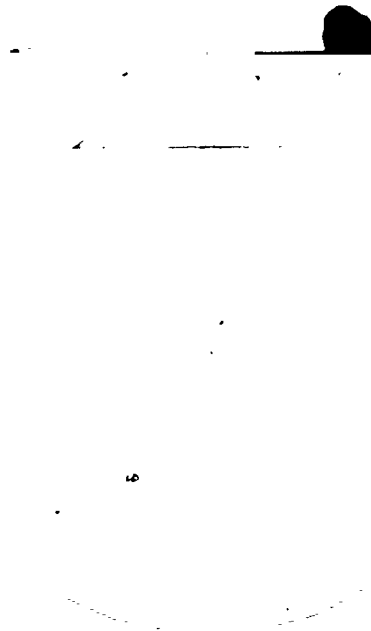
2. Apparatus

The basic 100-lb_r impinging-sheet injector element is illustrated in Fig. 8. In the geometry of its orifices and internal deflector surfaces, this injector was a scaled-up version of the earlier, 25-lb_r element. It was designed not only for equal fuel and oxidizer pressure drops, but also with a momentum ratio of unity, thus fulfilling the optimum mixing requirements for a pair of impinging streams of unlike liquids (Ref. 6).

Two identical stainless steel deflector/orifice combinations were mounted to the face of the injector so that the spacing between their knife-edges could be varied continuously between zero and 0.1 in. Each orifice had a 0.075-in. diameter, and a 100-diameter length to insure fully-developed turbulent flow with its reproducible hydraulic characteristics. The jet of liquid emerging from each orifice was introduced tangentially to a concave deflector surface of 0.58-in. radius, where it spread to form a thin, flat sheet. After being turned through angles of 45 deg on the deflectors, the sheets (one of fuel, the



(a) WITH PROPELLANT FLOW



(b) WITHOUT PROPELLANT FLOW

Fig. 7. Steel impinging-sheet injector element after throttled firing: (a) cooled by propellant flow, (b) without propellant flow

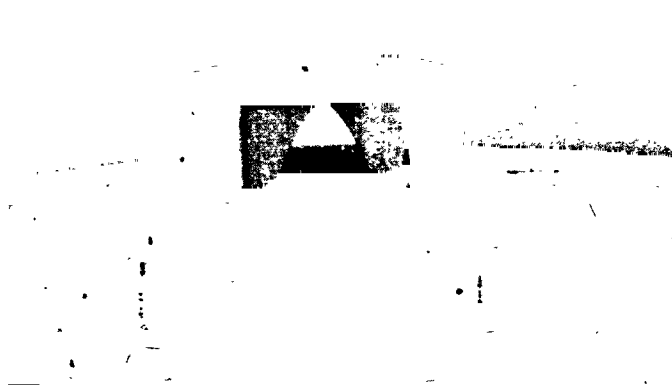


Fig. 8. Basic impinging-sheet injector element

other of oxidizer) impinged at 90 deg to form a finely divided spray.

To study the effects of burying this element within the face of an injector, a set of "sides" was made, which, when clamped in place around the basic element (Fig. 9)

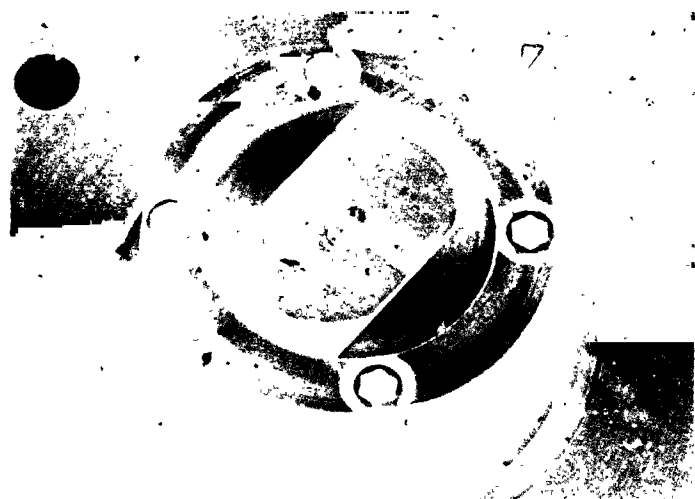


Fig. 9. Impinging-sheet element enclosed by unvened sides

completely enclosed the sheet formation devices. The only egress for the propellants was through the narrow slit between the two deflectors.

When similar impinging-sheet elements are flowed with inert propellant simulants, a small but not negligible amount of backspray emanates from the impingement line (Fig. 10). If such a backspray forms with the reactive propellants, some combustion might take place within the cavity between the two deflectors. This has caused no problems where the cavity is open to the surrounding chamber environment, as in the external installations shown in Fig. 8, but with the cavity completely enclosed as in Fig. 9, any gases produced by a reactive backspray could exit only through the slot between the deflectors. If this action interfered with the proper impingement of the propellant sheets, less efficient combustion could result. Accordingly, a second set of sides was fabricated, identical to the first except for vent passages designed to offer the least resistance to the escape of any backspray combustion products formed. These sides are shown in Fig. 11.



Fig. 10. Edge view of impinging liquid sheets showing backspray

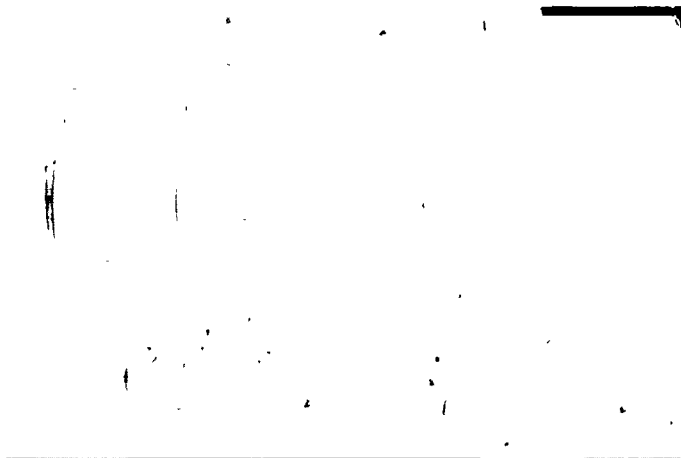


Fig. 11. Impinging-sheet element enclosed with vented sides

To determine the effect on performance of recirculation patterns in the vicinity of the impingement region the external surfaces of both deflectors were cut back, as shown in Fig. 12, a top view. This made the element similar to the 25-lb_r elements (Fig. 7) in external, as well as internal geometry.

The final configuration (Fig. 13, which also shows the cutback external deflector surfaces of Fig. 12) included an insert that considerably reduced the volume of the



Fig. 12. Impinging-sheet element with deflector sides cut back



Fig. 13. Impinging-sheet element with sides removed to show insert between deflectors

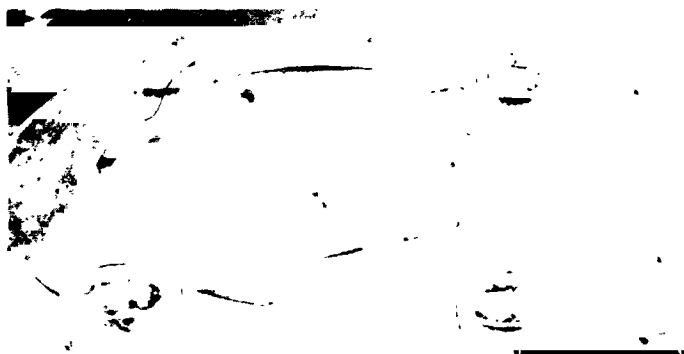


Fig. 14. Impinging-sheet element with unvented sides

cavity between the deflectors without interfering with the sheet-formation process. The element containing this insert was enclosed with a set of unvented sides (Fig. 14).

All of the injector configurations described above were fired in an uncooled, steel thrust chamber under ambient conditions 13.5 psia. Table 9 summarizes the chamber

Table 9. Summary of the 100-lb_r thrust chamber geometry

Measurement	Value
Cylindrical length ^a	4.4 in.
Diameter	2.0 in.
Throat area	0.5458 in. ²
Characteristic length ^a	28.6 in.
Expansion area ratio	2.134
Convergence half angle	40 deg
Divergence half angle	15 deg
^a With injector and accessories installed.	

geometry. In addition, the injector configuration shown in Fig. 12 was fired into a baffled chamber during a series of stream-separation experiments discussed later in this report. The baffled chamber was described in detail in SPS 37-31, Vol. IV, p. 192. It was similar to the chamber of Table 9, except for its greater characteristic chamber length L^* of 50 in., and the baffle and side spray apparatus customarily used in stream-separation firings.

3. Results and Discussion

Eight short-duration firings of 1 to 3 sec each were conducted to assess the effects of external element geometry on the performance of the 100-lb_r impinging-sheet injector. The propellants were N_2O_4/N_2H_4 at a nominal mixture ratio of 1.25, and chamber pressures that ranged between approximately 100 and 120 psia. The results are summarized in Table 10. Chamber pressures were measured at the entrance to the convergent section of the nozzle and corrected to stagnation values. The spacing between the two deflectors was constant at 0.05 in., except in Tests 6 and 7, where it was intentionally varied. Characteristic velocity efficiency (η_{c^*}), expressed in percent, was chosen as the comparative index of performance level, and is the ratio of the measured value of c^* to that theoretically attainable with full shifting equilibrium flow at the experimentally measured values of chamber pressure and mixture ratio. No corrections were made to the measured values of c^* because the results were to be used primarily for comparative purposes.

The results are presented in graphical form in Figs. 15 through 18. Where the basic injector configuration of

Table 10. The effects of external element geometry on performance of 100-lb_r impinging-sheet injector

Test No.	Injector configuration	Deflector spacing, in.	Chamber pressure (p_c), psia	Total flow rate ($\dot{w}_t$), lb _m /sec	Mixture ratio (r)	Characteristic velocity (c^*), ft/sec	Efficiency (η_{c^*}), %
1	Basic external installation (Fig. 8)	0.05	115	0.407	1.25	5090	87.7
2	Element enclosed with unvented sides (Fig. 9)	0.05	98	0.430	1.25	4010	69.5
3	Same as test no. 2	0.05	99	0.422	1.22	4130	71.2
4	Element enclosed with vented sides (Fig. 11)	0.05	113	0.416	1.27	4780	82.5
5	External surfaces of deflectors cut back, no sides (Fig. 12)	0.05	118	0.416	1.24	5000	86.4
6	Same as test no. 5	0.075	114	0.416	1.24	4830	83.4
7	Same as test no. 5	0.10	116	0.414	1.24	4920	84.9
8	Element enclosed with unvented sides and insert between deflectors (Fig. 14)	0.10	99	0.422	1.23	4130	71.3

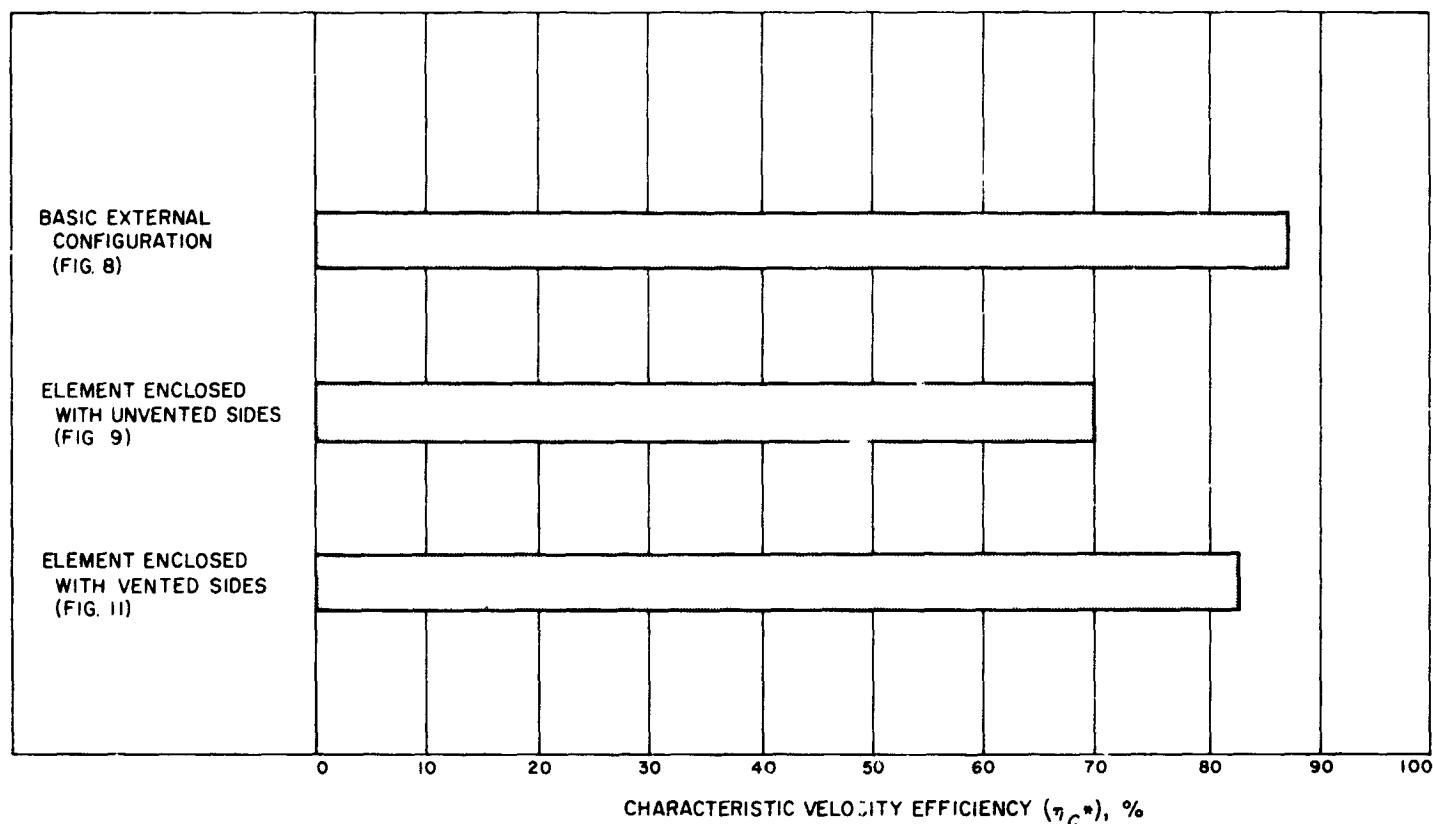


Fig. 15. The effect of side venting on performance of impinging-sheet injector (0.05-in. deflector spacing)

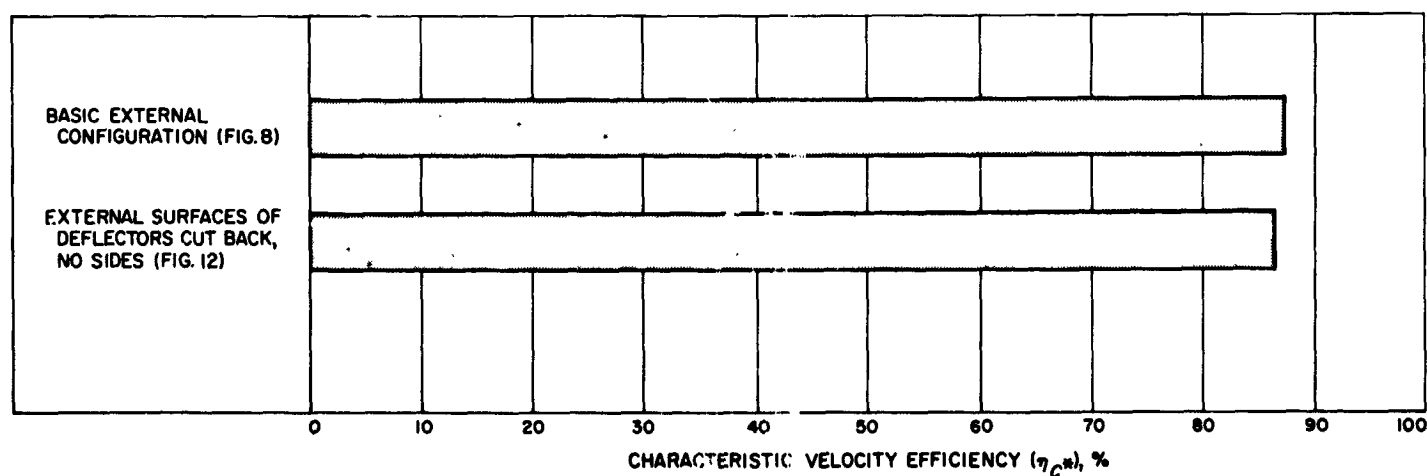


Fig. 16. The effect of external deflector geometry on performance of impinging-sheet injector (0.05-in. deflector spacing)

Fig. 8, which represents the typical external installation, was enclosed by sides (Fig. 9) simulating fully-recessed mounting, its performance index (η_{c*}) dropped from 87.7% to only 69.5% (Fig. 15). Yet when those sides were provided with vent passages, each with a cross-sectional area that was large relative to the area of the slot between

the deflectors, η_{c*} was restored to nearly its original value. No attempt was made to optimize the area or the contours of the vent passages in this brief program, but it seems reasonable that suitably-designed vents with fully buried elements would give the same performance as vents with completely exposed elements.

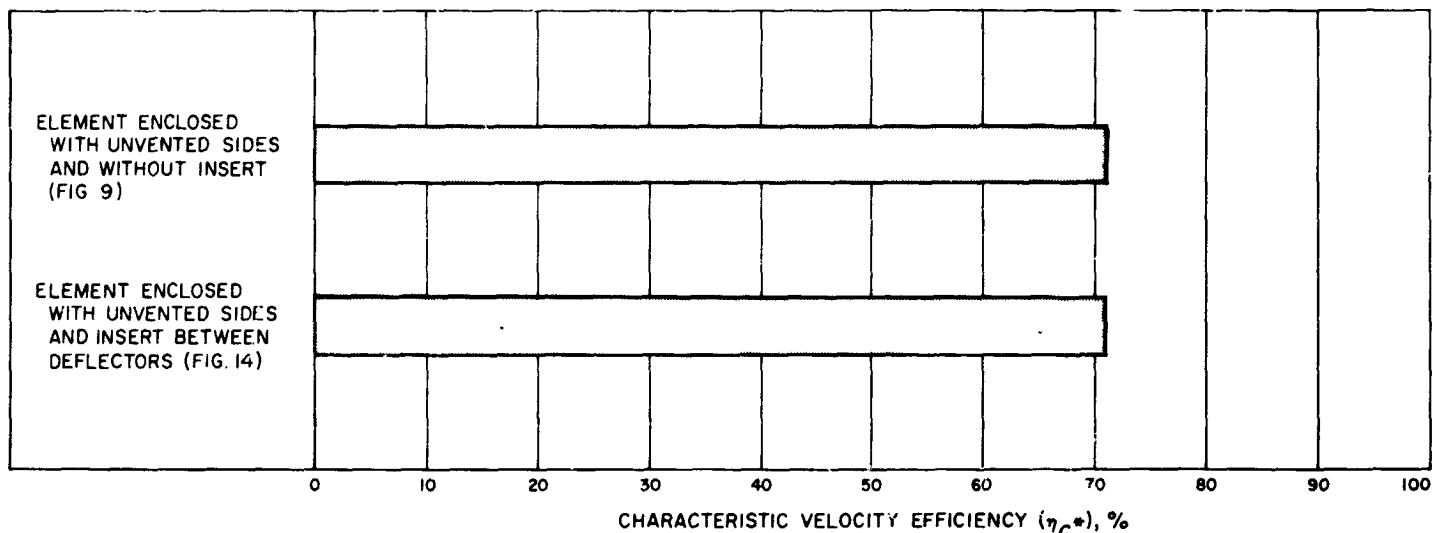


Fig. 17. The effect of internal cavity volume on performance of impinging-sheet injector (0.05-in. deflector spacing)

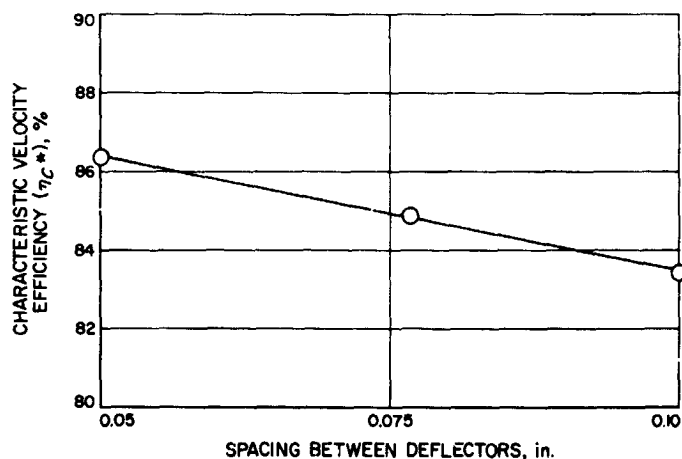


Fig. 18. The effect of deflector spacing on performance of impinging-sheet injector (the configuration of Fig. 12)

Fig. 16 indicates no change in performance with the external surfaces of the deflectors in Fig. 3 cut back to give the configuration of Fig. 7. If gas recirculation patterns had been present in the vicinity of the deflector knife-edges, they would have been debilitated by the geometry of Fig. 3 and enhanced by that of Fig. 7. Because the performance of both configurations is nearly identical, gas recirculation is either absent or does not play a significant role in the operation of the impinging-sheet injection element. Furthermore, the data in Fig. 16 indicate that a properly-vented element buried flush with the face of an injector should perform as well as one of the same scale that has cutback deflectors and is mounted externally. The degradation in performance of buried elements without vents is not fully understood at this time,

but may be the result of sheet disruption by gases from a reactive backspray discussed earlier.

To facilitate machining, the deflectors of impinging-sheet injectors are made usually with full internal radii, as shown in Fig. 8, although only a small portion of the resulting internal surface is actually used for sheet formation. This geometry results in a relatively large cavity between the deflectors. The solid insert (Fig. 13) reduced the volume of this cavity by about 75%, leaving only sufficient clearance for sheet formation, yet a buried, unvented element with this reduced cavity volume (Fig. 14) gave the same performance as a buried, unvented element with the full internal volume (Fig. 17). This would suggest that whatever performance degradation exists with buried, unvented elements is not greatly influenced by the internal cavity volume.

The effect of deflector spacing on performance is shown in Fig. 18; at least over the range of spacings investigated, closer spacing results in higher performance. This is in agreement with the earlier 25-lb., single element results reported in SPS 37-34, Vol. IV, p. 174, and in SPS 37-35, Vol. IV, p. 152. It is substantiated also by recent cold flow studies at JPL, the subject of a future SPS article, which have shown that deflector spacing exerts a major influence on the mixing efficiency of unlike-impinging sheets. Closer spacings result in higher values of the propellant distribution index (η_M), measured in percent. Of course, lower performance might be expected from extremely close spacings that interfere with the sheets themselves. This apparently did not take place in

firings at the 0.05-in. spacing because performance steadily decreased as the spacing increased to 0.1 in.

Some of the hardware modifications investigated were capable of delivering about the same performance as the baseline design, which was the externally-mounted element. However, none of these modifications to the basic concept resulted in improved performance, and, with this 100-lb_i impinging-sheet injector, it seems that 85 or 88% is an effective upper limit to the c^* efficiencies attainable in chambers with characteristic lengths of about 30 in. If this is true, the performance of impinging-sheet injectors decreases as their physical size increases, behavior that is similar to that of conventional impinging-jet injectors.

For most propellant combinations, regardless of the type of injector used, these primary processes must take place to ensure efficient combustion: (1) propellant mixing, (2) atomization and vaporization, and (3) chemical reaction. Normally, chemical reaction rates are quite rapid compared to the rates of mixing and atomization, so that one or both of the two physical processes usually controls the overall combustion rate. For impinging-jet injectors, which are generically similar to impinging-sheet injectors, propellant mixing is impaired as the diameters of the impinging streams are increased. The now classical experiments of Johnson (Ref. 7) and Stanford (SPS 37-31, Vol. IV and SPS 37-36, Vol. IV) showed that impinging jets of hypergolic, highly-reactive propellants such as N_2O_4/N_2H_4 tend to "blow apart" before complete mixing in the liquid-phase because of rapid chemical reactions at the impingement interface. Although these stream separation effects are negligible with jets of small diameter (~ 0.02 in.), they become noticeable at intermediate diameters (~ 0.065 in.), and seriously degrade performance with large (~ 0.25 in.) diameter jets.

To determine whether the impaired propellant mixing that resulted from combustion effects limited the performance of the 100-lb_i impinging-sheet element, a series of nine baffled chamber firings was made. The injector configuration shown in Fig. 12 was fired in a chamber with a characteristic length of 50 in., and the chamber was divided into two longitudinal channels by a baffle plate. The baffle was oriented to lie in a plane perpendicular to the plane in which the injector orifices lay. The top of the baffle was situated a short distance from the injector plate to avoid interference with the impingement process, and the bottom of the baffle was a short distance upstream of the throat. Two full-cone spray nozzles were located in the chamber wall, one on each side of the

baffle, and turbulence rings were mounted in the chamber, downstream of the spray nozzles. The objective of the experiment was to measure the difference in performance when the sprays from the side nozzles (one spraying fuel and the other oxidizer) were reversed.

The concept involved follows: if the propellants flowing from the main injector are repelled from each other to form a fuel-rich zone on the fuel orifice side of the chamber, and an oxidizer-rich zone on the oxidizer side, the baffle should prevent secondary mixing because of turbulence and diffusion. Thus, one channel should contain an oxidizer-rich flow, and the other a fuel-rich flow of gases. Spraying oxidizer into fuel-rich gases and fuel into the oxidizer-rich gases (termed "unlike" propellants), should increase performance, while spraying fuel into fuel-rich gases and oxidizer into oxidizer-rich gases (termed "like" propellants), should reduce performance. If the streams from the main injector do not blow apart, and a nearly uniform mixture-ratio distribution exists in the chamber, performance should remain relatively unchanged when the propellant sprays are reversed. If the streams penetrate each other, the fuel-rich and oxidizer-rich channels will be reversed, and the performance changes should indicate this condition, as well.

The apparatus and procedures were identical to those used by Stanford (SPS 37-31, Vol. IV), except for the injector, which was the present impinging-sheet model. Several series of firings were conducted in which propellants were injected through the main injector as well as the side-spray nozzles along the chamber walls. In some tests, like propellants were injected at the sides, while in other tests unlike propellants were injected. In each of these series, the side flows were varied from about 10 to nearly 30% by weight of the total flow rate, while the flow through the main element was held constant. To measure the datum performance level, the engine was fired at the design flow rate with no side injection.

The results are summarized in Table 11. Performance was calculated on the basis of the total flowrate $\dot{w}_t$ of propellants (from both the main element and the side streams) injected into the chamber. The comments and qualifications made with respect to Table 10 apply to Table 11 as well.

The results of these tests are shown in Fig. 19. For purposes of comparison, the data of Stanford (SPS 37-31, Vol. IV) for a 100-lb_i unlike impinging-jet injector fired

Table 11. Degree of stream separation of unlike impinging sheets in 100-lb_f injector fired in baffled-chamber

Test No.	Kind of side spray	Oxidizer flow, lb _m /sec		Fuel flow, lb _m /sec		Total flow rate ($\dot{w}_t$), lb _m /sec	Side flow, % by weight	Overall mixture ratio (r)	Chamber pressure (p _c), psia	Characteristic velocity (c*), ft/sec	Efficiency (η_{c^*}), %
		Main element	Side spray	Main element	Side spray						
1	Like	0.161	0.075	0.133	0.047	0.418	29.7	1.30	108	4350	75.5
2	Like	0.160	0.036	0.133	0.035	0.369	19.2	1.17	97	4440	76.5
3	Like	0.158	0.017	0.131	0.022	0.328	11.9	1.15	95	4980	85.7
4	None	0.165	0.0	0.131	0.0	0.296	0.0	1.26	90	5140	89.0
5	Unlike	0.158	0.017	0.131	0.018	0.323	10.7	1.17	100	5240	90.2
6	Unlike	0.160	0.033	0.134	0.021	0.346	15.6	1.25	104	5110	88.3
7	Unlike	0.160	0.035	0.133	0.023	0.351	19.6	1.24	106	5040	87.0
8	Unlike	0.158	0.038	0.130	0.032	0.358	19.6	1.21	109	5130	88.5
9	Unlike	0.164	0.067	0.135	0.047	0.413	27.7	1.27	121	4940	85.4

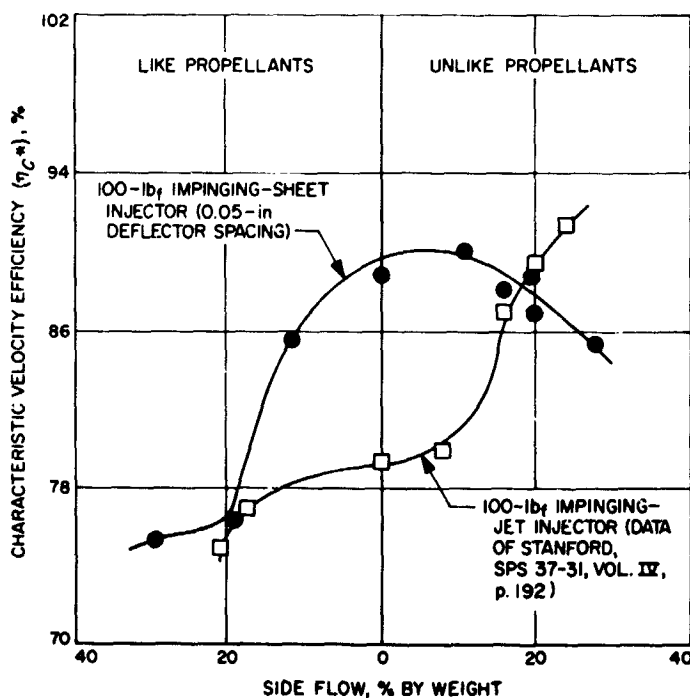


Fig. 19. Variation of c^* efficiency with side flow in baffled chamber experiments using two 100-lb_f single-element injectors

in the same apparatus* are included. The maximum performance ($\eta_{c^*} \approx 90\%$) with impinging sheets was realized at only about 5% unlike side flow, which indicates, at

*Stanford also studied an early version of the impinging-sheet injector, but in both its geometry and its spacing between deflectors (0.72 in.) it bore little resemblance to present designs. Accordingly, valid comparisons probably cannot be made between its performance and that of the present injector.

most, a very slight degree of stream separation. This performance level is slightly higher than that attained in the unbaffled chamber, probably the result of the greater L^* (50 in.) of the baffled chamber. Increasing the percentage of side flow, either like or unlike, lowered the performance, probably by the amount of heat absorbed by the uncombined propellant. Thus, for all practical purposes, the impinging sheets neither blew apart nor penetrated each other. Rather, they mixed well and produced a nearly uniform mixture ratio distribution in the chamber.

This behavior may be contrasted to that of the impinging jets, where performance increased by over twelve percentage points at maximum unlike sidespray, indicating severe blowing apart of the main element's unlike propellant streams. It would seem, then, that while rapid interfacial chemical reactions at the impingement surface interface considerably with the efficient mixing of a pair of round jets at the 100-lb_f thrust level, their effects can be virtually eliminated by transforming those streams into thin, flat sheets prior to impingement.

Because propellant mixing effects probably are not the cause of performance degradation as the physical size of impinging-sheet elements is increased, the atomization process is logically suspect. Most modern combustion models recognize the influence of propellant drop size on vaporization, and therefore on combustion. Some years ago, Ingebo (Ref. 8) studied the atomization of like impinging jets of heptane in air streams. He published an equation relating the volumetric mean droplet diameter ($\bar{D}_{3v}$, measured in microns), to the jet diameter (D_j , measured in in.), jet velocity (V_j , measured either in in./sec

or ft/sec), and to the difference between the airstream and jet velocities (ΔV). That equation may be simplified to

$$\bar{D}_{30} \sim \left(\frac{D_j}{V_j} \right)^{1/2} \quad (1)$$

by assuming $\Delta V = 0$. This relationship indicates that the degree of atomization that results from the self-impingement of a pair of round jets decreases as the jet diameter increases, or as the injection velocity decreases. Although Eq. (1) applies strictly to the like impingement of heptane jets only, several attempts have been made to extend it to the *unlike* impingement of jets of other liquids. A detailed discussion of this subject is clearly beyond the scope of this report. It is sufficient to say that good correlations have been obtained between

$\bar{D}_{30}$ and $(D_j/V_j)^{1/2}$ in cold flow drop-size experiments with like impinging jets of water and other liquids, and between η_{cs} and $(D_j/V_j)^{1/2}$ in test firings where atomization controlled the combustion process. The data for unlike impinging streams do not correlate quite as well, but do show the same general trend. In brief, then, the degree of atomization in the spray produced by a pair of impinging jets decreases as the diameter of the jets increases, assuming a constant injection velocity. It would not be surprising to find similar effects because of physical size in the atomization of impinging sheets. This, and the demonstrated absence of mixing effects would suggest that impinging-sheet injection elements are atomization-limited at higher levels of thrust-per-element. In general, the controlling parameter may prove to be the stream cross-sectional area rather than the diameter, which is meaningful only for round jets.

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XIV. Lunar and Planetary Instruments

SPACE SCIENCES DIVISION

A. Mass Spectrometer Gaussmeter, J. R. Locke

1. Introduction

In support of mass spectrometer instrumentation, a gaussmeter has been designed and is presently being tested. This gaussmeter will be used to monitor the field of a 5-kilogauss permanent magnet essential to mass identification. To detect masses in the 10 to 120 range, it is necessary to know the permanent magnet's field strength to better than 0.5%.

A Hall-effect probe is used to monitor the field. The probe is driven by a transformer-coupled, square-wave-modulated, field-strength-dependent current source. The output of the Hall-effect probe is ac-coupled into a voltage gain stage, synchronously demodulated, filtered, dc-amplified, and fed back to control the current source.

2. Accuracy Consideration

To understand the significance of the magnetic field in connection with the mass spectrometer, consider Fig. 1, which is a simplified diagram demonstrating the principle that is involved. The potential energy of the ionized particle of mass m and charge q is converted into kinetic energy by

$$\frac{1}{2} mv_f^2 = qV \quad (1)$$

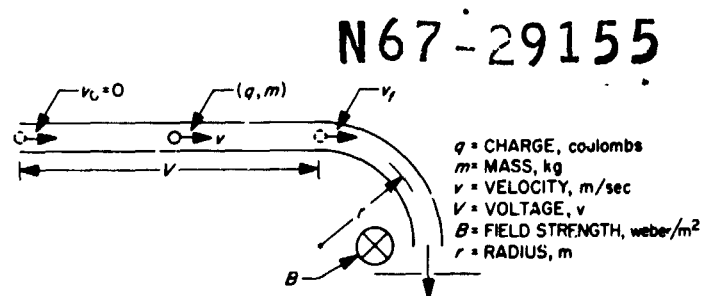


Fig. 1. Mass spectrometer principle

The B -field causes the charged particle to pass through a radius r :

$$r = \frac{mv_f}{qB} \quad (2)$$

$$\frac{m}{q} = \frac{2V}{v_f^2} \quad (1a)$$

$$v_f^2 = \left[\frac{rB}{\left(\frac{m}{q}\right)} \right]^2 \quad (2a)$$

Substituting Eq. (2a) into Eq. (1a),

$$\frac{m}{q} = \frac{r^2 B^2}{2V}$$

Considering only monovalent substances, $q = e$, where e is the charge of one electron, and

$$m = \frac{er^2 B^2}{2V}$$

Since only the mass and B -field relationship is considered, let

$$\frac{er^2}{2V} = \phi$$

Then

$$m = \phi B^2$$

$$\frac{\partial m}{\partial B} = 2\phi B$$

$$\frac{\partial m}{m} = \frac{2 \partial B}{B}$$

Therefore, with all other factors constant, to resolve 1 part in 120, it is necessary to know the B -field 1 part in 240, or to 0.42%

3. Instrumentation

Fig. 2 is a block diagram of the gaussmeter and Fig. 3 is the schematic identifying the components that will be referred to in the following analyses.

a. The Hall-effect probe. If an electric field is imposed upon a conductor or semiconductor, the electrons (or

holes) drift from one current electrode to the other. If a magnetic field B is imposed in the transverse direction, the electrons are deflected by the Lorentz force acting upon them and a charge builds up on the boundary of the current-carrying element until the repulsive force, due to the charge, balances the Lorentz force. The resultant electrostatic field across the width of the conductor (or semiconductor) gives rise to a voltage which can be expressed as:

$$V_H = \frac{R_H}{t} (I_c B \sin \Theta) 10^{-8} \text{ v}$$

where

R_H = Hall coefficient, $\text{cm}^3/\text{coulomb}$

t = element thickness, cm

I_c = control current, amp

B = magnetic field density, gauss

Θ = angle between I_c and B

If $\Theta = 90$ deg, then $V_H = K I_c B$ where $K = \text{v/amp-kilogauss}$.

b. Design considerations. The Hall-effect probe chosen for this application is the Bell BH-200. This device has a sensitivity constant of 0.08 v/amp-kilogauss and is operated at a nominal control current of 100 ma. In a 5-kilogauss field, the resultant voltage is 40 mv. To measure this level within 0.42% ($170 \mu\text{v}$) precludes making a straight dc voltage measurement because of amplifier dc offset current and offset voltage variation with temperature. The alternatives are to use a dc control current

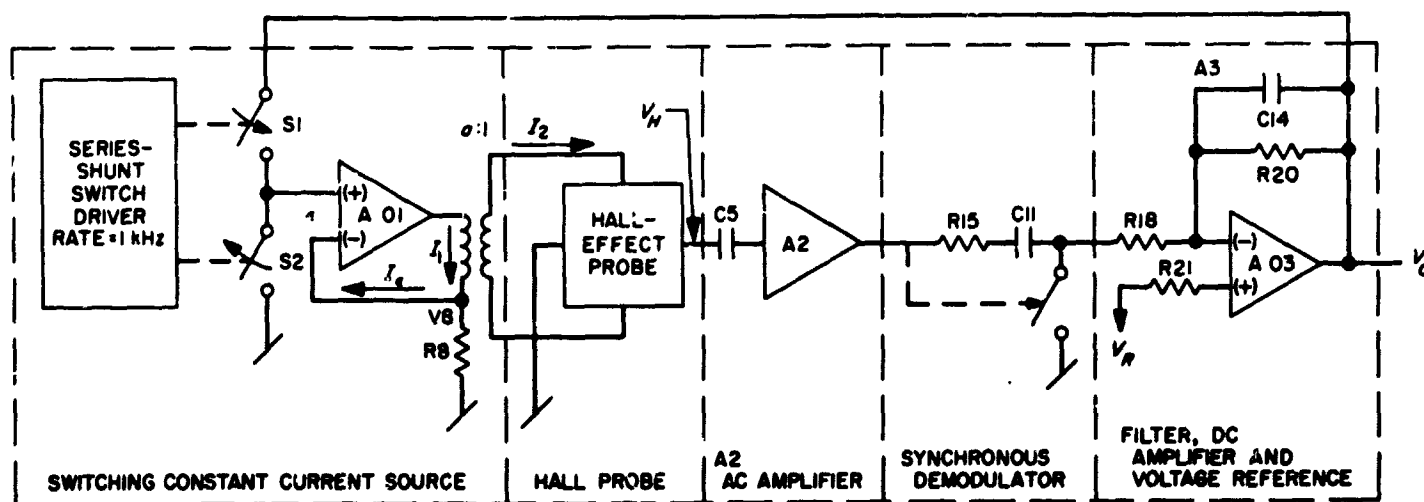
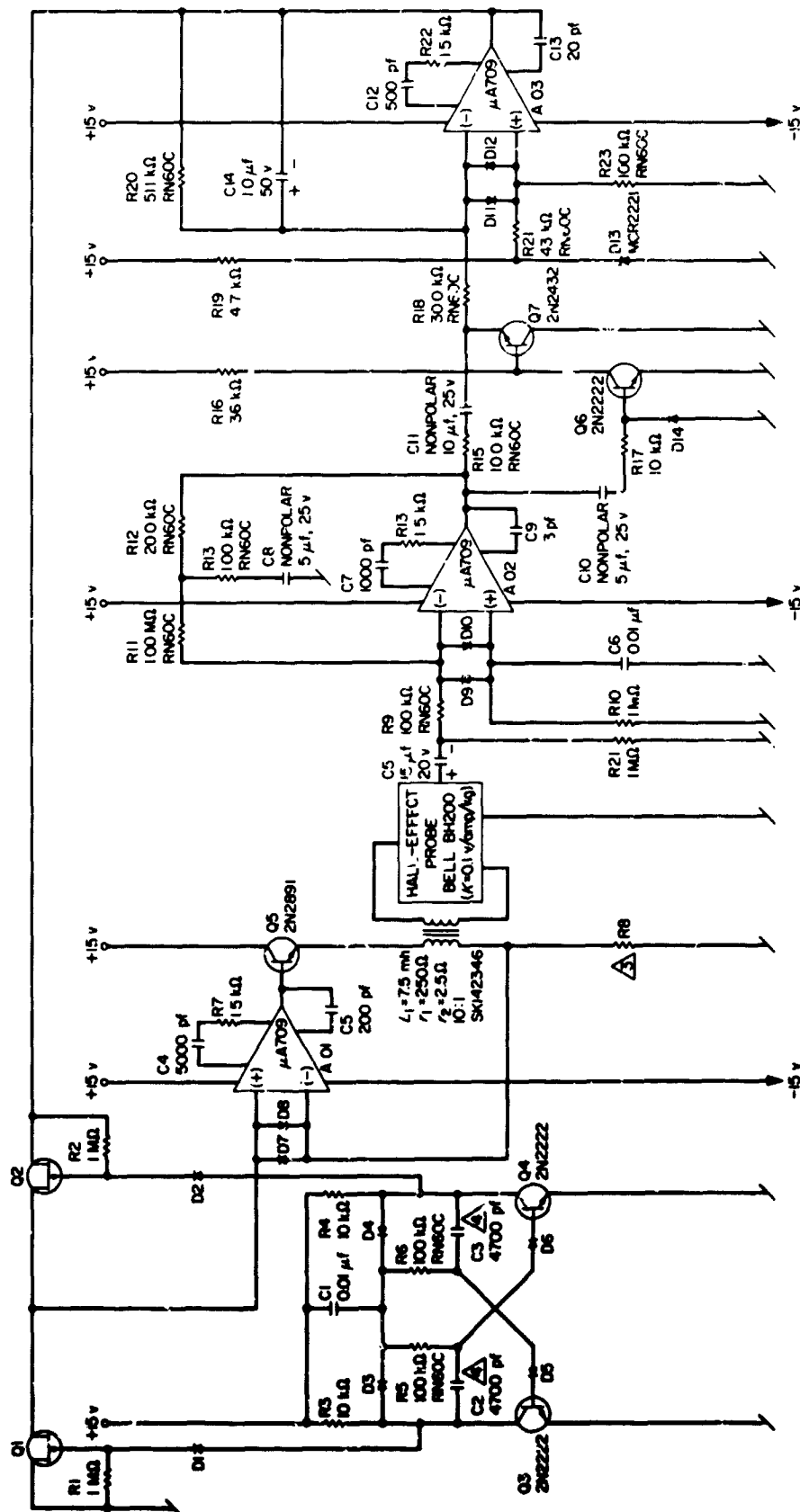


Fig. 2. Block diagram of gaussmeter



- NOTES:
1. ALL DIODES IN966 UNLESS OTHERWISE NOTED
 2. ALL RESISTORS ARE 5%, 1/4 W UNLESS OTHERWISE SPECIFIED
- △ CHOOSE R8 TO TRACK THE TEMPERATURE COEFFICIENT OF THE HALL-EFFECT PROBE
- △ C2 AND C3 CHOSEN TO TRACK EACH OTHER WITHIN 10 ppm/°C

Fig. 3. Mass spectrometer gaussmeter

and a chopper stabilized amplifier, or an ac control current and an ac amplifier with demodulator. The latter approach was decided upon for the following reasons:

- (1) The lowest mass spectrometer system supply voltages are ± 15 v, and it is therefore desirable to transformer-couple the control current into the probe to minimize power consumption.
- (2) Transformer coupling provides required isolation between the control current terminals and the voltage output terminals.

4. Design Theory

a. The transfer function. This section is devoted to developing the transfer function that relates the gauss-meter output voltage to the magnetic field. Then, using this transfer function, an analysis will be made of the error contributions.

Consider the Hall probe voltage expression:

$$V_H = K I_c B \quad (3)$$

Referring to Fig. 2,

$$I_2 = I_c = a I_1 = a \left(I_\epsilon + \frac{V_o - \epsilon}{R_s} \right) \approx \frac{a V_o}{R_s} \quad (4)$$

since $I_\epsilon \approx 0.5 \mu\text{a}$ (compared with 10 ma) and $\epsilon \approx 1.0$ mv (compared with 7.0 v) for switch 1 closed and switch 2 open. For switch 1 open and switch 2 closed, $I_c = 0$.

Because the Hall probe is ac-coupled into amplifier A2, the peak-to-peak input voltage is

$$V_{p-p} = V_H = \frac{K a V_o B}{R_s} \quad (5)$$

This voltage is amplified by ac amplifier A2 and becomes the demodulator input voltage

$$(V_{in})_{DEM} = \frac{K a V_o B A_2}{R_s} \quad (6)$$

To simplify the analysis of the demodulator, the following assumptions are made:

- (1) Chopper transistor Q7 switches synchronously with the input signal $(V_{in})_{DEM}$.

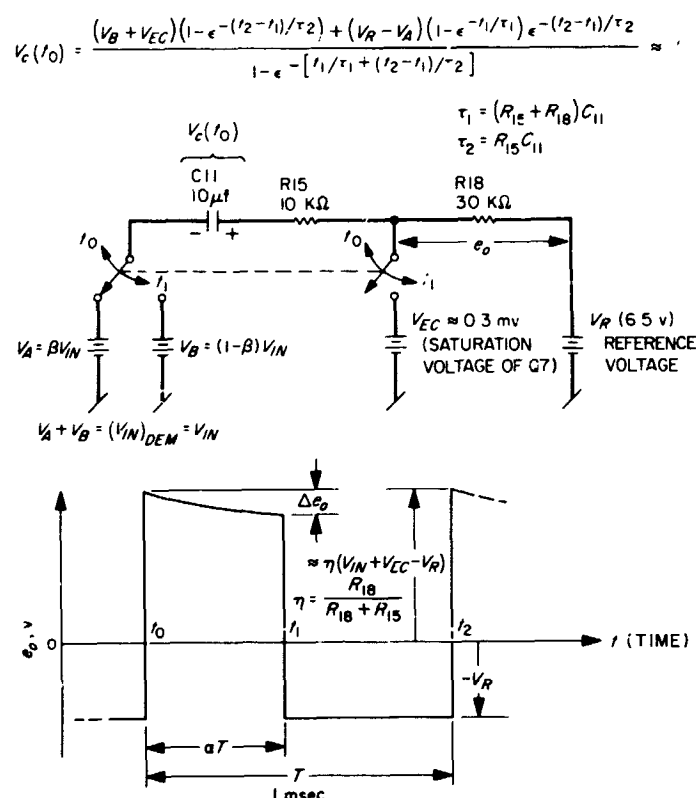


Fig. 4. Demodulator circuit model and output waveform

- (2) The input voltage $(V_{in})_{DEM}$ is idealized as a perfect square wave (i.e.; no droop, $t_{FALL} = t_{RISE} = 0$).
- (3) The summing point of amplifier A3 is at the same voltage as V_R .
- (4) The output impedance of amplifier A2 and the saturation resistance of the chopper transistor Q7 can be neglected.

Using the indicated assumptions, the demodulator may be represented by the model shown in Fig. 4. Fig. 4 also indicates the output voltage that will be developed across the input resistor (R18) of amplifier A3.

To further simplify the analysis, the exponential decay of e_o will be linearized. Then

$$\Delta V_c = \frac{i_c}{C_{11}} \Delta T \quad (7)$$

where ΔV_c is the voltage change across C_{11} during the interval t_0 to t_1 .

The point at which the demodulator analysis begins, defined as t_0 , assumes that capacitor C11 has charged to its steady state value and that the voltage across it is approximately V_B . Then

$$\begin{aligned} i_c &\approx \frac{V_{in} + V_{EC} - V_R}{R_{17} + R_{18}} \\ \Delta T &= \alpha T \\ \Delta V_c &\approx \frac{V_{in} + V_{EC} - V_R}{(R_{17} + R_{18}) C_{11}} \alpha T \\ \Delta V_c &\approx (V_{in} + V_{EC} - V_R) \alpha \frac{T}{\tau_1} \end{aligned} \quad (8)$$

where

$$\tau_1 = (R_{17} + R_{18}) C_{11}$$

To establish a relationship that relates the peak-to-peak input voltage of the demodulator to the dc (average) voltage across resistor R18, the following relationship is stated:

$$\psi \equiv \frac{(e_o)_{AVG}}{V_{in}} = \frac{1}{V_{in} T} \int_{t_0}^{t_1} e_o dt + \frac{1}{V_{in} T} \int_{t_1}^{t_2} e_o dt \quad (9)$$

$$\begin{aligned} \psi &\approx \eta \alpha \left(1 + \alpha \frac{T}{2\tau_1} \right) + \frac{1}{V_{in}} \left\{ \eta V_{EC} \alpha \left(1 - \alpha \frac{T}{2\tau_1} \right) \right. \\ &\quad \left. + V_R \left[\eta \alpha^2 \frac{T}{2\tau_1} + \alpha (1 - \eta) - 1 \right] \right\} \end{aligned} \quad (10)$$

For convenience of handling, ψ will be expressed as

$$\psi = D + \frac{H}{V_{in}} \quad (11)$$

It is desirable that Eq. (10) be simplified. To do this, the following parameter values will be utilized:

$$\begin{aligned} \eta &= \frac{R_{18}}{R_{18} + R_{17}} = 0.75 \\ \alpha &= 0.5 \\ T &= 1 \text{ msec} \\ \tau_1 &= 400 \text{ msec} \\ V_{EC} &= 1 \text{ mv (max)} \\ V_R &= 4.6 \text{ v} \end{aligned}$$

Substituting these values into Eq. (10) results in the following:

$$\begin{aligned} \psi &= (0.75)(0.5) \left[1 + \frac{(0.5)(1 \times 10^{-3} \text{ sec})}{2(400 \times 10^{-3} \text{ sec})} \right] \\ &\quad + \frac{1}{V_{in}} \left\{ (0.75)(1 \times 10^{-3} \text{ v})(0.5) \left[1 - \frac{0.5(1 \times 10^{-3} \text{ sec})}{2(400 \times 10^{-3} \text{ sec})} \right] \right. \\ &\quad \left. + 4.6 \text{ v} \left[\frac{(0.75)(0.25)(1 \times 10^{-3} \text{ sec})}{2(400 \times 10^{-3} \text{ sec})} + 0.5 \times 0.25 - 1 \right] \right\} \end{aligned} \quad (12)$$

By inspecting corresponding terms, it can be seen that Eq. (10) can be written as follows with very little loss in accuracy:

$$\psi = \eta \alpha + \frac{V_R}{V_{in}} [\alpha(1 - \eta) - 1] = \frac{(e_o)_{AVG}}{V_{in}} \quad (10a)$$

Having established a relationship between the output of amplifier A2 and the input of amplifier A3, the system output voltage may be written using Eq. (6) as the input to the demodulator. Thus,

$$(V_{in})_{DEM} = \frac{K_a V_o B A_2}{R_s} \quad (6)$$

$$(e_o)_{AVG} = (V_{in})_{DEM} \psi = V_{in} D + H \quad (11a)$$

where the output of amplifier A3 is given as

$$V_o = V_R + A_3 (e_o)_{AVG} \quad (13)$$

Combining Eqs. (6), (11a), and (13) yields

$$V_o = V_R + A_3 \frac{K_a V_o B A_2 D}{R_s} + A_3 H$$

Collecting terms, solving for V_o , and substituting the values for D and H determined in Eq. (10a) gives the transfer function relating the gaussmeter output to the B-field:

$$V_o = \frac{V_R \{1 - A_3 [1 - \alpha(1 - \eta)]\}}{1 - \frac{A_3 A_2 K_a B \eta \alpha}{R_s}} \quad (14)$$

b. Thermal analysis. For purposes of future reference the following nominal values will be used:

$$\begin{aligned} V_R &= 4.6 \text{ v} \\ A_1 &= -17 \\ A_2 &= 210 \\ V_D &= 0 \text{ v} \\ K &= 0.08 \text{ v/amp-kilogauss} \\ a &= 10 \\ B &= 5 \text{ kilogauss} \\ \eta &= 0.75 \\ \alpha &= 0.50 \\ R_s &= 700 \Omega \\ V_o &= 7.2 \text{ v} \end{aligned}$$

To simplify the following analyses, Eq. (14) can be approximated as

$$V_o \approx \frac{\lambda \{V_R [1 - \alpha(1 - \eta)] + V_D\}}{A_2 a B \eta \alpha} \quad (14a)$$

where

$$\lambda \equiv \frac{R_s}{K}$$

without introducing any substantial error. V_D has been added to represent the voltage and current offset error associated with amplifier A3.

Since all of the parameters in Eq. (14a) may be considered temperature sensitive, with the exception of a and η which will be neglected, we may then write

$$\begin{aligned} \Delta V_o &\approx \left(\frac{dV_o}{dT} \right) \Delta T \\ &= \left(\frac{\partial V_o}{\partial V_R} \frac{dV_R}{dT} + \frac{\partial V_o}{\partial \alpha} \frac{d\alpha}{dT} \right. \\ &\quad + \frac{\partial V_o}{\partial \lambda} \frac{d\lambda}{dT} + \frac{\partial V_o}{\partial A_2} \frac{dA_2}{dT} + \frac{\partial V_o}{\partial V_D} \frac{dV_D}{dT} \\ &\quad \left. + \frac{\partial V_o}{\partial B} \frac{dB}{dT} \right) \Delta T \end{aligned} \quad (15)$$

The mass spectrometer is required to operate from 0 to +50°C with a maximum allowable uncertainty in the

B -field of 0.42%; therefore

$$\begin{aligned} \frac{\Delta V_o}{V_o} \times 100 &= 0.42\% \\ &= \frac{100}{V_o} \left(\frac{\partial V_o}{\partial V_R} \frac{dV_R}{dT} \right. \\ &\quad + \frac{\partial V_o}{\partial \alpha} \frac{d\alpha}{dT} + \frac{\partial V_o}{\partial \lambda} \frac{d\lambda}{dT} \\ &\quad \left. + \frac{\partial V_o}{\partial A_2} \frac{dA_2}{dT} + \frac{\partial V_o}{\partial V_D} \frac{dV_D}{dT} \right) \Delta T \end{aligned} \quad (16)$$

with $\frac{\partial V_o}{\partial B} \frac{dB}{dT}$ excluded.

c. A3 input offset error V_D . The amplifier model shown in Fig. 5 was used to analyze the offset voltage and current drift. The resultant output drift is given by

$$\Delta V_D = \left(\Delta I_{os} \pm \frac{\Delta V_{os}}{R_s} \right) (R_{1s} + R'_{21}) \quad (17)$$

where

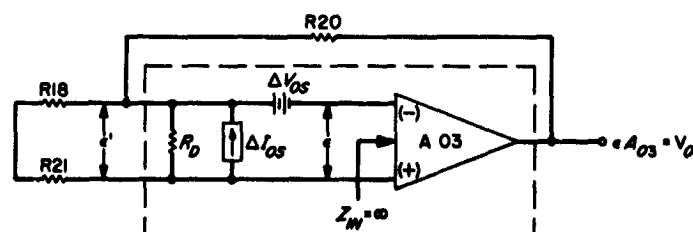
$$R_s \equiv (R_{1s} + R'_{21}) \parallel R_D \parallel R_{20} \approx 45 \text{ k}\Omega, R'_{21} = R_{21} \parallel R_{23}$$

From the Fairchild μA 709 data sheet,

$$\Delta I_{os} = 20 \text{ na, and } \Delta V_{os} = 150 \mu\text{v for } T = 0 \text{ to } +50^\circ\text{C.}$$

Therefore,

$$\begin{aligned} \frac{dV_D}{dT} \Delta T &\approx \Delta V_D = \pm \left(20 \text{ na} + \frac{150 \mu\text{v}}{45 \text{ k}\Omega} \right) 60 \text{ k} = \pm 1.40 \text{ mv} \\ \delta V_D &\approx \frac{\partial V_o}{\partial V_D} \Delta V_D \end{aligned} \quad (18)$$



ΔI_{os} AND ΔV_{os} ARE THE CHANGES IN DIFFERENTIAL OFFSET CURRENT AND VOLTAGE THAT RESULT FOR A CHANGE IN TEMPERATURE ΔT (POLARITIES SHOWN ARE ARBITRARY)

Fig. 5. Amplifier drift model

where

$$\frac{\partial V_o}{\partial V_D} \approx \frac{-V_o}{V_R [1 - \alpha (1 - \eta)] + V_D}$$

Therefore,

$$\delta V_D \approx \frac{(7.2 \text{ v}) (\pm 1.40 \text{ mv})}{(4.6 \text{ v}) (0.875)} = \pm 2.50 \text{ mv}$$

d. Reference voltage error V_R . The reference voltage is fixed by dividing down the Zener diode (D13) voltage V_Z by resistors R21 and R23. Then

$$V_R = kV_Z$$

where

$$k = \frac{R_{21}}{R_{21} + R_{23}}$$

If R23 has a maximum temperature coefficient (TC) of +50 ppm/°C and R21 has the smaller TC and tracks R23 to within 30 ppm/°C, the error caused at the output (ΔV_o) is 14 mv and may therefore be neglected. The Zener diode is then the primary error determiner. The Zener diode chosen (MCR2221) has a maximum TC of ± 5 ppm/°C.

For $\Delta T = 50^\circ\text{C}$,

$$\Delta V_R = k \Delta V_Z = (0.7) (6.5 \text{ v}) (\pm 5 \times 10^{-4} / ^\circ\text{C}) (50^\circ\text{C}) = \pm 1.14 \text{ mv}$$

$$\frac{\partial V_o}{\partial V_R} = \frac{V_o}{V_R}$$

$$\delta V_o = \frac{V_o}{V_R} \Delta V_R = \frac{7.2 \text{ v}}{4.6 \text{ v}} \times \pm 1.14 \text{ mv} = \pm 1.78 \text{ mv} \quad (19)$$

e. Switching time symmetry α . The switching symmetry is determined by how well matched and how closely the timing components of the multivibrator track their counterparts. Eq. (20) is an expression derived to calculate the half-period of oscillation of the multivibrator:

$$\frac{T}{2} = R_5 C_2 \ln \left[\frac{2E_{BR}K_1 - 2K_1V_{D4} + (K_2 - 2K_1K_2 - 1)V_{D5} + V_{BE3}(K_2 - 2K_1K_2 - 1)}{E_{RB}K_1 + (1 - K_1)V_{D4} + (K_2 - K_1K_2)V_{D5} + V_{BE3}(K_2 - K_1K_2) - V_{D6} - V_{BE4}} \right] \quad (20)$$

where

$$K_1 = \frac{R_5 \parallel R_6}{R_5 \parallel R_6 + R_4}$$

and

$$K_2 = \frac{R_5}{R_5 + R_6}$$

The K terms (K_1 and K_2) are relatively temperature insensitive and can be ignored. More significant, however, is the tracking of V_{D5} with V_{D6} and V_{BE3} with V_{BE4} , since they appear only in the denominator of the timing expression. By far the most significant is the matching of R5 with R6 and C2 with C3. Assuming most of the dissymmetry is caused by the tracking of C2 and C3, they are chosen so as to have not more than 0.05% differential change. Using this figure,

$$\frac{\partial V_o}{\partial \alpha} = - \frac{1}{[1 - \alpha (1 - \eta)]} \frac{V_o}{\alpha} \quad (21)$$

and

$$\delta \alpha \approx \frac{\partial V_o}{\partial \alpha} \Delta \alpha = - \frac{1}{[1 - \alpha (1 - \eta)]} \frac{\Delta \alpha}{\alpha} V_o$$

Then

$$\delta \alpha \approx \frac{-1}{1 - 0.5(1 - 0.75)} \times \pm 5 \times 10^{-4} \times 7.2 \text{ v} = \pm 4.1 \text{ mv}$$

f. Amplifier A2 error. Let the amplifier error of A2 be expressed as

$$\delta_{A1} = \frac{\partial V_o}{\partial A_2} \frac{dA_2}{dT} \Delta T \quad (22)$$

$$\frac{\partial V_o}{\partial A_2} = \frac{-V_o}{A_2} \quad (23)$$

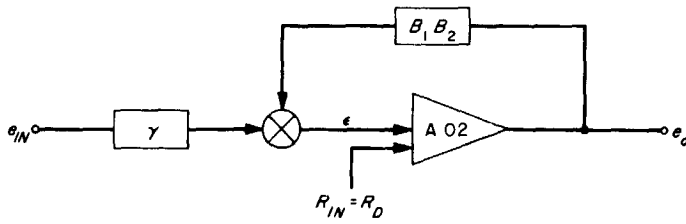


Fig. 6. Gain analysis model

Fig. 6 is the amplifier model used in deriving the closed-loop midband gain expression for A2. The summing point error voltage is given by

$$\epsilon = \frac{e_o}{A_{o2}} = e_{in} \gamma + B_1 B_2 e_o \quad (24)$$

where

$$\gamma = \frac{R_D \parallel R_{11}}{R_D \parallel R_{11} + R_9} = \frac{R_9 \parallel R_D \parallel R_{11}}{R_9}$$

$$B_1 = \frac{R_D \parallel R_9}{R_D \parallel R_9 + R_{11}} = \frac{R_9 \parallel R_D \parallel R_{11}}{R_{11}}$$

$$B_2 = \frac{R_{13}}{R_{11} + R_{12}}$$

assuming that $R_{11} \gg R_{12} \parallel R_{13}$.

Substituting the values of γ , B_1 , and B_2 into Eq. (5) and solving for e_o/e_{in} (where $e_o/e_{in} \equiv A_2$) yields

$$A_2 = \frac{R_{11}}{R_9} \left(\frac{R_{13} + R_{12}}{R_{13}} \right) \left(\frac{F}{1-F} \right) = G \left(\frac{F}{1-F} \right) \quad (25)$$

where

$$F = A_{o2} \frac{R_{13}}{R_{12} + R_{13}} \left(\frac{R_9 \parallel R_D \parallel R_{11}}{R_{11}} \right)$$

It will be assumed that the temperature coefficients and tracking of resistors R9, R11, R12, and R13 are such that their contribution to the gain change of A2 can be ignored and that the principal causes of change are due to A_{o2} and R_D .

$$\frac{dA_2}{dT} = \frac{G}{(1-F)^2} \frac{dF}{dT} \quad (26)$$

$$\frac{\Delta A_2}{A_2} \approx \frac{1}{(1-F)F} \Delta F$$

$$\Delta A \approx A_2 \frac{\Delta F}{(1-F)F} \approx \left(\frac{dA}{dT} \right) \Delta T$$

Therefore,

$$\delta A \approx \frac{-V_o}{A_2} A_2 \frac{\Delta F}{(1-F)F} = \frac{-V_o}{(1-F)} \frac{\Delta F}{F} \quad (27)$$

From temperature data taken on the Fairchild $\mu A709$, it is found that

$$A_{o2}(0^\circ\text{C}) = -50 \times 10^3 \text{ and } R_D(0^\circ\text{C}) = 300 \text{ k}\Omega$$

$$A_{o2}(50^\circ\text{C}) = -40 \times 10^3 \text{ and } R_D(50^\circ\text{C}) = 550 \text{ k}\Omega$$

$$F(0^\circ\text{C}) = -168, F(+50^\circ\text{C}) = -149, F(25^\circ\text{C}) \approx -158$$

Then

$$\delta A \approx -5.4 \text{ mv}$$

g. Hall probe tracking error λ . The sensitivity constant K of the Hall probe has a reasonably large temperature coefficient ($-800 \text{ ppm}/^\circ\text{C}$). For this reason the other error contributors have been calculated first, so that the allowable probe tracking error would be fixed by the difference of the maximum allowable error and the sum of the other contributors. Thus,

$$\delta \lambda = V_o \left(\frac{0.42\%}{100} \right) - (\delta V_R + \delta V_D + \delta \alpha + \delta A_2)$$

$$= -30 \text{ mv} + 13.8 \text{ mv} \quad (28)$$

Therefore,

$$\delta \lambda \approx V_o \frac{\Delta \lambda}{\lambda} = -16.2 \text{ mv}$$

The actual tracking requirement is placed upon resistor R8. Assume that for a given increase in temperature the B -field of the magnet was to remain constant, but K decreased. Referring to the Hall voltage expression $V_H = KI_c B$, it can be seen that if the current were to increase in a manner that kept the KI_c product constant, no spurious signal would be introduced. This can be accomplished by decreasing R_s , since

$$KI_c = \frac{KV_o a}{R_s}$$

where V_o remains constant for V_H constant.

Therefore, assuming that the curve of R_s tracks the curve of K over the specified temperature range within

the indicated tolerance, a 0.42% mass spectrometer gauss-meter seems attainable.

XV. Space Instruments

SPACE SCIENCES DIVISION

N67-29156

A. Range of Applicability for the Photometric Method of Topographic Mapping, T. Rindfleisch

1. Summary

In Ref. 1 a method is discussed for extracting topographic information about the Moon for monoscopic photographs using a knowledge of the lunar photometric properties (Ref. 2). This method is attractive for space applications since it does not require the redundant photographic coverage used in stereoscopy, but requires instead a detailed knowledge of the surface photometric properties. In applying the method to the lunar surface, it was necessary to use certain symmetry properties of the lunar photometric function to integrate the general differential equation. The purpose of this note is to examine the applicability of the method to surfaces with photometric properties different from those of the Moon. The conclusion will be that only a slight generalization in photometric function symmetries from those of the Moon is possible while still being able to integrate the basic equation.

2. Discussion

Representing points on an object surface by position vectors with respect to the center of the camera lens, it

can be shown (Ref. 1) that the directional derivative of the length of the vector to a point with respect to distance in the image plane is given by

$$\frac{d}{dS} (\ln r) = \frac{1}{F} \frac{(\hat{\mathbf{r}} \cdot \hat{\mathbf{z}})}{(\hat{\mathbf{N}} \cdot \hat{\mathbf{r}})} [\hat{\mathbf{r}} \times (\hat{\mathbf{N}} \times \hat{\mathbf{r}})] \cdot \hat{\mathbf{S}}_r \quad (1)$$

The symbols in Eq. (1) are defined (see Fig. 1) as

$\mathbf{r}$ = the position vector to an object point (to be determined)

r = the length of $\mathbf{r}$ (to be determined)

$\hat{\mathbf{r}}$ = a unit vector along $-\mathbf{r}$; $\hat{\mathbf{r}} = -\mathbf{r}/r$ (known from image geometry)

F = the focal length of the camera lens (known)

$\hat{\mathbf{z}}$ = a unit vector along the optical axis (known)

$\hat{\mathbf{S}}_r$ = a unit vector in the image plane along the direction of differentiation (arbitrary)

S = a distance variable in the image plane (along $\hat{\mathbf{S}}_r$)

$\hat{\mathbf{N}}$ = a local unit normal to the object surface (unknown except through the surface photometric properties and the recorded picture brightness)

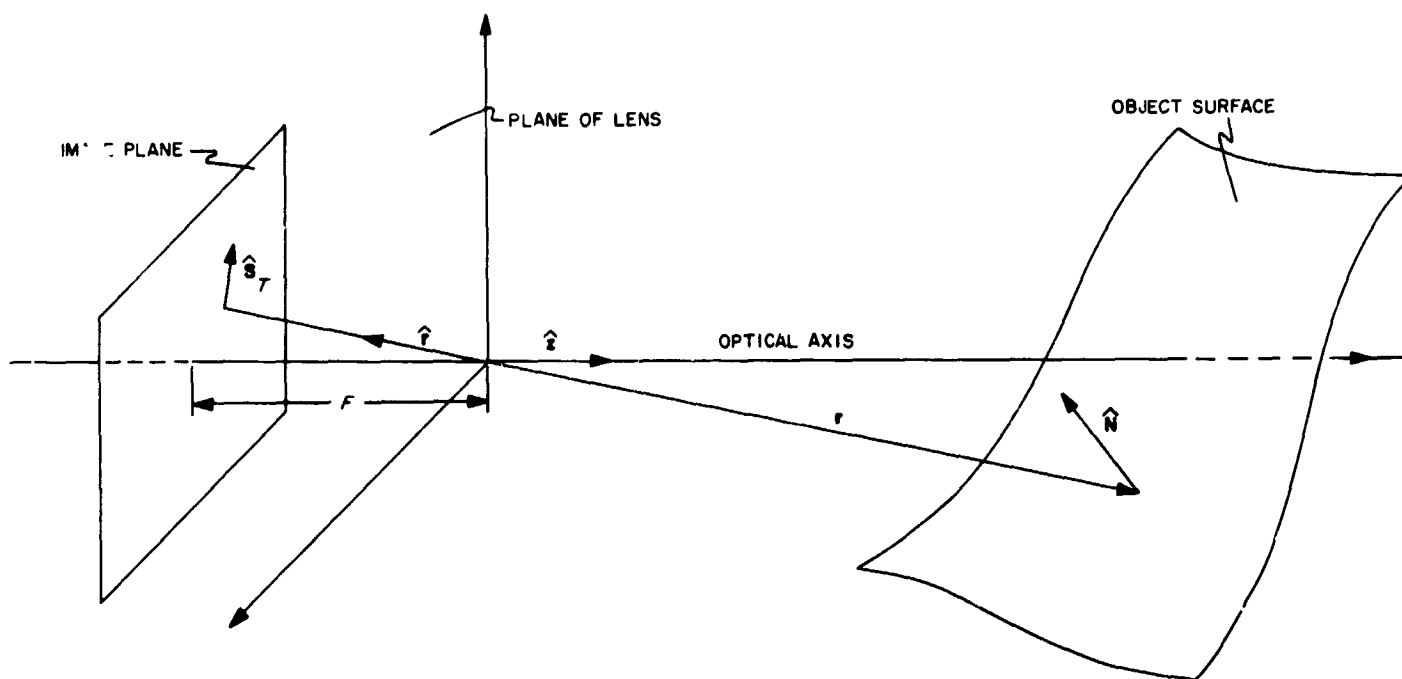


Fig. 1. Definition of image geometry

If the direction $\hat{N}$ were known, the right side of Eq. (1) could be evaluated for any direction $\hat{S}_r$ in the image plane. However, the only information about $\hat{N}$ that is actually available comes through the surface photometric function

$$b = b(\hat{N}, \hat{r}, \hat{R}) \quad (2)$$

which expresses the apparent brightness b as a function of the *relative* orientations of the surface normal direction $\hat{N}$, the viewing direction $\hat{r}$, and the illumination direction $\hat{R}$ (it is assumed the illumination is uniform, collimated, and from a known direction).

Since it takes two angles to specify the direction $\hat{N}$, Eq. (2) and a given observed surface brightness b are insufficient to completely determine $\hat{N}$. Thus the right side of Eq. (1) cannot be evaluated for an arbitrary differentiation direction $\hat{S}_r$ using *only* photometric information. The problem is to determine the conditions under which a direction $\hat{S}_r$ can be found so that Eq. (2) can be used to integrate Eq. (1). (Ref. 1 discusses the solution for the lunar case.)

It is convenient to describe the photometric geometry in terms of the angles α , β , and g used in Ref. 1 and shown in Fig. 2.

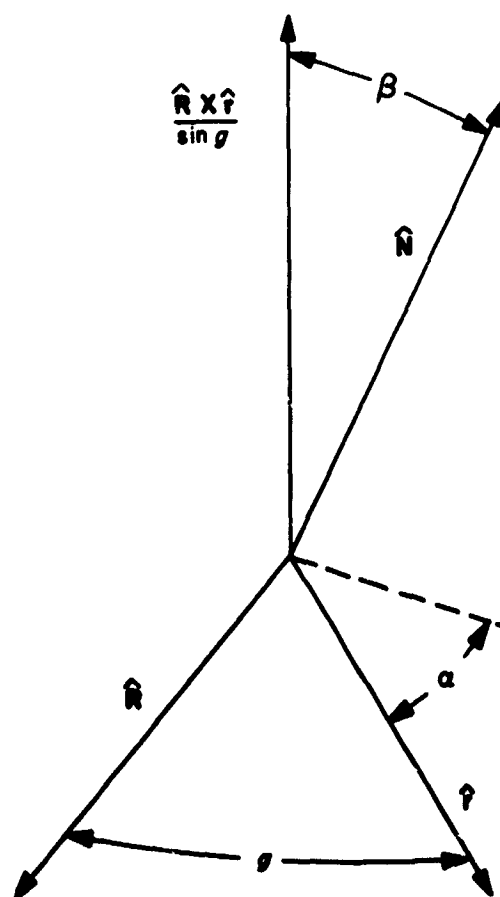


Fig. 2. Definition of photometric angles

Using these angles, Eqs. (1) and (2) become

$$\frac{d}{dS} (\ln r) = \frac{1}{F} (\hat{\mathbf{r}} \cdot \hat{\mathbf{z}}) \left\{ \frac{\cot \beta}{\cos \alpha} \left(\frac{\hat{\mathbf{R}} \times \hat{\mathbf{r}}}{\sin g} \right) - \tan \alpha \right. \\ \left. \times \left[\hat{\mathbf{r}} \times \left(\frac{\hat{\mathbf{R}} \times \hat{\mathbf{r}}}{\sin g} \right) \right] \right\} \cdot \hat{\mathbf{S}}_r \quad (1a)$$

and

$$b = b(\alpha, \beta, g) \quad (2a)$$

It is also convenient to decompose the vector $\hat{\mathbf{S}}_r$ along a coordinate system with basis vectors $\hat{\mathbf{r}}$, $(\hat{\mathbf{R}} \times \hat{\mathbf{r}})/\sin g$, and $[\hat{\mathbf{r}} \times (\hat{\mathbf{R}} \times \hat{\mathbf{r}})]/\sin g$ as shown in Fig. 3. Letting δ be a measure of the direction of $\hat{\mathbf{S}}_r$ (i.e., the angle between the component of $\hat{\mathbf{S}}_r$ perpendicular to $\hat{\mathbf{r}}$ and the basis vector $[\hat{\mathbf{r}} \times (\hat{\mathbf{R}} \times \hat{\mathbf{r}})]/\sin g$), one can write

$$\hat{\mathbf{S}}_r = (\hat{\mathbf{S}}_r \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} + \sqrt{1 - (\hat{\mathbf{S}}_r \cdot \hat{\mathbf{r}})^2} \left[\sin \delta \left(\frac{\hat{\mathbf{R}} \times \hat{\mathbf{r}}}{\sin g} \right) \right. \\ \left. + \cos \delta \left(\frac{\hat{\mathbf{r}} \times (\hat{\mathbf{R}} \times \hat{\mathbf{r}})}{\sin g} \right) \right] \quad (3)$$

where

$$\hat{\mathbf{S}}_r \cdot \hat{\mathbf{r}} = \frac{\hat{\mathbf{z}} \cdot \left[\sin \delta \left(\frac{\hat{\mathbf{R}} \times \hat{\mathbf{r}}}{\sin g} \right) + \cos \delta \left(\frac{\hat{\mathbf{r}} \times (\hat{\mathbf{R}} \times \hat{\mathbf{r}})}{\sin g} \right) \right]}{\left\{ (\hat{\mathbf{r}} \cdot \hat{\mathbf{z}})^2 + \left[\sin \delta \frac{\hat{\mathbf{z}} \cdot (\hat{\mathbf{R}} \times \hat{\mathbf{r}})}{\sin g} + \cos \delta \frac{\hat{\mathbf{z}} \cdot [\hat{\mathbf{r}} \times (\hat{\mathbf{R}} \times \hat{\mathbf{r}})]}{\sin g} \right]^2 \right\}^{1/2}}$$

Substitution of Eq. (3) into Eq. (1a) yields

$$\frac{d}{dS} (\ln r) = \frac{1}{F} (\hat{\mathbf{r}} \cdot \hat{\mathbf{z}}) \sqrt{1 - (\hat{\mathbf{S}}_r \cdot \hat{\mathbf{r}})^2} \left[\frac{\cot \beta}{\cos \alpha} \sin \delta - \tan \alpha \cos \delta \right] \quad (4)$$

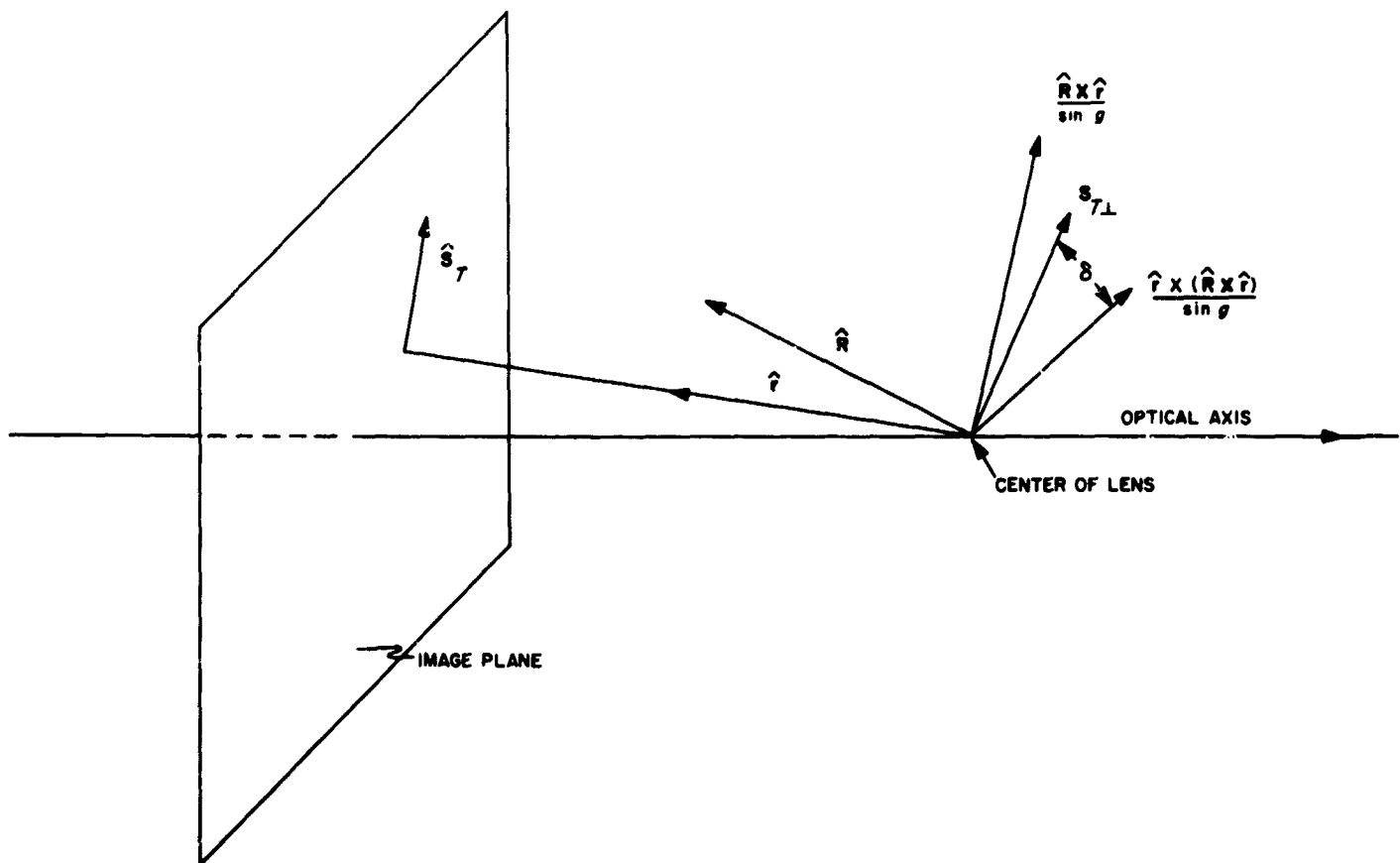


Fig. 3. Coordinate decomposition of $\hat{\mathbf{S}}_r$

Since the topography is independent of the camera orientation, in order to integrate Eq. (4), both the angle δ and the quantity ϕ , where

$$\phi = \frac{\cot \beta}{\cos \alpha} \sin \delta - \tan \alpha \cos \delta$$

must be derivable from the known phase angle g and the measured brightness b related through Eq. (2a). Thus, to proceed with the integration, there must exist relations of the form

$$\phi = \phi(b, g)$$

and

$$\delta = \delta(b, g)$$

which can, in principle, be inverted to give

$$\left. \begin{aligned} b &= b(\phi, g) \\ \delta &= \delta(\phi, g) \end{aligned} \right\} \quad (5)$$

Thus, for constants b and g , ϕ and δ must be constant so that isophotes are given by

$$\frac{\cot \beta}{\cos \alpha} \sin \delta - \tan \alpha \cos \delta = \text{constant} \quad (6)$$

and

$$g, \delta = \text{constants}$$

Eq. (6) determines the locus of α and β for which the hypothetical surface must exhibit constant brightness. It is straightforward geometry to show that in a coordinate system rotated an angle δ opposite to a right-hand screw about the axis $\hat{r}$ of Fig. 2, Eq. (6) takes on the simple form

$$\tan \alpha' = \text{constant} \quad (6a)$$

where the angle α' is the azimuth of $\hat{N}$ in the rotated system (see Fig. 4).

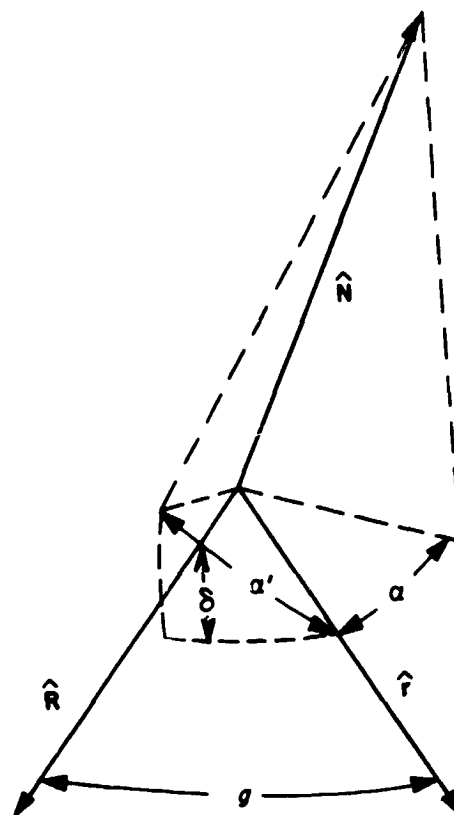


Fig. 4. Rotated photometric geometry

It will be noted that this situation is a generalization of the lunar case for which $\delta \equiv 0$, $\phi \equiv -\tan \alpha$, and, as observed experimentally (Ref. 2),

$$b = b(\alpha, g)$$

The physical consequences of a case different from the Moon ($\delta = 0$) are most easily visualized by considering the topology of isophotes on a sphere having the hypothetical photometric properties (see Fig. 5). For given illumination and viewing directions and assuming $\delta \neq 0$, isophotes will be great circles inclined to the plane containing g and intersecting the terminator (note that for $\delta = 0$, the isophotes run parallel to the terminator in a spherical sense; i.e., the isophotes are a family of great circles with a common diameter). The isophotes must also be symmetric above and below the plane of g , as is clear from the previous discussion.

Certainly, Lambertian-like matt surfaces do not exhibit such properties. Whether or not surfaces of bodies of the solar system other than the Moon exhibit this type of symmetry is an open question, since little detailed photometric information is available because of poor resolution, angular variation, and atmospheres.

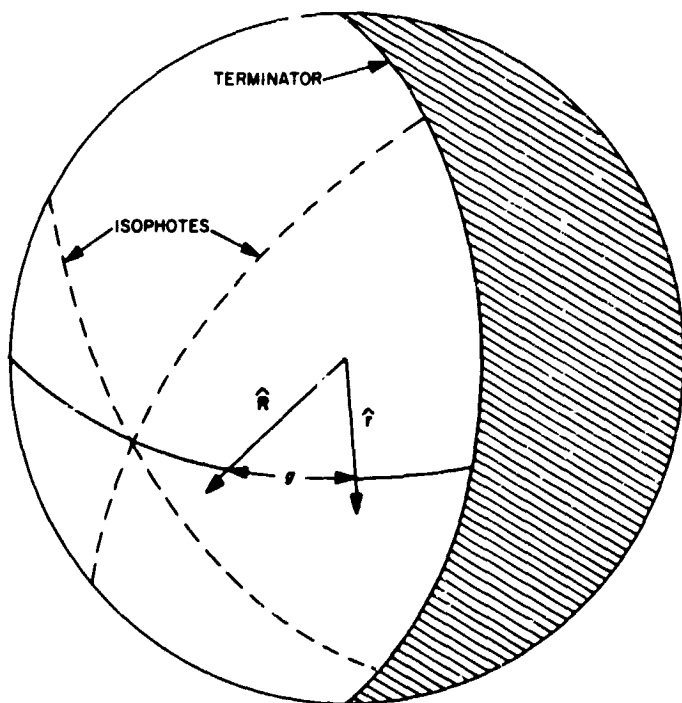


Fig. 5. Isophotes on a sphere with generalized photometric properties

3. Conclusion

It has been shown that the geometrical equation derived in Ref. 1 can be integrated photometrically only for a class of surfaces having reflectance properties that are a simple generalization of those of the Moon. Clearly excluded from this class are Lambertian-type matt surfaces typified by finely divided materials. It is not well known if lunar photometric properties are common to other soils of the solar system. Earth-type soils do not exhibit photometric properties similar to those of the Moon, indicating that erosion processes may be important. In any case, the applicability of the photometric method for topographic mapping appears to be severely limited.

B. Residual Signal and Image Storage of Several Slow-Scan Vidicons, L. R. Baker

1. Introduction

As a part of the photosensor evaluation supporting research program,¹ a series of tests were performed on various slow-scan vidicons. The purpose of these tests was to obtain a better understanding of the physical problems of vidicon operation at slow scanning rates.

¹NASA Work Unit 125-24-01-03.

Ever since television systems have been required to operate at data rates lower than standard television rates (525 line, 60 fields/sec), problems have arisen with the image sensor. The very low data transmission rate from planetary spacecraft places very special requirements on image sensors to be used in scientific instruments. In particular, residual signal (erase) and image storage are very important image quality characteristics of a slow-scan vidicon. Residual signal, which refers to the signal remaining after one *expose-read* cycle, is important since residual signal from a previous exposure will appear in the picture as coherent noise and decrease the signal-to-noise ratio. Image storage, which refers to the signal remaining after a long period of time between exposure and readout, is important since the vidicon signal output at the bottom of the frame should be equal to the signal at the top of the frame (excluding beam landing errors and optical vignetting effects). Additionally, the preservation of fine detail for long periods is associated with image storage.

2. Test Procedure

The following procedure was used to evaluate image storage and erase of nine slow-scan vidicons. The procedure was designed to determine long-term image storage capability and the ability of the photoconductor to recharge to cathode potential so as to erase the previously exposed image.

The vidicon was operated in a 950-line, 8.33-msec/line slow-scan mode using a calibrated shutter of 60-msec exposure time. The video bandwidth was set at 100 kHz.

a. Residual signal tests. A high-contrast, small area, black bar pattern was imaged on the vidicon target, using a calibrated slide illuminator and a calibrated 50-mm lens. The faceplate illumination was calculated to be 0.076-ft-c sec at $f/5.6$. The photoconductor was allowed to completely recharge; then one single exposure was taken. Single-line photographs of the center of the raster were taken of the initial, second, third, and fourth readouts. The ratio of signal remaining to initial signal was measured from the photographs.

b. Image storage tests. The same test pattern was used as in the residual signal test, and the faceplate illumination was 0.076-ft-c sec. The photoconductor was allowed to completely recharge; then the beam current was cut off by applying -140 v to the vidicon control grid, and a single exposure was taken. After waiting a predetermined length of time, the control grid bias was set to normal.

Table 1. Test results

Vidicon serial No.	Use	Photoconductor	Residual, %			Image storage, %				Vidicon manufacturer	Vidicon type
			2nd read	3rd read	4th read	30	60	90	120		
Z07731	Experimental sterilizable	Modified ASOS	35	23	16	42	35	32	26	RCA	7735a
Z12227	Experimental sterilizable	Modified ASOS	75	53	30	70	47	43	23	RCA	7735a
2125858	Rangers VI-IX	ASOS	31	15	10	21	0	0	0	RCA	C74072
9-23-271	General slow-scan	Se	16	0	0	100	92	88	87	West	WL7290
3-6-675	Experimental Ranger	Se	13	0	0	100	64	51	41	West	WX4783
2538436	General slow-scan	Modified Se	19	0	0	63	44	34	32	GEC	TD1368
2538450	General slow-scan	Modified Se	34	13	8	100	92	83	80	GEC	TD1368
0473122	Mariner Mars 1969	Modified Se	23	9.6	0	100	100	89	89	GEC	TD1342-011
0473132	Mariner Mars 1969	Modified Se	39	12.2	9.7	98	102	98	90	GEC	TD1342-011

A photograph of a single line of video was taken from the center of the raster. The signal amplitude at the end of the image storage period was compared to the non-stored signal amplitude. The storage times used were 30, 60, 90 and 120 sec. The results of these tests are given in Table 1.

3. Summary and Conclusions

A number of conclusions can be drawn from the test results. The physical operation of antimony-sulfide-oxide-sulfide (ASOS) and selenium-based photoconductors is substantially different. ASOS is a modification of the standard Sb_2S_3 photoconductor and operates as a very "laggy" vidicon; it does not exhibit true charge storage. The selenium-based photoconductors generally show a lower residual signal and better image storage, indicating an operation closer to true charge storage. The effect is probably due to higher resistivity and fewer electron traps in selenium-based photoconductors. The percentage of signal removed during each scan is shown in Table 2. These data show that ASOS photoconductors have less signal removed per scan than the other photoconductors, verifying the "lag" and "stickiness" of ASOS. In other words, ASOS not only has a higher residual signal, but retains the residual signal for a longer period of time.

Table 2. Signal removal via scanning

Vidicon serial No.	Residual signal removal, %		
	2nd to 3rd read	3rd to 4th read	2nd to 4th read
Z07731	34	31	54
Z12227	27	43	59
2125858	52	33	68
9-23-271	100	—	—
3-6-675	100	—	—
2538436	100	—	—
2538450	62	38	77
0473122	58	100	—
0473132	69	21	75

The tests discussed in this report were carried out on a small sample of vidicons and are not entirely conclusive. However, additional tests are not expected to show any change in the existing trends. The low storage and high residual image characteristics of ASOS are parameters that have caused and will continue to cause great difficulties in the design of future television instruments.

It should be noted that although selenium has very attractive image quality characteristics, it has an absolute upper temperature limitation of 45°C. This is too low for planetary spacecraft, where temperature control is a

problem. Modified selenium will go to 55°C and has good image characteristics, and is therefore a photoconductor that finds acceptance and application to interplanetary photography

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XVI. Science Data Systems

SPACE SCIENCES DIVISION

N67-29157

A. Near-Maximal-Length Cycles With Linear Feedback Shift Registers, M. Perlman

1. Introduction

The behavior of synchronously operated shift registers with linear logic feedback has been studied in detail (e.g., Ref. 1). An r -stage linear feedback shift register (FSR) can be used to realize cycle lengths of $2^r - 1$, which are termed maximal. The simplest of these, in terms of the complexity of the feedback logic, are those with two-tap feedback which satisfy the linear recurrence relationship

$$a_n = a_{n-i} \oplus a_{n-r} \quad (1)$$

The subscript n in Eq. (1) refers to the clock pulse time. The bit being fed back at time n is a_n , the modulo 2 sum (i.e., EXCLUSIVE-OR) of the contents of the i th and r th stages at time n . The initial state of the i th stage is a_{-i} , where $n = 0$.

Unfortunately, there are many values of r for which maximal-length cycles cannot be realized with two feed-

back taps (see Ref. 2). In these cases, four or a higher even number of taps are required. As the number of feedback taps increases, the complexity of the feedback function grows sharply. The question then arises: Are near-maximal-length cycles realizable with linear FSRs having feedback functions less complicated than a four-input modulo 2 summer? It will be shown that cycle lengths of $2^r - 2$ and $2^r - 4$ can be realized with s -stage linear FSRs. The feedback functions are (with few exceptions) effectively a three-input modulo 2 summer.

2. Generalized r -Stage FSR With Linear Feedback

In Fig. 1, the stages of the register are designated (left to right) $S_1, S_2, \dots, S_r$. The output of stage S_i is connected to the input of the modulo 2 summer when $C_{r-i} = 1$. C_0 is always 1, otherwise fewer than r stages would be in use. The external input e is a Boolean constant.

Let x_i represent the present state and X_i the next state of stage S_i . The next state of each stage may be expressed as a linear Boolean function of the present state of one or more stages.

$$\left. \begin{aligned} X_1 &= C_{r-1} x_1 \oplus C_{r-2} x_2 \oplus \cdots \oplus C_1 x_{r-1} \oplus x_r \oplus e \\ X_2 &= x_1 \\ X_3 &= x_2 \\ &\vdots \\ X_r &= x_{r-1} \end{aligned} \right\} \quad (2)$$

This may be expressed as

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_r \end{bmatrix} = \begin{bmatrix} C_{r-1} & C_{r-2} & \cdots & 1 & 0 \\ 1 & 0 & \cdots & C_1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_r \end{bmatrix} \oplus \begin{bmatrix} e \\ \vdots \\ 0 \end{bmatrix} \quad (3)$$

or

$$\mathbf{X} = T\mathbf{x} \oplus \mathbf{L} \quad (4)$$

Rules of modulo 2 arithmetic are used in determining $\mathbf{X}$. The $r \times r$ Boolean matrix T is nonsingular since its row (column) vectors are linearly independent (Ref. 3). It is termed an *associated matrix*. T represents the linear transformation of an r component vector (present state of the register) into another r component vector (next state of the register). $\mathbf{L}$ represents a translation. When nonzero (i.e., $e = 1$), the modulo 2 sum of the column vector $\mathbf{L}$ and $T\mathbf{x}$ represents the complementation of the bit being fed back. A linear transformation T followed by a translation is called an affine transformation (Ref. 3). Every translation is one-to-one and has an inverse, and the linear transformation T is one-to-one and has an inverse. Hence, the affine transformation $T\mathbf{x} \oplus \mathbf{L}$ is one-to-one and has an inverse. This is another way of saying that each state has a unique predecessor (or equivalently, distinct states have distinct successors).

Of primary interest are the feedback combinations that yield the longest possible cycle length.

CASE I:

$$e = 0$$

Therefore,

$$\mathbf{L} = 0$$

and

$$\mathbf{X} = T\mathbf{x}$$

This case has been thoroughly analyzed (see Refs. 1 and 4) and is summarized here. The smallest value of k for which $T^k = I$ is the length of the longest possible cycle.

The divisibility properties of $\phi(\lambda)$, the characteristic polynomial of T , and k are related as follows:

The smallest value of k for which

$$\phi(\lambda) \mid \lambda^k - 1 \quad (5)$$

is the length of the longest cycle which always contains the state $00 \cdots 01$. In general,

$$\phi(\lambda) = |T - \lambda I| = \lambda^r + C_{r-1} \lambda^{r-1} + \cdots + C_1 \lambda + 1 \quad (6)$$

For simplicity, $+$ is used to represent modulo 2 addition. Also, -1 appears as $+1$ since $-1 \equiv 1 \pmod{2}$. In accordance with the Caley-Hamilton theorem (Ref. 3),

$$\phi(T) = T^k - I = 0$$

and

$$T^k = I$$

Thus, if $\phi(\lambda)$ divides $\lambda^k - 1$ [i.e., $\phi(\lambda)$ is a factor of $\lambda^k - 1$], T satisfies $\lambda^k - 1$ and $T^k = I$. The polynomial of the lowest degree which is satisfied by a square matrix A is the minimal polynomial $m(\lambda)$ of A , and it is unique. Fortunately, as will be shown, $\phi(\lambda) = m(\lambda)$ for the *associated matrix* T .

When $\phi(\lambda)$ is irreducible, the smallest k for which (5) holds is termed the exponent to which $\phi(\lambda)$ belongs. To obtain the longest possible cycle length of an r -stage FSR, one must find an irreducible $\phi(\lambda)$ of degree r which belongs to a maximum exponent. The maximum cycle length is

$$k = 2^r - 1$$

The exponent of an irreducible polynomial of degree r which is not maximum divides $2^r - 1$. For every positive integer r , there are $[\varphi(2^r - 1)]/r$ polynomials of degree r that belong to a maximum exponent of $2^r - 1$. The Euler phi-function $\varphi(n)$ is the number of positive integers no greater than the integer n that are relatively prime to n .

Irreducibility is a necessary, but not sufficient, condition for $\phi(\lambda)$ to belong to a maximum exponent. A $\phi(\lambda)$ of degree r that belongs to a maximum exponent characterizes an r -stage maximal length FSR.

Cycle lengths for irreducible polynomials through degree 19 are given in Ref. 5. Irreducible polynomials of degree $r > 1$ will always have an odd number of terms, otherwise $\phi(\lambda)$ will contain $\lambda + 1$ as a factor. Irreducible trinomials of maximum exponent characterize maximal length FSRs with the simplest feedback logic; namely, a two-input modulo 2 summer. As previously stated, trinomials are not always among the $[\varphi(2^r - 1)]/r$ irreducible polynomials of maximum exponent. A conjecture,¹ which has since been proven true, states that every trinomial of degree 2^m ($m = 1, 2, \dots$) is reducible. This is a sufficient, but not necessary, condition for a trinomial to be reducible.

For repeated irreducible factors such as $\phi(\lambda) = [g(\lambda)]^\mu$,

$$k_\phi = e(\nu)k_g$$

where $e(\nu) = 2^i$ and i is an integer such that $1 \leq (2^i/\nu) < 2$.

Tabulated below are values of $e(\nu)$ for ν from 1 through 10.

ν	$e(\nu)$	ν	$e(\nu)$
1	1	6	8
2	2	7	8
3	4	8	8
4	4	9	16
5	8	10	16

In general, for

$$\phi(\lambda) = [g_1(\lambda)]^{\mu_1} [g_2(\lambda)]^{\mu_2} \cdots [g_m(\lambda)]^{\mu_m}$$

¹Made by S. W. Golomb, formerly of JPL Section 331.

where $g_1(\lambda), g_2(\lambda), \dots, g_m(\lambda)$ are irreducible,

$$k_\phi = \text{LCM} [e(\nu_1) k_{g_1}, e(\nu_2) k_{g_2}, \dots, e(\nu_m) k_{g_m}]$$

where LCM denotes the least common multiple.

CASE II:

$$e = 1$$

Then

$$\mathbf{L} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

and

$$\mathbf{X} = T\mathbf{x} \oplus \mathbf{L}$$

The transformation T and the translation $\mathbf{L}$ may be combined by bordering T with $\mathbf{L}$ to the right as a column, below with r zeros, and below and to the right by a single entry one (Ref. 3). The matrix equation (Eq. 3) can thus be written as

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_r \\ 1 \end{bmatrix} = \begin{bmatrix} C_{r-1} & C_{r-2} & \cdots & C_1 & 1 & 1 \\ 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_r \\ 1 \end{bmatrix} \quad (7)$$

The present state $\mathbf{x}$ is bordered with a one to make it conformal with the bordered T matrix and to perform the necessary complementation (i.e., translation) in the feedback to S_1 in Fig. 1. The next state vector $\mathbf{X}$ is also bordered just as $\mathbf{x}$. The one in the last row appears for each successive transformation. The matrix

$$\mathbf{A} = \begin{bmatrix} T & \mathbf{L} \\ 0 & 1 \end{bmatrix} \quad (8)$$

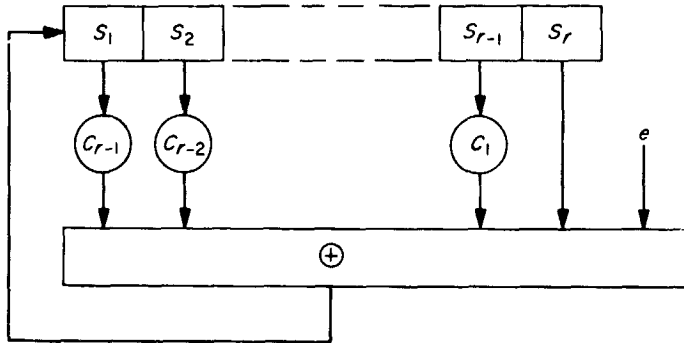


Fig. 1. Generalized r -stage FSR with linear feedback

is of the order $r + 1$ by $r + 1$ and includes the translation L . The characteristic polynomial of A is

$$\theta(\lambda) = \begin{vmatrix} T & \lambda I & L \\ 0 & 1 - \lambda & 0 \end{vmatrix} \quad (9)$$

$$\theta(\lambda) = (\lambda + 1) \phi(\lambda) \quad (10)$$

where $\phi(\lambda)$ is the characteristic polynomial of T . Just as T , A is nonsingular for all combinations of values of C_i , where $1 \leq i < r$ and $C_r = 1$. It will be shown that $\theta(\lambda)$ is minimal for A , just as $\phi(\lambda) = m(\lambda)$ for T . Note that the $\lambda + 1$ in Eq. (10) accounts for the complementation of the bit being fed back. The degree of $\phi(\lambda)$ determines the number of stages required. When the feedback of an FSR characterized by $\phi(\lambda)$ is complemented, it will be designated by $\phi(\lambda)^*$ where $\phi(\lambda)^* = [\theta(\lambda)]/(\lambda + 1)$.

3. The Minimal Polynomial of the T and A Matrices

Every square matrix satisfies a unique polynomial, called the minimal polynomial. The minimal polynomial $m(\lambda)$ of a square matrix B is the polynomial of lowest degree for which $m(B) = 0$. Furthermore, $m(\lambda)$ divides every polynomial which is satisfied by B (Ref. 3). Therefore, $m(\lambda) | \phi(\lambda)$ where $\phi(\lambda)$ is the characteristic polynomial of B .

The length of the longest cycle of an FSR with linear logic feedback is related to the divisibility properties of $m(\lambda)$. Only when $\phi(\lambda) = m(\lambda)$, can $\phi(\lambda)$ be used to determine the length of all cycles (Ref. 1). To justify the use of $\phi(\lambda)$ or $\theta(\lambda)$ in determining the longest cycle length of FSRs associated with the T and A matrices, it must be shown that their characteristic and minimal polynomials are equal.

For any $n \times n$ matrix B , there exist elementary polynomial matrices $P(\lambda)$ and $Q(\lambda)$, such that

$$[P(\lambda)] [B - \lambda I] [Q(\lambda)] = \begin{bmatrix} d_1(\lambda) & 0 & \dots & 0 \\ 0 & d_2(\lambda) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & d_n(\lambda) \end{bmatrix} \quad (11)$$

where $d_1(\lambda), d_2(\lambda), \dots, d_n(\lambda)$ are monic polynomials (see Ref. 6). The matrix $[B - \lambda I]$ satisfies (i.e., is a root of)

$$D(\lambda) = d_1(\lambda) d_2(\lambda), \dots, d_n(\lambda) \quad (12)$$

in which $d_i(\lambda) | d_{i+1}(\lambda)$ for $i = 1, \dots, n-1$. The diagonal matrix (Eq. 11) is the *Smith canonical form* of B and $d_i(\lambda)$ for all i is a *similarity invariant* of B . The minimal polynomial of B is $d_n(\lambda)$. The characteristic polynomial of B is $D(\lambda)$. When $D(\lambda) = d_n(\lambda)$, B is said to be non-derogatory.

The *Smith canonical form* can be derived from $[B - \lambda I]$ without explicitly determining $P(\lambda)$ and $Q(\lambda)$. The *Smith canonical form* is derived as follows for the T and A matrices.

a. *The T matrix.* Given the 4×4 $[T - \lambda I]$ matrix,

$$\begin{bmatrix} \lambda + C_3 & C_2 & C_1 & 1 \\ 1 & \lambda & 0 & 0 \\ 0 & 1 & \lambda & 0 \\ 0 & 0 & 1 & \lambda \end{bmatrix} \quad (13)$$

Let the elementary transformations induced by $P(\lambda)$ or $Q(\lambda)$ be denoted as follows:

C_{ij} is the interchange of columns i and j

$C_{ij}(k)$ is the replacement of column i by column j plus k times column j

$r_{ij}, r_{ij}(k)$ are corresponding row operations

Note that all arithmetic operations are reduced modulo 2. The sequence of elementary operations

$$\left. \begin{array}{l} (1) C_{21}(\lambda) \\ (2) r_{12}(\lambda + C_1) \\ (3) C_{32}(\lambda) \\ (4) r_{11}(\lambda^2 + C_2\lambda + C_1) \\ (5) C_{41}(\lambda) \\ (6) r_{11}(\lambda^3 + C_3\lambda^2 + C_2\lambda + C_1) \\ (7) r_{12} \\ (8) r_{23} \\ (9) r_{34} \end{array} \right\} \quad (14)$$

reduces (13) to the Smith form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \phi(\lambda) \end{bmatrix} \quad (15)$$

where $D(\lambda) = \phi(\lambda) = m(\lambda) = \lambda^4 + C_3\lambda^3 + C_2\lambda^2 + C_1\lambda + 1$.

This procedure is readily extended to any $r \times r$ T matrix as shown in Eq. (3) of Section 2.

b. The A matrix. Given the 5×5 $[A - \lambda I]$ matrix,

$$\begin{bmatrix} \lambda + C_3 & C_2 & C_1 & 1 & 1 \\ 1 & \lambda & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 \\ 0 & 0 & 1 & \lambda & 0 \\ 0 & 0 & 0 & 0 & \lambda + 1 \end{bmatrix} \quad (16)$$

The sequence of elementary transformations from 1 through 6 as shown in (14) results in

$$\begin{bmatrix} 0 & 0 & 0 & \phi(\lambda) & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda + 1 \end{bmatrix} \quad (17)$$

and continuing with

$$\begin{array}{l} (7) C_{15}[\phi(\lambda)] \\ (8) r_{51}(\lambda + 1) \\ (9) C_{15} \\ (10) C_{31} \\ (11) C_{21} \\ (12) C_{12} \end{array}$$

reduces the matrix of (17) to

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & (\lambda + 1)\phi(\lambda) \end{bmatrix}$$

4. Feedback Configuration for Near-Maximal Linear FSRs

a. Derivative of a characteristic polynomial for a cycle length of $2^r - 2$. Given the characteristic polynomial $g(\lambda)$ of degree r associated with a maximal-length r -stage linear FSR, then

$$\theta(\lambda) = (\lambda + 1)^2 g(\lambda)$$

is the characteristic polynomial of an $(r+2)$ -stage linear FSR with a major cycle length of

$$2(2^r - 1) \text{ or } 2^{r+1} - 2$$

Since complementation of the feedback has the effect of introducing a factor of $\lambda + 1$ in the characteristic polynomial, a cycle length of $2^{r+1} - 2$ can be realized with an $(r+1)$ -stage FSR, where

$$\phi(\lambda)^* = (\lambda + 1) g(\lambda)$$

For many values of r , a $g(\lambda)$ of degree r can be found such that $\phi(\lambda)^*$ is a tetranomial.

EXAMPLE 1:

$$g(\lambda) = \lambda^7 + \lambda^5 + \lambda^4 + \lambda^3 + \lambda^2 + \lambda + 1$$

$$\phi(\lambda)^* = (\lambda + 1) g(\lambda) = \lambda^8 + \lambda^7 + \lambda^6 + 1 \quad (18)$$

or

$$\left. \begin{aligned} a_n &= 1 \oplus a_{n-1} \oplus a_{n-2} \oplus a_{n-3} \\ a_n &= a_{n-1} \oplus a_{n-2} \oplus a'_{n-3} \end{aligned} \right\} \quad (19)$$

The characteristic polynomial (Eq. 18), or equivalently, the linear recurrence relationship (Eq. 19), characterizes a major cycle length of 254 (and a minor cycle length of 2) of an eight-stage linear FSR.

The binary coefficients (1 0 1 1 1 1 1 in Example 1) may be determined from Ref. 5 for every maximal-length feedback configuration. When a $g(\lambda)$ is selected such that the binary sequence of coefficients starts and ends with a run of ones separated by a run of zeros,

$$\phi(\lambda)^* = (\lambda + 1) g(\lambda)$$

results in a tetranomial.

In Example 1, $(\lambda + 1) g(\lambda)$ can be determined from

$$g(\lambda) + \lambda g(\lambda)$$

as follows:

$$\begin{array}{r} 10111111 \\ 10111111 \\ \hline 111000001 \end{array} \quad \begin{array}{r} \lambda g(\lambda) \\ g(\lambda) \\ \hline (\lambda + 1) g(\lambda) \end{array}$$

Table 1. Linear feedback configurations for FSR cycle lengths of $2^r - 2$ and 2

s	i	j	$2^r - 2$
4	1	2	14
5	1	3	30
6	1	2	62
7	1	5	126
8	1	2	254
9	2	6	510
10	2	3	1022
11	1	3	2046
12	2	7	4094
13	—	—	—
14	1	2	16328
15	3	5	32766
16	1	2	65534
17	1	11	131070
18	1	12	262142
19	1	7	524286
20	1	14	1048574

As shown in Table 1, there is an s -stage linear three-tap FSR with a major cycle length of $2^s - 2$ for every value of s from 4 through 20. The only exception is for an s of 13.

Where possible, it is desirable to have the stage storing a_{n-1} connected to the feedback. This allows a simplification in the implementation of the feedback when using RS flip-flops as memory elements. If the leading run of ones in the binary coefficient of $g(\lambda)$ contains a single one as in Example 1, the feedback function for $(\lambda + 1) g(\lambda)$ will be

$$a_{n-1} \oplus a_n \oplus a'_{n-3}$$

Each feedback configuration tabulated in Table 1 has a minor cycle of length 2. The states of the minor cycles are:

S even	S odd
0 1 0 1 . . . 0 1	0 1 0 1 . . . 0 1 0
1 0 1 0 . . . 1 0	1 0 1 0 . . . 1 0 1

b. Derivation of a characteristic polynomial for a cycle length of $2^r - 4$. The characteristic polynomial

$$\theta(\lambda) = (\lambda + 1)^3 g(\lambda)$$

where $g(\lambda)$ is of degree r and maximal is the characteristic polynomial of an $(r+3)$ -stage linear FSR with a major cycle length of

$$4(2^r - 1) \text{ or } 2^{r+2} - 4$$

A major cycle length of $2^{r+2} - 4$ can be realized with an $(r+2)$ -stage linear FSR. By complementing the feedback, a factor of $\lambda + 1$ is introduced. Thus,

$$\phi(\lambda)^* = (\lambda + 1)^2 g(\lambda) = (\lambda^2 + 1) g(\lambda)$$

characterizes a linear FSR with a major cycle length of $2^{r+2} - 4$ and a minor cycle length of 4.

EXAMPLE 3:

$$g(\lambda) = \lambda^8 + \lambda^7 + \lambda^6 + \lambda^5 + \lambda^4 + \lambda^3 + 1$$

$$\phi(\lambda)^* = (\lambda^2 + 1) g(\lambda) = \lambda^{10} + \lambda^9 + \lambda^8 + 1 \quad (20)$$

or

$$\left. \begin{aligned} a_n &= 1 \oplus a_{n-1} \oplus a_{n-5} \oplus a_{n-10} \\ a_n &= a_{n-1} \oplus a_{n-5} \oplus a'_{n-10} \end{aligned} \right\} \quad (21)$$

The characteristic polynomial (Eq. 20), or equivalently, the linear recurrence relationship (Eq. 21), characterizes a major cycle length of 1020 (and a minor cycle length of 4) of 10-stage linear FSR.

When a $g(\lambda)$ is selected such that the binary sequence of coefficients either starts with a run of *ones* and ends with alternating *zeros* and *ones* (i.e., 1 1 0 1 . . . 0 1), or starts and ends with alternating subsequences separated by a run of *zeros* or *ones*, $\phi(\lambda)^* = (\lambda + 1)^2 g(\lambda)$ results in a tetranomial. A $g(\lambda)$ of the first form yields a feedback configuration in which a_n is fed back.

In Example 2, $(\lambda^2 + 1)g(\lambda)$ can be determined from $g(\lambda) + \lambda^2 g(\lambda)$ as follows:

$$\begin{array}{r} 111110101 \\ 111110101 \\ \hline 11000100001 \end{array} \quad \begin{array}{l} \lambda^2 g(\lambda) \\ g(\lambda) \\ \hline (\lambda^2 + 1)g(\lambda) \end{array}$$

Linear s -stage FSRs with a three-tap feedback configuration and a major cycle length of $2^s - 4$ are tabulated in Table 2. Values of s from 4 through 21 are included,

Table 2. Linear feedback configurations for FSR cycle lengths of $2^s - 4$ and 4

s	i	j	$2^s - 4$
4	1	3	12
5	1	2	28
6	—	—	—
7	1	4	124
8	—	—	—
9	1	2	508
10	1	5	1020
11	1	4	2044
12	1	3	4092
13	1	2	8188
14	—	—	—
15	1	12	32764
16	1	7	65532
17	1	14	131068
18	5	9	262140
19	7	10	524284
20	5	7	1048572
21	1	6	2097148

with the exception of 6, 8, and 14 which do not exist with three feedback taps (i.e., which can be characterized with tetranomials).

Each feedback configuration tabulated in Table 2 has a minor cycle of length 4. The states of the minor cycle are:

$$\begin{array}{l} 00110011 \dots 0011 \dots \\ 10011001 \dots 1001 \dots \\ 11001100 \dots 1100 \dots \\ 01100110 \dots 0110 \dots \end{array}$$

c. Implementation of near-maximal linear FSRs. As shown in Tables 1 and 2, a near-maximal cycle length can be realized with a feedback function of the form

$$a_{n-1} \oplus a_{n-s} \oplus a'_{n-s}$$

for values of s from 4 through 21.

Substituting q_i for a_{n-i} , the next state of the leftmost memory element may be expressed as

$$Q_1 = q_1 \oplus q_j \oplus q'_s \quad (22)$$

Given RS flip-flops with the characteristic equation

$$Q = S' + Rq$$

where

$$R'S' = 0$$

the minimized R_1 and S_1 inputs for the flip-flops whose next state is Q_1 are:

$$(1) \quad i = 1$$

$$R_1 = (q_1 q'_j q'_s + q_1 q_j q_s)'$$

$$S_1 = (q'_1 q'_j q'_s + q'_1 q_j q_s)'$$

$$(2) \quad i \neq 1$$

$$R_1 = q'_1 q_j q_s + q_1 q'_j q_s + q_1 q_j q'_s + q'_1 q'_j q'_s$$

$$S_1 = R'_1$$

Thus, the cost of the feedback network is four NAND gates when $i = 1$ and five NAND gates when $i \neq 1$.

Provision for common collector operation (i.e., NAND-AND) is assumed.

When a maximal-length cycle of $2^r - 1$ cannot be realized with r stages, a near-maximal length of $2^r - 2$ or $2^r - 4$ may be realized with r stages and as few as four NAND gates which comprise the feedback network. (Two-tap feedback networks for maximal-length linear

FSRs require two NAND gates when q_1 is fed back, or three NAND gates otherwise.)

For example, there is no 12-stage, two-tap feedback configuration that yields a maximal length of 4095 (see Ref. 2). However, the near-maximal length of 4092 can be realized with 12 memory elements and four NAND gates (see Table 2).

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XVII. Lunar and Planetary Sciences

SPACE SCIENCES DIVISION

N67-29158

A. An Experiment for the Determination of Martian Surface and Cloud-Top Elevations,

R. A. McCluthey

This experiment depends on the absorption of solar radiation in the Martian atmosphere by CO₂ molecules in both the 2- and 2.7- μ bands. In this spectral region thermal emission is negligible compared with reflected solar radiation; therefore, by pointing a spectrometer toward the Martian surface, the atmospheric absorption spectrum can be measured. The measured absorption will depend on the total amount of CO₂ in the radiation path as well as on the atmospheric pressure. If the mean atmospheric surface pressure is known and it is assumed that CO₂ is uniformly mixed, the anticipated absorption for a given surface elevation can be computed. Such calculations can then be compared with the measured absorption spectrum. As this experiment requires a measurement of absorption *relative* to the continuum, changes in surface reflectance with different kinds of underlying surface material do not affect the results.

Uncertainty in the value of the mean surface pressure does *not* render the experiment meaningless. It places uncertainty on the absolute elevation determinations, but relative elevation determinations are unaffected. Accurate measurements of absorption at these wavelengths would,

as a by-product, lead to a better estimate of the mean surface pressure.

Although this experiment could be performed from either a flyby spacecraft or an orbiter, the latter would be preferable from the standpoint that detailed area coverage would be obtained and temporal variations could be measured.

In addition to elevation differences, several other factors will affect the atmospheric absorption. The most important are:

- (1) Temporal changes—e.g., tidal effects and pressure fluctuations resulting from moving pressure systems; also, possible seasonal changes resulting from the changing size of the polar caps (if they are composed of CO₂).
- (2) Spatial changes—e.g., a correlation between absorption and latitude could measure the planetary oblateness; mean atmospheric temperature changes will affect the absorption, but the atmospheric emission in the 4.3- μ CO₂ band should yield the mean temperature. Other spatial variations would be due to surface elevation differences.

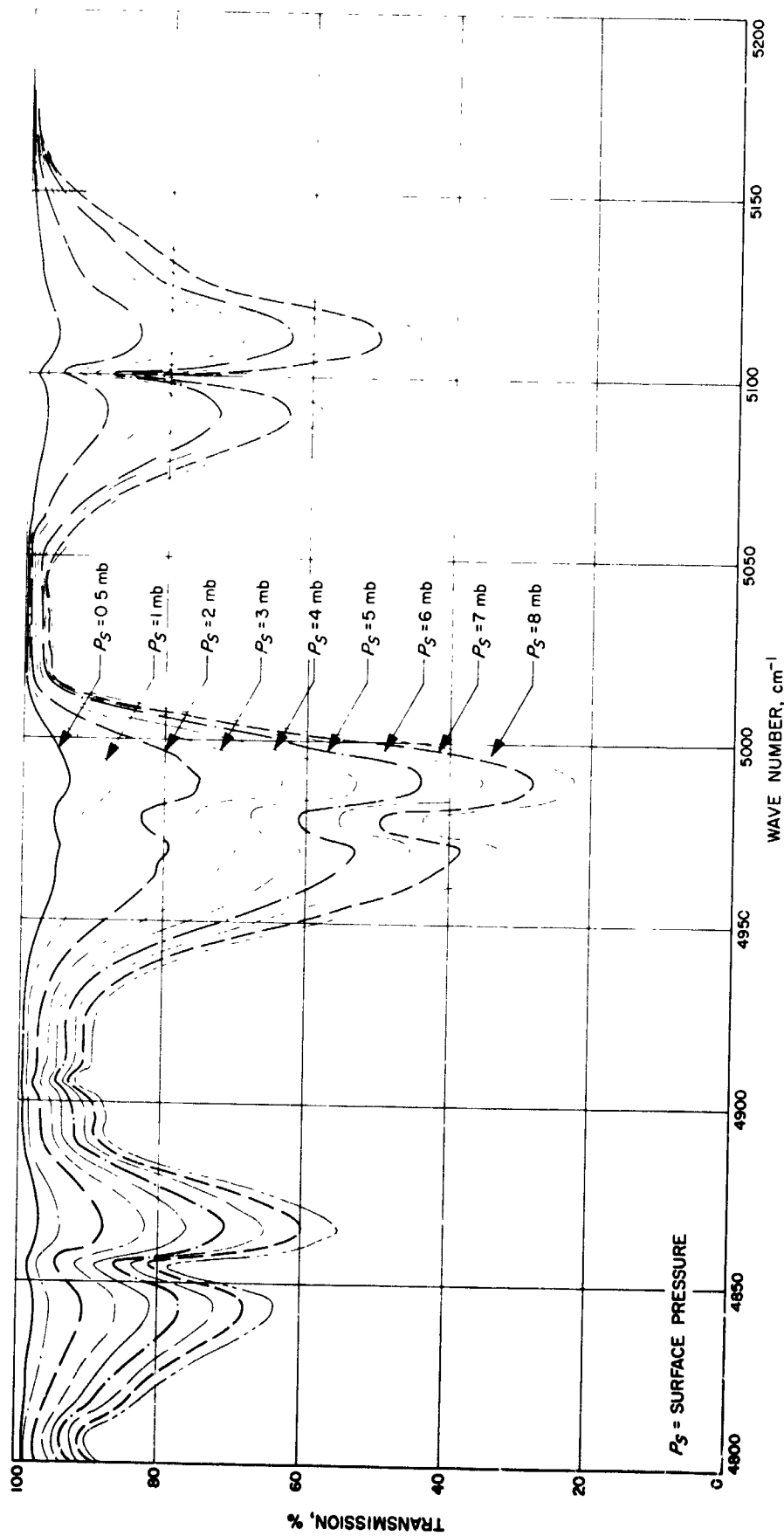


Fig. 1. Transmission of an assumed Mars atmosphere near 2μ

Most of the above-mentioned effects are expected to be much smaller and of different spatial and temporal extent than elevation differences of the order of 1 km or more. Tidal effects may be of the order of 10%. The Mars circulation results generated by Mintz's program (Ref. 1) indicate that moving weather systems may cause pressure variations as great as 20%. The work of Leighton and Murray (Ref. 2) shows that polar caps could contain as much as one-third of the total atmosphere. The planetary oblateness is not well known, but de Vaucouleurs (Ref. 3) indicates that a flattening in excess of that associated with a geopotential surface seems to exist. The anomalous flattening suggests an increase in atmospheric mass in the polar region of the order of a scale height. The uncertainty in this result can be resolved as part of this experiment. If such an anomaly is not detected, this controversy may be dismissed.

Temporal and spatial correlations of the absorption measurements should yield information on some or all of the above effects. In this way, this experiment provides

much more than a simple elevation determination. After the removal of temporal and low-amplitude spatial variations, elevation differences should be determinable to about 1 km. The following discussion supports this assertion.

Figs. 1 and 2 express the absorption by CO_2 of solar radiation that has twice traversed a hypothetical Martian atmosphere normal to the surface. The computer program and molecular spectroscopy used in deriving these curves were done by L. Gray.¹ The accuracy of the calculation has been verified by comparing calculated spectra with laboratory measurements (Ref. 4). The calculated absorption near $2\ \mu$ results from 45 different vibrational transitions; absorption near $2.7\ \mu$ is based on 67 different vibrational transitions. Figs. 1 and 2 have been drawn for 100% CO_2 atmospheres having surface pressures up to 8 mb. The successive curves correspond to reflection levels existing at lower pressures in 1-mb intervals; the top curve corresponds to reflection of solar radiation at

¹On leave of absence from JPL Section 325.

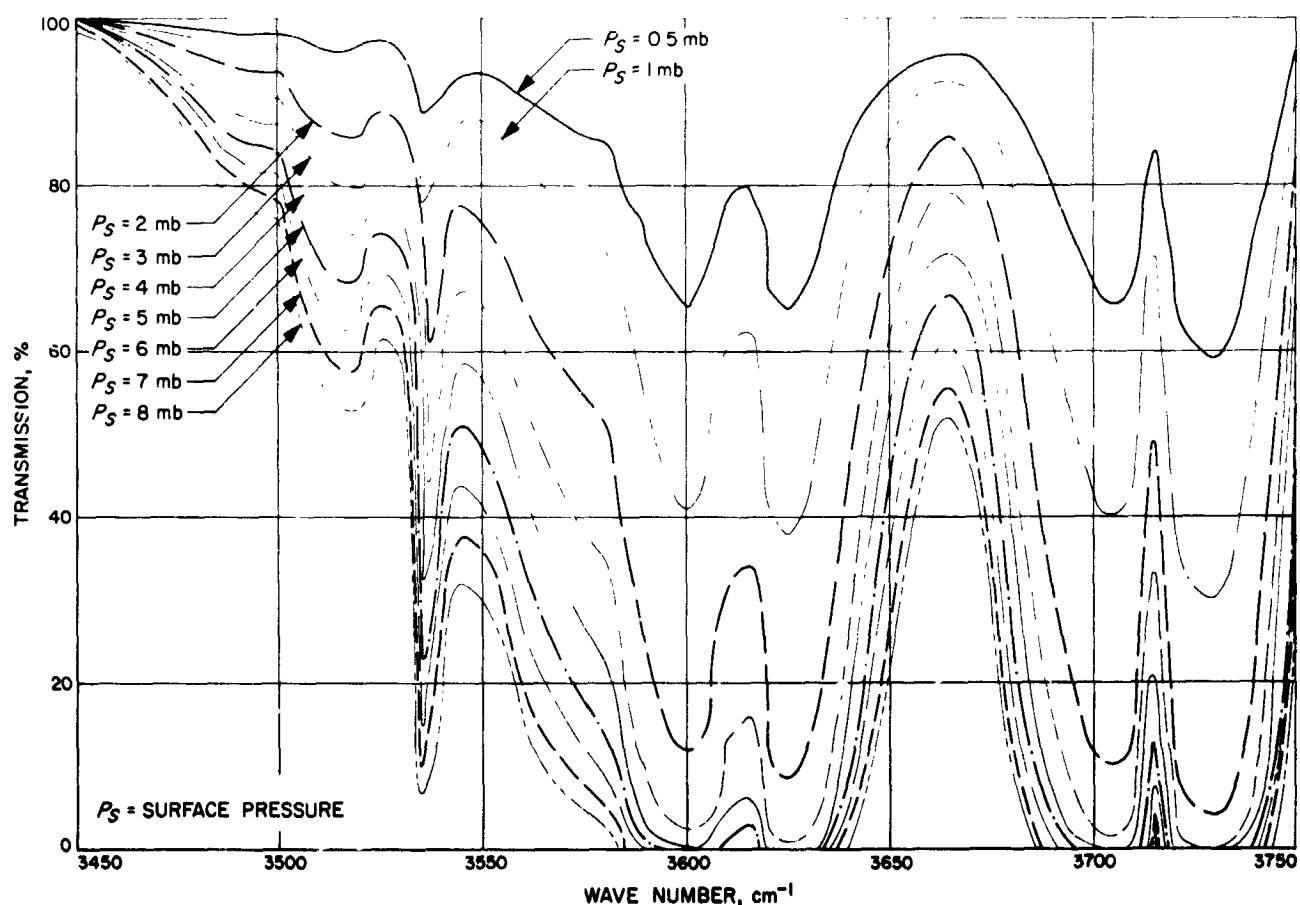


Fig. 2. Transmission of an assumed Mars atmosphere near $2.7\ \mu$

the 0.5-mb level. The mean transmittance over a 10-cm^{-1} interval located near the peak of the P branch centered at 4980 cm^{-1} would be increased from 38 to 44% if an elevation difference of 1 out of 8 mb was encountered (under the assumptions of this atmospheric model). A change in surface pressure from 8 to 7 mb corresponds to a rise in elevation of about 1.5 km. An instrument having the capability of making a 1%-of-full-scale measurement could do better than this. It would be able to distinguish elevation differences somewhat less than 1 km.

Detectors placed in both the $2.7\text{-}\mu$ and $2\text{-}\mu$ spectral regions would be capable of measuring a wide range of absorption, corresponding to almost any conceivable model of the Martian atmosphere and to any conceivable elevation differences. The $2.7\text{-}\mu$ band is considerably stronger than the $2\text{-}\mu$ band and thus would offer a more detailed elevation determination should the mean surface pressure be considerably lower than anticipated. An examination of Figs. 1 and 2 dictates the detector wavelength settings required to carry out this experiment. There is much flexibility in the number and location of detectors, but detectors centered at 4970, 4980, 4990, 5040, 5090, 5180, 3450, 3515, 3600 wave numbers should yield useful results.

Detectors centered at 5040, 5180, and 3450 are used for background measurements to establish the continuum. It would be ideal to have at least these nine detectors; however, some useful information could be obtained with a lesser number.

It should be stressed that these calculations represent normal incidence and emergence of radiation. Although there is no such constraint, it is anticipated that measurements would be made looking straight down, normal to the planetary surface. However, it is necessary to know the direction in which the instrument is pointing as well as the angle the Sun makes with the local vertical. Calculations could easily be made for slant paths without seriously affecting the elevation determination, as discussed above. It would also be preferable to look down the local vertical in order to optimize the spatial resolution.

The above discussion applies with slight modification to the determination of the heights of the cloud tops. Although it is assumed that TV coverage of the areas being observed with the spectrometer will be available, it is suggested that an independent detection of visible radiation be made to establish the existence or absence of clouds in the field of view. The cloud top will act

almost as a reflecting surface, but, in general, some correction for scattering within the cloud should be made. Saiedy, et al.,² find corrections ranging up to 38% when using the oxygen band near 7600 Å for similar measurements in the terrestrial atmosphere. It is anticipated that smaller corrections would be required at the longer wavelengths considered here, but there is strong dependence on particle size and optical thickness of the cloud. Detailed photographic coverage of the planet from the same spacecraft might yield valuable information concerning the nature of the clouds; this would aid in making corrections to the first-order estimate of cloud height (assuming that the clouds act as simple diffuse reflectors). It should be kept in mind, however, that the first-order estimate will be systematically an underestimate of the elevation of the cloud top. Multiple scattering in the clouds causes a greater increase in the radiation path length for absorption than would be observed under no-scattering conditions. This would be interpreted as a lower reflecting layer (i.e., at higher pressure) than actually exists.

Possible absorption due to any other molecules having absorption bands in the same spectral region should be considered. One such molecule that should be investigated is H_2O . Absorption due to 10 to $20\text{ }\mu$ of precipitable water in the $2\text{-}\mu$ region should be negligible (see Table 3-2 of Burch, et al., Ref. 5), but in the $2.7\text{-}\mu$ region it may not be quite so small. Fig. 3 is a plot (taken from Ref. 4) of the transmission of radiation through $16.9\text{ }\mu$ of H_2O vapor at a pressure of 200 mb. The conditions of the experiment are *not* the same as those in the Martian atmosphere. They should yield an overestimate of the absorption due to the high pressure in the absorption cell. The overestimate in absorption should be by about a factor of 5. This effect can be removed by measuring the absorption in an independent H_2O band, thus establishing the amount of water vapor in the radiation path. Absorption due to other more exotic molecules is expected to be small, and could be removed when positive identification of such molecules has been made and their abundances established by measuring absorption in other parts of the spectrum. In any event, as long as other molecules absorbing in the same spectral region as the CO_2 absorption are uniformly distributed in the atmosphere, relative elevation differences and cloud heights will be accurately determined.

This technique offers a straightforward method for establishing differences in elevation. Simplicity is its

² Saiedy, F., Jacobowitz, H., and Wark, D. Q., *On Cloud-Top Determination From Gemini-5*, personal communication.

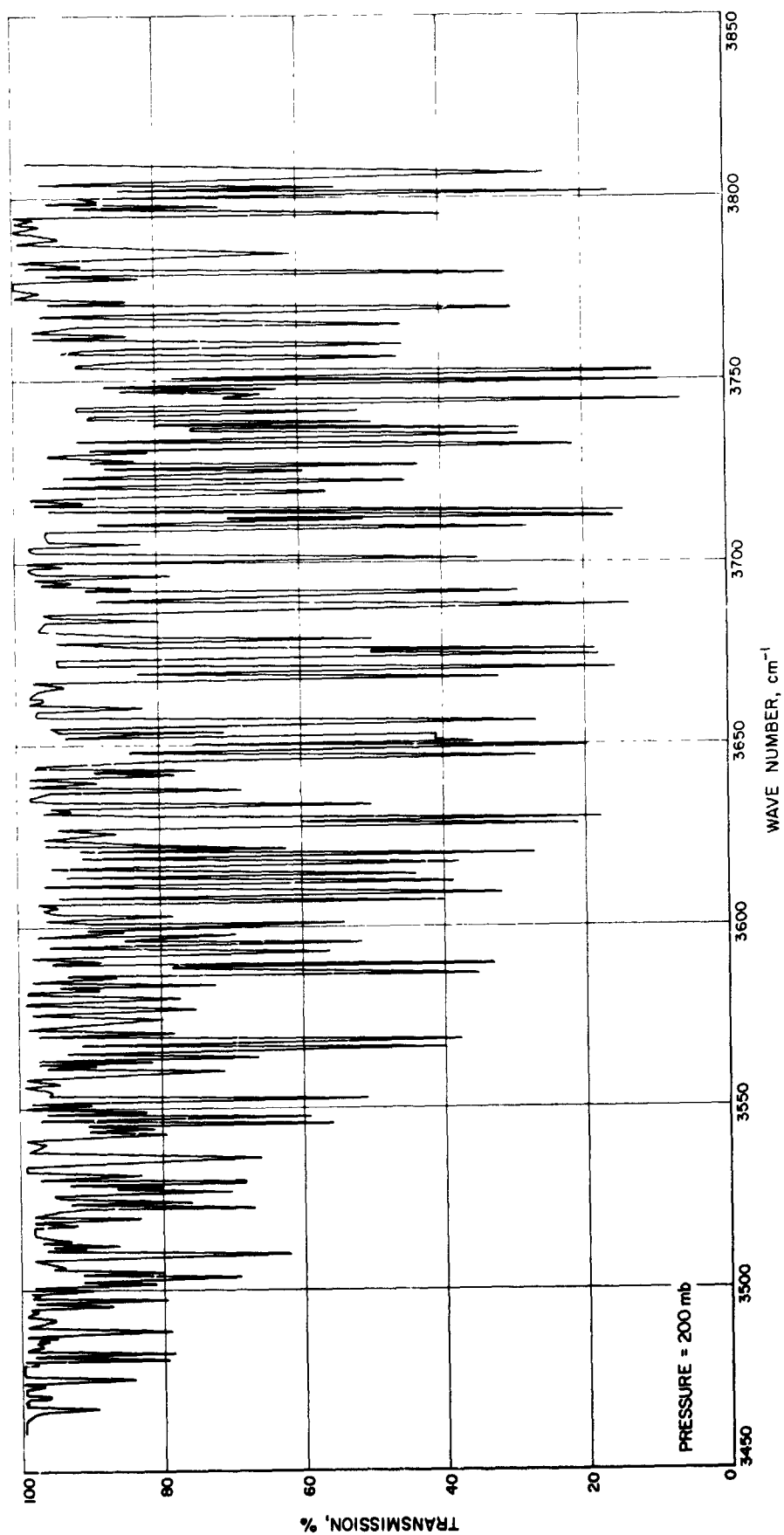


Fig. 3. Transmission of radiation through 16.9μ of H_2O vapor

greatest asset. A glance at the results will yield valuable information, more detailed interpretation would serve to refine the initial estimate and yield additional important results. Although the cloud-height problem is not quite so straightforward, even there a glance will yield significant information that is not obtainable by any other means.

B. Balloon Observations of the Radiance of the Earth Between 2100 and 2700 cm^{-1} , J. H. Shaw, R. A. McClatchey, and P. W. Schaper¹

The retrieval of atmospheric temperature information from observations of the outgoing thermal radiation from a planet in a spectral region occupied by a strong band of an atmospheric gas whose distribution with height is known has been described by Kaplan (Ref. 6). Hilleary, et al., (Ref. 7) have successfully used this method to determine an atmospheric temperature profile to 30 km by measuring the upward radiance from the ground and the CO_2 in the atmosphere in narrow spectral intervals near $15\ \mu$ with a balloon-borne grating spectrometer.

Carbon dioxide is the only strongly IR-absorbing atmospheric constituent whose distribution with height is known with some certainty. This molecule has strong

bands at 4.3 and $15\ \mu$. The $15\text{-}\mu$ band is the weaker of the two, but it occurs at a wavelength near which the outgoing radiance from the Earth is at a maximum. The $4.3\text{-}\mu$ band lies in a region where the outgoing thermal radiation flux is small, but strongly temperature dependent. This band can also be used to determine atmospheric temperatures. McClatchey (Ref. 8) has shown that suitable experimental data for such an analysis can be obtained with a spectrometer that has a spectral resolution of 100 and can determine the flux in this region from a 200°K blackbody with a signal-to-noise ratio greater than 10.

A grating spectrometer capable of measuring small radiation fluxes with a spectral resolution of 95 at $4.3\ \mu$ has been constructed and flown on a balloon from Sioux Falls, South Dakota. Bands of CO_2 , N_2O , and O_3 are identified in spectra of the Earth and lower atmosphere between 2100 and $2700\ \text{cm}^{-1}$. Measurements were made at an altitude of 30 km. Scattering of solar radiation by clouds was observed between 2400 and $2700\ \text{cm}^{-1}$. A temperature profile of the atmosphere to 30 km determined from an analysis of the measurements in the region of the $4.3\text{-}\mu$ CO_2 band was compared with radiosonde observations made during the flight.

A more complete account of the instrumentation and the data is given in *Applied Optics*, Vol. 6, No. 2, p. 227, February 1967.

¹Ohio State University, Department of Physics, Columbus, Ohio.
²JPL Section 323.

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XVIII. Bioscience

SPACE SCIENCES DIVISION

N67-29159

A. Soil Studies — Desert Microflora. XIV. Soil Properties and Abundance of Microflora From a Soil Profile in McKelvey Valley, Antarctica,¹ **R. E. Cameron**

1. Introduction

The Antarctic polar region is one of the coldest high-altitude deserts on Earth. Studies of the soil microflora of this region are of interest in connection with the detection and characterization of possible extraterrestrial microorganisms. They are also relevant to the Mars quarantine problem. Climate, which is one of the important factors for the development of soils and their microflora, is more rigorous and severe in Antarctica than in any other land area on this planet (Ref. 1). Organisms are either scarce, absent, or present only at certain seasons.

Antarctica is characterized by a dry atmosphere; the lowest mean temperatures on Earth; strong, persistent,

¹Logistic support and facilities for this study in Antarctica were supplied by the National Science Foundation. H. P. Conrow and G. B. Blank of JPL Section 328 provided assistance in collection of samples and in measuring *in situ* properties.

cold, dry winds; and five months of continual darkness. Except in coastal areas, nearly all of the Antarctic mainland is covered by an ice sheet of 11.5×10^6 km², with a mean thickness of 2000 to 2500 m (Ref. 2). Precipitation (such as snow) is generally very low, and the relative humidity is also low, about one-tenth that of middle-latitude regions. The air contains relatively little dust or pollutants.

Other factors that can be of biological significance include the surface ozone concentration and soil surface temperatures. Surface ozone values, about 45 $\mu\text{g}/\text{m}^3$ of air, are about 25% higher than in middle latitudes of the Northern Hemisphere (Ref. 2). Relatively warm soil conditions can exist in the summer along the coast, and there is a significant difference in the radiation balance of rock and soil surface (Refs. 2-5).

Antarctic soils are primarily and typically desertic and immature, with very little distinctive morphogenesis or distinct horizons. Therefore they do not have a true profile. They are generally sandy, salty, very low in organic matter, and underlaid by permafrost (Ref. 4). Desert pavement of gravel, pebbles, cobbles and boulders, and frost-crack polygons are common surface features.

The biotic factor has a negligible influence on exposed Antarctic land surfaces (Refs. 3-6). Plants are generally found only in some favorable habitats such as shallow ponds, around the faces of glaciers, in glacial outflow, and on some soil and rock surfaces. An investigation of these habitats shows that relatively warm, sheltered, moist microclimates or microenvironments at the air-surface interface are important for maintaining life. Primary factors for the survival and reproduction of organisms in the Antarctic environment include the ability to resist long periods of extreme desiccation, cold, and lack of solar energy. Masses of dead algae (without a single living alga) can be found around ponds, which attest to periods when survival was not possible in the rigorous Antarctic environment.

Lichens, algae, mosses, and fungi (the cryptogamic plants) are the primary life in Antarctica (Refs. 3, 5, and 6). Lichens and algae are the most abundant. The lichens appear to have morphological and physiological adaptations that enable them to best withstand the cold, desiccation, and winter dark period (Ref. 7). Algae and lichens are also the most abundant cryptogams in other desert areas (Ref. 8).

A review of microbiology of Antarctica (Ref. 9) indicates that a variety of microorganisms have been recovered from some habitats such as air, snow, ice, ponds, sea water, marine sediments, and Antarctic sea animals who spend part of their lives on the land or ice mass (Ref. 9). All of the different habitats, including the gastrointestinal microflora of warm-blooded animals (Ref. 9), showed a very low bacterial content. The cryptogamic flora provided a favorable habitat for microorganisms, but most soil samples contained relatively low abundances of populations (Ref. 9). In some soils there is a significant abundance of bacteria and fungi (molds); for other soils no or very little growth could be obtained using culture media for a wide variety of physiological groups (Ref. 10).

The study of Antarctic soils and microorganisms is a continuation of desert soil microbial ecology relative to possible extraterrestrial life in Martian soil. For this study, 26 soil sites and 76 samples were selected along the coast, primarily in "dry valley" areas of southern Victoria Land (Fig. 1). Two camps were established during the Antarctic austral summer, 1966-67. This report presents some of the results on soil properties and microflora determinations from one soil site at the McKelvey Valley camp, which existed from November 28 to December 11, 1966 (Fig. 2).

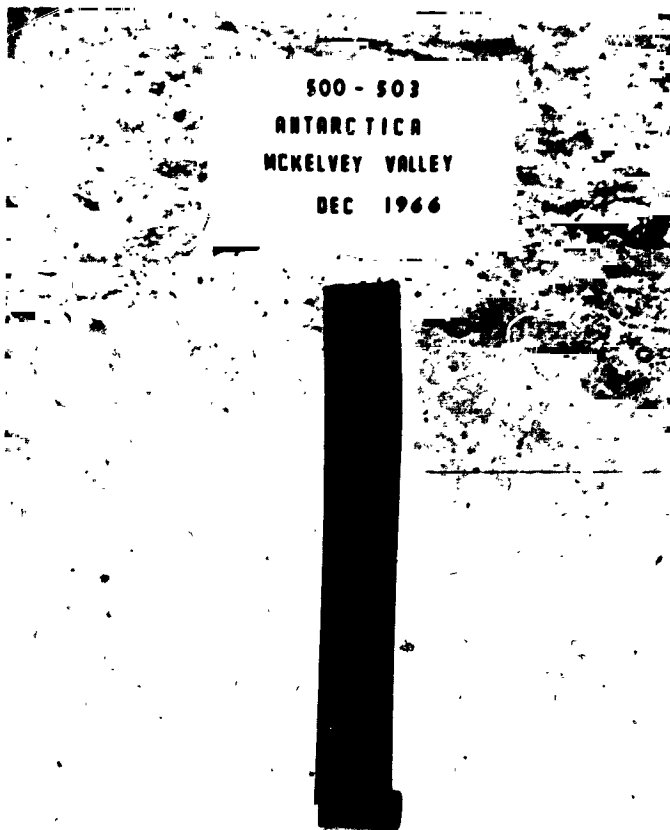


Fig. 2. Soil site and environmental instruments in McKelvey Valley (2730-ft elevation) with Olympus range in background. Approximately 80 mi from McMurdo Sound Station, Antarctica

2. Materials and Methods

By means of an aseptic technique previously developed for collecting and examining desert soils (Ref. 11), samples were taken from the surface 2-in. (No. 500), 2- to 6-in. level (No. 501), 1-ft level (No. 502), and the lowest depth at 1.5 ft (No. 503) (Fig. 3). Care was taken to sterilize the collecting instruments (Fig. 4) before and after each sample was obtained from the pit (Fig. 5). All samples were placed in previously sterilized plastic-lined canvas sacks. Additional samples were taken for soil moisture, porosity, and bulk density, as described previously (Ref. 11). Soil temperature and relative humidity values were recorded to a depth of 1 ft. A few bacteriological fallout culture plates (trypticase soy agar and Sabouraud dextrose agar) were exposed at the soil surface for 2 hr (Fig. 6). Small amounts of fresh soil from each depth were also sprinkled over the surface of other culture plates (Fig. 7). All of these culture plates were incubated in the field and later examined at McMurdo Station.

For most purposes, samples were homogenized by passing them through a 2-mm sieve (No. 10 mesh). An aliquote of the sieved sample was powdered by mortar and pestle for appropriate physical and chemical analyses at JPL.



Official U.S. Navy photograph by C. L. Abramson, USN.

Fig. 3. Gravelly frozen soil in pit excavated in McKelvey Valley



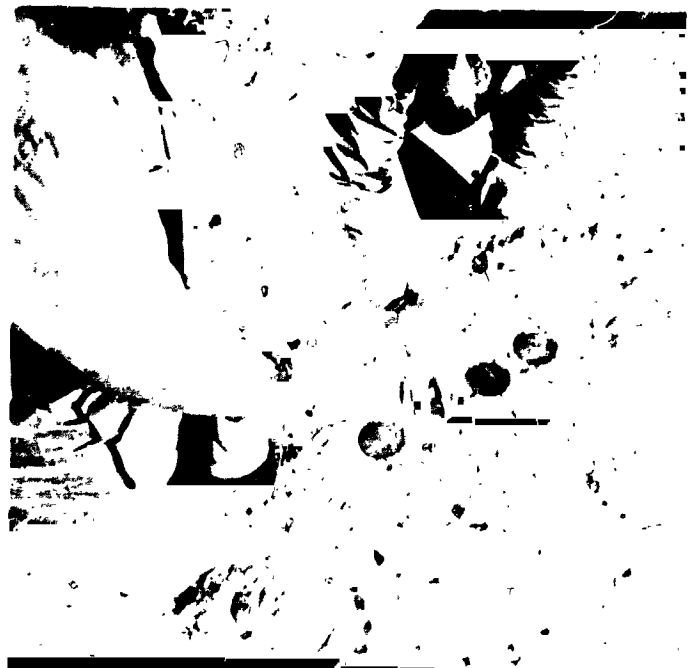
Official U.S. Navy photograph by C. L. Abramson, USN.

Fig. 4. Sterilization of sample-collecting tool with propane torch prior to collection of soil samples



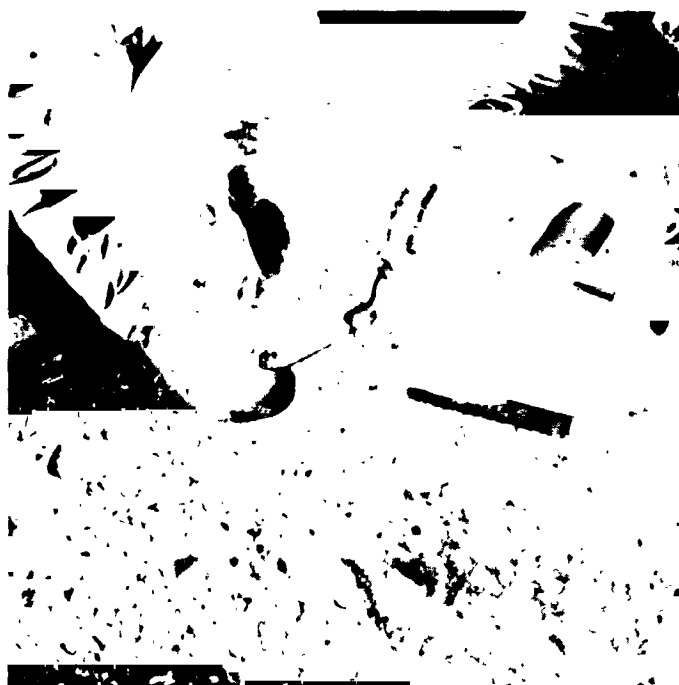
Official U.S. Navy photograph by C. L. Abramson, USN.

Fig. 5. Collecting soil sample from side of pit



Official U.S. Navy photograph by C. L. Abramson, USN.

Fig. 6. Placing microbiological fallout plates on soil surface near site



Official U.S. Navy photograph by C. L. Abramson, USN.

Fig. 7. Sprinkling freshly procured surface soil on agar plates at soil site to determine presence of bacteria and fungi

a. Physical analyses. Soil color and Munsell notation were determined on both *in situ* (wet) and sieved (dry) soils by reference to Munsell Soil Color Charts. Textural class was estimated in the field by the standard method of feeling the soil. *In situ* moisture was determined gravimetrically by infrared drying to constant weight at 105°C. Bulk density and porosity, except for hard permafrost, were calculated from samples collected in standard soil-moisture tins.

b. Physicochemical analyses. Soil pH, Eh, and electrical conductivity were determined on the saturated soil paste (Ref. 12). Combustible materials were determined² by igniting the dry soil in air at 750°C for 30 min. The cation exchange capacity values were obtained by the modified barium chloride-triethanolamine procedure for calcareous soils (Ref. 13).² Mineralogical analyses³ were obtained by X-ray diffraction with a scan speed of 0.5 deg/min.

c. Chemical analyses. Total carbon, hydrogen, and nitrogen analyses were performed by ignition in Coleman

C-H and N-analyzers, as described previously (Ref. 14).⁴ Organic C and carbonate C were obtained by the Allison method, utilizing acid digestion and gravimetric measurement of released CO₂ (Ref. 12).⁴ The Kjeldahl method was used to obtain organic N (Ref. 12).⁴ Total organic matter was estimated by using the factor $1.724 \times \text{wt } \% \text{ organic C}$. Sulfur and P were determined on HClO₄ extracts. Sulfur was determined by Na₂CO₃ fusion and P by the Truog-Meyer method (Ref. 13).⁴ Standard colorimetric procedures or flame photometric detection were used to determine the presence or absence of available ions on 1:5, soil:water extracts. These ions included NH₄⁺, NO₂⁻, NO₃⁻, Cl⁻, Mn⁺⁺, SO₄²⁻, K⁺, Na⁺, Mg⁺⁺, Ca⁺⁺, and PO₄³⁻. Spectrographic analyses were performed for all the other reported total elements by igniting soil (≥ 40 mesh) mixed with graphite using a dc arc.⁵

d. Microbiological analyses. Ten-gram quantities of sieved soil samples containing *in situ* moisture were suspended and dispersed by a reciprocating shaker for 20 min in deionized water. Deionized water was obtained by filtering distilled water through a Barnstead demineralizer. Following dispersion of the sample, the suspension was diluted serially by a 1:10 ratio and appropriate 1-cc aliquotes were pipetted into dilution tubes of selective media or disposable plastic petri dishes for pour plates.

For the various groups of microflora, this included the following media. (1) aerobic bacteria, trypticase soy agar; (2) microaerophilic bacteria, fluid thioglycollate medium; (3) anaerobes, trypticase soy agar in a reduced atmosphere obtained by mixing pyrogalllic acid with concentrated NaOH; (4) actinomycetes, actinomycete agar; (5) sulfate reducers, Van Delden's sulfate reduction agar (Ref. 15); (6) fungi, Rose Bengal agar without antibiotics and Sabouraud dextrose agar; and (7) algae, Thornton's medium without organics (Ref. 15). Most of the media were prepared according to standard procedures, and can be recommended for various chemoautotrophic, heterotrophic, aerobic, anaerobic, and microaerophilic bacteria, actinomycetes, fungi, and algae. All incubations were carried out at variable room temperatures, at + 55 or + 4°C. All cultures were incubated at the NSF Biology Laboratory, McMurdo Sound Station, Antarctica, between December 19, 1966 and January 31, 1967.

⁴Analyses by Elek Microanalytical Laboratories, Torrance, California.

⁵Analyses by Pacific Spectro Chemical Laboratory, Los Angeles, California.

²Analyses by L. Taylor, JPL Bioscience Section (326).

³Analyses by Dr. E. Bolln, JPL Bioscience Section (326).

3. Results and Discussion

a. Contamination. McKelvey Valley presented a unique opportunity to investigate a cold, barren, windswept, glacier-cut area which had not been previously studied by soil scientists or microbiologists. In terms of geological time, this valley was relatively recently glaciated. It shows evidence of a third glaciation and may have been free of ice for approximately 10,000 years, although it is not yet possible to accurately date the stages of glacial recession in this area (Ref. 16).

Although McKelvey Valley was traversed in several directions for this study, at no time was there any evidence of previous human contamination; e.g., footprints, remains of camps, or refuse. However, on one day for about 15 min, a skua (gull-like sea bird) circled over the camp (Fig. 8) and rested for a few minutes about $\frac{1}{8}$ mi from the soil site. On another occasion, while on traverse from another site, a desiccated piece of seal skin was found approximately 1 mi northeast of camp (Fig. 9).



Fig. 8. Skua circling over McKelvey site—possible source of contamination

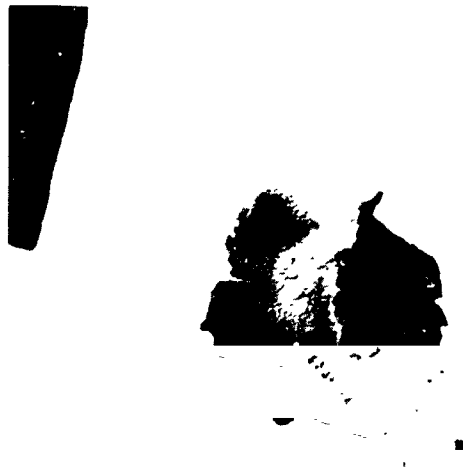


Fig. 9. Desiccated piece of seal skin found about 1 mi northeast of camp (length of skin = ca. 4 in.)

There was no other evidence of seals, although desiccated and mummified seals were observed in other valleys (e.g.; Taylor Valley).

Before leaving the McKelvey camp, no discernible growth was noted on either the bacteriological fallout plates (Fig. 6) or the plates sprinkled with soil from the collection site (Fig. 7). Incubation period was from December 4 to 11, 1966 at variable temperatures, from below freezing to $+11^{\circ}\text{C}$. There was growth on some of the agar plates after arrival back at McMurdo, where incubation was continued at higher room temperatures, $+15$ to $+23^{\circ}\text{C}$. Some of the control plates were contaminated, which made interpretation difficult, but a comparison with cultures made from processed McKelvey Valley soil at McMurdo showed that most of the microorganisms were similar. At nearly all times the wind was blowing in the valley, usually 5 to 15 mph, primarily from the northeast off the sea. There was undoubtedly a continual influx, although not necessarily an abundance, of airborne contaminants into the valley. The wind also scoured the valley floor, resulting in a desert pavement (Figs. 10 and 11); when walked upon or otherwise mechanically disturbed, fine particles of soil blew into the air and across the soil surface, serving as contamination for other windswept areas. Helicopter landings near the camp were an additional source responsible for spreading airborne contaminants or fine soil particles. Thirty- to forty-mph winds were created by the wash of helicopters, as recorded by the wind system at the camp site.



Fig. 10. Rocky, gravelly, and bouldery desert pavement at soil site

b. Soil analyses. A summary of the characteristics of the four soils is given in Tables 1-6. Physical determinations (Table 1) showed a narrow range of greyish, yellowish, or brownish sandy soils, which were composed primarily of quartz, and secondarily of oligoclase. Both of these minerals are common in rock formations and soils. With depth of soil, the amount of coarse materials ≥ 2 mm was found to increase. There was a slight increase of moisture with depth of soil, and subsurface soils appeared and felt

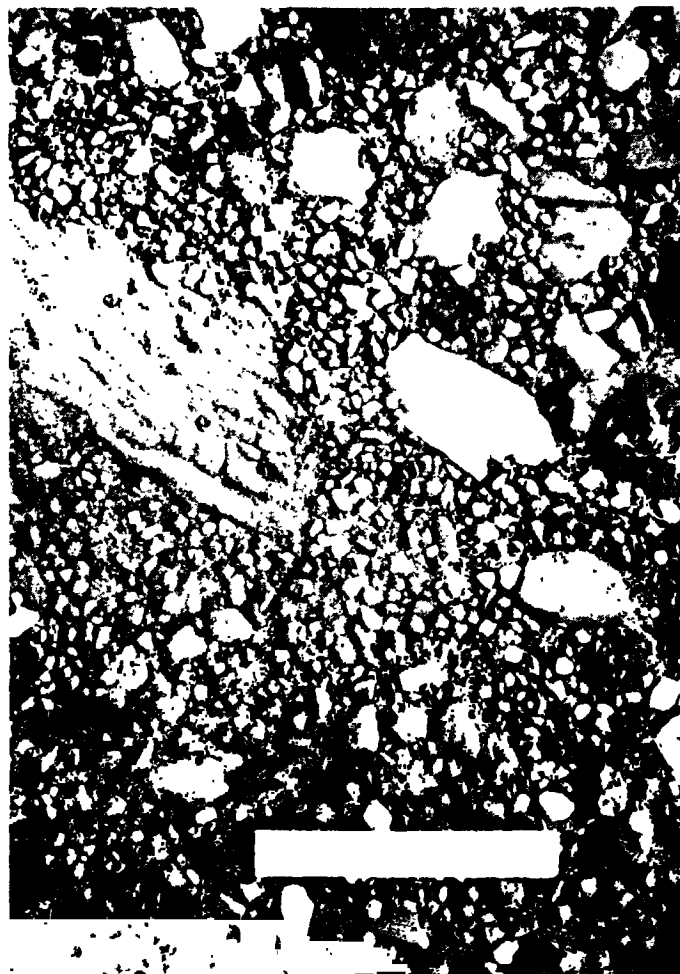


Fig. 11. Close-up of sharp, angular, and weathered rocks and gravel overlaying soil

Table 1. Antarctic soils: physical determinations

Soil No.	Soil depth	Color & Munsell notation		Textural class	<i>In situ</i> moisture, % wt	Bulk density, g/cc	Porosity, %	Unconfined strength, kg/cm ²	Temperature, °C		Relative humidity, %	
		<i>In situ</i>	Dry						max	min	max	min
500	Surface 2 in.	7.5 YR 6/6 pinkish grey	10 YR 6/4 light yellowish brown	Sand	1.4	1.69	36.2	<0.25 to 0.75	+2.0 ^b	-5.5 ^b	73.0	50.5
501	2 to 6 in.	7.5 YR 6/6 pinkish grey	10 YR 6/4 light yellowish brown	Sand	1.3	1.68	36.6	1.75 to >4.5	-4	-7.5	87.0	78.0
502	1 ft	7.5 YR 6/6 pinkish grey	10 YR 6/6 brownish yellow	Sand	1.6	1.62	38.9	2.0 to 3.75	-6	-9.0	86.5	81.0
503	1.5 ft	2.5 YR 6/2 light brownish grey	5 YR 6/4 pale olive	Sand	10.2	— ^a	— ^a	>4.5	— ^a	— ^a	— ^a	— ^a

^aHard, icy permafrost, temperature = -11°C.

^bTemperature at 2-in. depth.

Table 2. Antarctic soils: physicochemical determinations

Soil No.	Soil depth	pH	Eh	Electrical conductivity	Loss on ignition	Cation exchange capacity
		Saturated paste	Saturated paste, + mv ^a	Saturated paste ECX 10 ⁶ mho/cm at 25°C	30 min at 750°C, % wt	meq/100 gm soil
500	Surface 2 in.	8.0	403	3360	1.56	2.56
501	2 to 6 in.	7.6	416	3040	1.07	2.50
502	1 ft	7.6	439	4160	1.19	2.42
503	1.5 ft	8.3	491	1920	1.26	2.76

^aIncludes cell K = +243 mv.

Table 3. Antarctic soils: chemical determinations, % of dry weight

Soil No.	Soil depth	Total organic matter	Total carbon	Total nitrogen	Organic carbon	Organic nitrogen	Carbonate carbon	Total hydrogen	C:N total	N:H total	H:C total
500	Surface 2 in.	0.153	0.096	0.041	0.087	0.007	0.011	0.153	2.3	0.27	1.6
501	2 to 6 in.	0.233	0.163	0.033	0.135	0.012	0.008	0.142	4.9	0.23	0.9
502	1 ft	0.160	0.109	0.028	0.093	0.007	0.011	0.118	3.9	0.24	1.1
503	1.5 ft	0.308	0.185	0.018	0.179	0.007	0.017	0.134	10.3	0.13	0.7

moist to the 1.5-ft depth, where icy, hard permafrost was encountered. Bulk density decreased slightly with depth and varied inversely with porosity values. Bulk density and porosity values were comparable to those of other sandy or loamy desert soils. Soil hardness also increased with depth, exceeding 4.5 kg/cm² at the depth of hard, icy permafrost. With depth of profile, the soil became increasingly more difficult to dislodge.

The soil temperature decreased and relative humidity increased with depth below the surface. The soil was continually frozen below the 4-in. depth, and never reached the melting point of the liquid phase. It is doubtful that the relatively higher subsurface humidity values provided any available vapor phase for the activity of microorganisms. They would also have had to overcome the continual below freezing conditions.

Physicochemical determinations are indicated in Table 2. The pH values were typical for saline soils, which ordinarily have pH values less than 8.5. These measurements were similar to pH values obtained for other Antarctic soils (Refs. 4, 10, and 17). Eh values were also typical, which is not uncommon for oxidized desert soils when analyses are performed on processed samples in the lab-

oratory. However, *in situ* Eh measurements were made, with values recorded between + 670 and + 780 mv (uncompensated). These latter values were extraordinarily high and have not been previously encountered in our determinations of *in situ* redox potentials in desert soils.⁴ If these values are valid (they were determined at two soil pits in McKelvey Valley on two separate occasions), this environment could be limiting for some microorganisms (Ref. 18). Cation exchange capacities were quite low and were similar to those obtained for the Valley of 10,000 Smokes, Alaska; Kau Desert, Hawaii; and Eastern Sahara Desert, Egypt; as well as the Atacama Desert, Chile (SPS 37-38, Vol. IV).⁶ Cation exchange capacities were lower than for those determined for Southwestern United States desert soils (Ref. 19).

Electrical conductivity and cation exchange capacity values also indicated a saline soil, although weight loss upon ignition was typical of many desert soils low in organic matter and clay which contain comparatively little other volatile materials, except "bound" water or salts. Electrical conductivity values for soil pastes, which were generally one-half to two-thirds lower for 1:5,

⁴JPL Desert Microflora Program, unreported results. 1961 to date.

Table 4. Antarctic soils: abundance of chemical elements, % of dry weight

Element	Soil number			
	500	501	502	503
O	49.74	50.74	51.0	50.9
Si	28.5	28.5	32.0	32.0
Al	6.0	5.8	4.25	4.65
Ca	5.6	5.0	4.6	4.75
Fe	4.05	4.7	3.55	3.4
Mg	2.45	2.4	2.25	2.25
Na	1.6	1.3	0.96	0.83
S	0.723	0.209	0.297	0.055
Ti	0.425	0.36	0.25	0.22
K	0.4 ^a	0.4 ^a	0.4 ^a	0.4 ^a
H	0.153	0.142	0.118	0.134
C	0.096	0.163	0.109	0.185
Mn	0.085	0.110	0.099	0.081
N	0.041	0.033	0.028	0.018
P	0.037	0.029	0.027	0.024
Cr	0.031	0.034	0.0235	0.0275
Sr	0.0155	0.0205	0.0165	0.0220
Zr	0.0120	0.0150	0.0135	0.0130
Ni	0.0103	0.0110	0.0086	0.00635
Va	0.0088	0.0093	0.00555	0.00945
Cu	0.0079	0.0079	0.0057	0.00595
Co	0.0053	0.0059	0.0057	0.00435
Ga	0.003 ^b	0.003 ^b	0.003 ^b	0.003 ^b
Ag	0.00094	0.00358	0	0
Sn	0.002 ^c	0.002 ^c	0	0

^a0.4% or less.
^b0.003% or less.
^c0.002% or less.

soil:water extracts, would be restrictive for the growth of some plants, and would also be more favorable for the growth of halophiles.

Chemical determinations for C, N, and H are indicated in Table 3. As shown in this table, C, N, and H were relatively evenly distributed with depth of soil profile. These values compared favorably with similar analyses of typically organically poor Southwestern United States desert soils (Ref. 14). Compared with analyses of soils from the Chilean Atacama Desert, the values were also similar, except that total hydrogen was lower than for any of the 39 Atacama Desert samples. Organic carbon has not been detected in some Antarctic soils (Ref. 4), but this depends to some extent on the level of sensitivity of the method of analysis. Low hydrogen values were consistent with the mineralogy of the Antarctic samples, as well as with the low organic matter and clay content.

Table 5. Antarctic soils: "available" ions

Cation or anion	Soil number				Concentration, ppm
	500	501	502	503	
Ca ⁺⁺	+	+	+	+	> 5
Na ⁺	+	+	+	+	> 1
Mg ⁺⁺	—	—	—	—	< 100
K ⁺	—	—	—	—	< 10
Mn ⁺⁺	—	—	—	—	< 10
NH ₄ ⁺	Tr	Tr	—	—	= 10 for Tr
SO ₄ ⁼	+	+	+	+	> 10
Cl ⁻	+	+	+	+	> 1
NO ₃ ⁻	+	+	+	+	> 100
PO ₄	—	—	—	—	< 10
NO ₂ ⁻	Tr	Tr	Tr	Tr	=0.1 for Tr

+ = presence
 — = not detectable
 Tr = detectable trace

C:N values, as based on total carbon and nitrogen, also compared favorably with values obtained for other desert soils with narrow ratios. The organic C:organic N values were two to three times as wide as the total C:N, and would indicate that the C and N in these samples (except for soil 503) were constituents of microbial cells or of relatively decomposed materials. With such narrow C:N values, there is probably very little or no organic matter that would not be readily utilizable or available in a more favorable solclimate (soil-temperature-moisture regime). Total N:H values and H:C values showed a similarity to Chilean Atacama Desert soils with a significant amount of NO₃⁻ or NH₄⁺.

The "available" or water-soluble cations and anions in the 1:5, soil:water extracts are given in Table 4. As is indicative of saline soils, the chief anions are SO₄⁼ and Cl⁻, although NO₃⁻ and a trace of NO₂⁻ are also present. Ca⁺⁺ and Na⁺ were the chief cations. Absence of detectable Mg⁺⁺, K⁺, Mn⁺⁺, and PO₄⁼ is quite significant since the availability of these ions is essential for growth and reproduction of organisms. A comparison of the concentration of these ions in Table 4 with the abundance of elements in Table 5 indicates the relative availability status of these elements.

Table 6. Antarctic soils: microbiological determinations

Soil No.	Soil depth	Incubation temperatures, °C	Microorganisms per gram of soil									
			Aerobes	Anaerobes	Micro-aerophiles ^b dilutions			Fungi	Algae ^h dilutions		Actinomycetes	Sulfate reducers
					10	10 ²	10 ³		10	10 ²		
500	Surface 2 in.	Room temperature ^a + 4 +55	30 8 0	25 10	+	- +	- -	1.5	+ - ^c	- - ^c	0 0	0 0
501	2 to 6 in.	Room temperature ^a + 4 +55	13 27 0	10 30	+	+	- + ^d	1	- - ^c	- - ^c	0 0	0 0
502	1 in.	Room temperature ^a + 4 +55	14 18 0	10 35	+	+ +	- -	1	- - ^c	- - ^c	0 0	0 0
503	1.5 ft	Room temperature ^a + 4 +55	43 10 2	<10 <10	+	+ +	- -	0	- - ^c	- - ^c	0 0	0 0
Culture medium			Trypticase soy agar	Trypticase soy agar (Pyrogallic acid + NaOH)	Fluid thioglycollate			Rose Bengal agar and Sabouraud dextrose agar	Thornton's medium less organics	Bacto actinomycete agar	Van Delden's sulfate reduction agar	

^aRoom temperature incubations usually varied between +15 to +23°C.
^bChecked microscopically for growth (+ = growth; - = no growth).
^cIncubation temperatures usually varied between -2 to +15°C.
^dOne discrete colony present.

All incubations conducted in the National Science Foundation Biology Lab., McMurdo Sound Station, Antarctica, between Dec. 19, 1966 and Jan. 31, 1967.

There was a slight or significant decrease in abundance of most elements with depth of soil. This decrease in elements was counterbalanced by an increase in Si. Although some elements—S, Cr, Ni, and Co—were of higher concentration than in some desert soils, the low abundance of K was notable, when compared with some Hawaiian desert soils.⁶

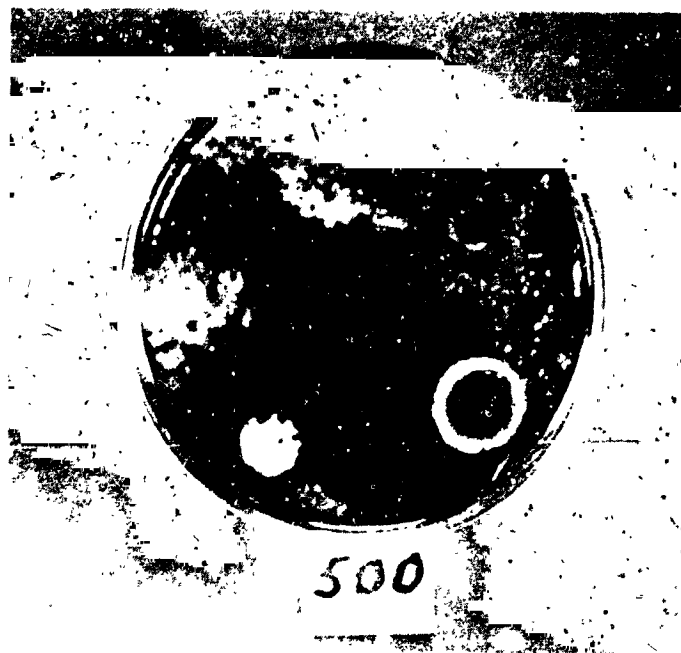
Abundance of common groups of microflora found in these soils is shown in Table 6. In all four soil samples, there were aerobic, anaerobic, and microaerophilic bacteria and/or yeasts. The abundance of these groups of microflora was quite low, much lower than for typical U.S. desert soils, and comparable to the lowest abundances encountered in some of the soils from the Valley of 10,000 Smokes, Alaska; Kau Desert, Hawaii; and Atacama Desert, Chile. However, in some of the saline Atacama Desert samples, the actinomycete (streptomycete) abundance in fluid thioglycollate (verified microscopically) approached values of 10⁶/g soil! The abundances of algae, fungi, and sulfate-reducing microorganisms were among the lowest values encountered in desert soils, and were also

comparable to the same levels, or absence, of these groups of microflora in soil samples from the Valley of 10,000 Smokes, Alaska; Kau Desert, Hawaii; Eastern Sahara Desert, Egypt; and the Atacama Desert, Chile.⁶ A filamentous, nonnitrogen-fixing blue-green alga, *Schizothrix calcicola* (Fig. 12), grew in only one triplicate 10-g sample of surface soil 500. A mold, *Penicillium* sp. (Fig. 13), was also found, except in sample 503. Several other molds were observed on the culture plates, but they were probably laboratory contaminants.

Most of the cultures grew slowly; they contained non-pigmented rather than pigmented colonies, and spreading rather than discrete colonies in the surface soils (Fig. 14). On agar plates, the colonies usually grew out from soil particles distributed on the surface of the agar (Fig. 15). Microorganisms did not grow well when the soil was diluted, whether in agar or in tubes of culture media. If growth did occur in the 10² dilution of 1 g of soil, it was all below the surface of the culture medium, indicating that growth took place under microaerophilic rather than aerobic conditions.

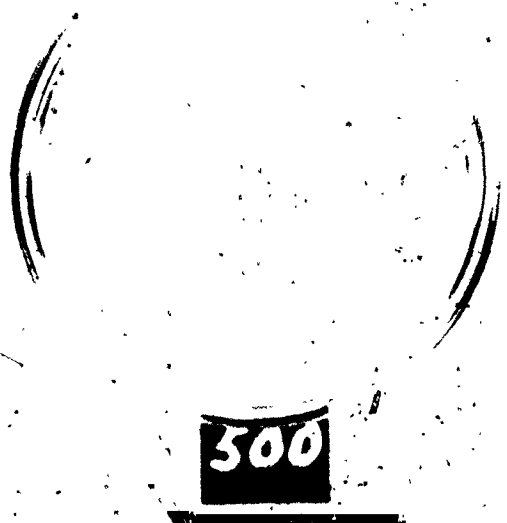


Fig. 12. Photomicrograph ($\times 1250$) of blue-green alga, *Schizothrix calcicola*, cultured from 10 g of surface soil sample 500



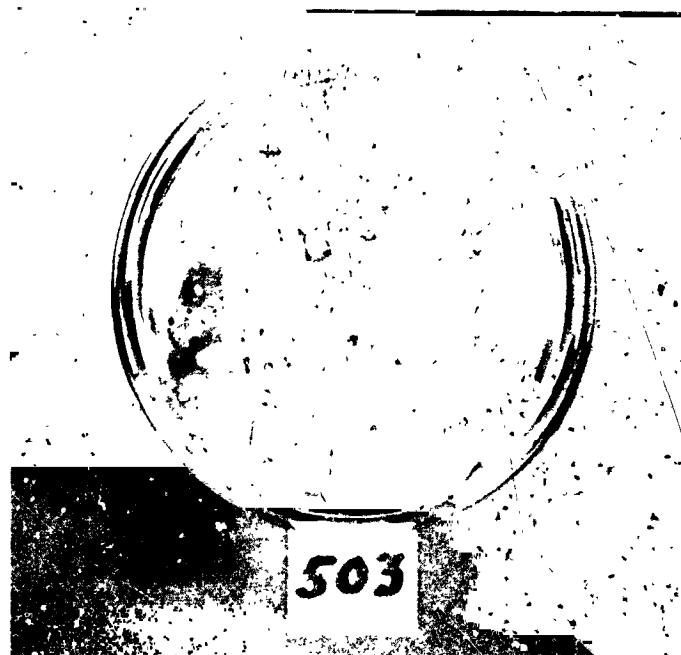
Photograph by C. L. Hall.

Fig. 14. Spreading pigmented and nonpigmented bacterial colonies (and one mold) cultured from 1 g of soil 500 sprinkled on surface of trypticase soil agar



Photograph by C. L. Hall, Virginia Polytechnic Institute, Blacksburg, at NSF Biology Laboratory, McMurdo Sound, Antarctica.

Fig. 13. Two colonies of *Penicillium* sp. cultured from 1 g of soil 500 sprinkled on surface of Rose Bengal agar



Photograph by C. L. Hall.

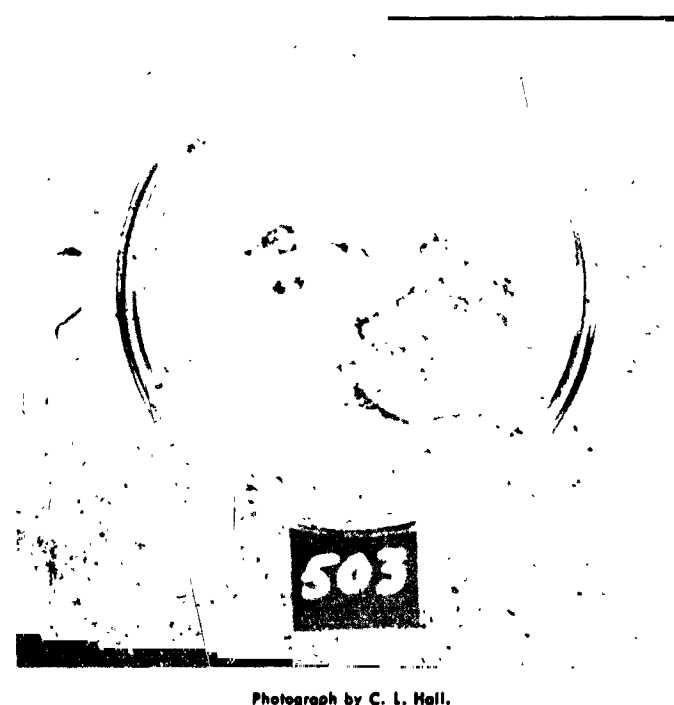
Fig. 15. Glistening bacterial colonies growing out from 1 g of soil particles sprinkled on surface of trypticase soy agar. Agar plate cultured at $+4^{\circ}\text{C}$ under anaerobic conditions

Although there were mesophiles which grew at room temperatures (Fig. 14), as well as microorganisms capable of growing at -4°C , the difference in abundance was not appreciable, as shown in Table 6. A thermophile capable of growing at $+55^{\circ}\text{C}$ (Fig. 16) was actually found in sample 503, from a depth of 1.5 ft. Since this was the only thermophile encountered, it may possibly be a contaminant that entered the sample during collection procedures. Some of the microorganisms may be heat-sensitive, and for subsurface soils 502 and 503, more bacteria were obtained by streak plates than by pour plates with warm agar (approximately 40°C).⁷

As shown in Table 6, there are as many, if not more, microorganisms below the surface than in the surface sample 500. This abundance of microorganisms was more apparent with fluid thioglycollate medium than with other media and was also evident when comparing streak plates with pour plates for these four soils.⁷

Gram stains of microorganisms cultured from the soil samples showed gram-positive, -negative, and -variable

⁷Streak plates prepared by R. E. Benoit, Virginia Polytechnic Institute, Blacksburg, Virginia.



Photograph by C. L. Hall.

Fig. 16. White bacterial colonies cultured at $+55^{\circ}\text{C}$ and growing out from 1 g of soil particles sprinkled on surface of trypticase soil agar

bacterial rods (Figs. 17 and 18) and gram-positive and gram-negative cocci (Fig. 19), especially gram-positive diplococci. Yeasts also made up a significant proportion of the microflora. Soil diptheroids and actinomycetes, which have been found in many other desert soils examined for this program, were not found in the four soil samples from McKelvey Valley.

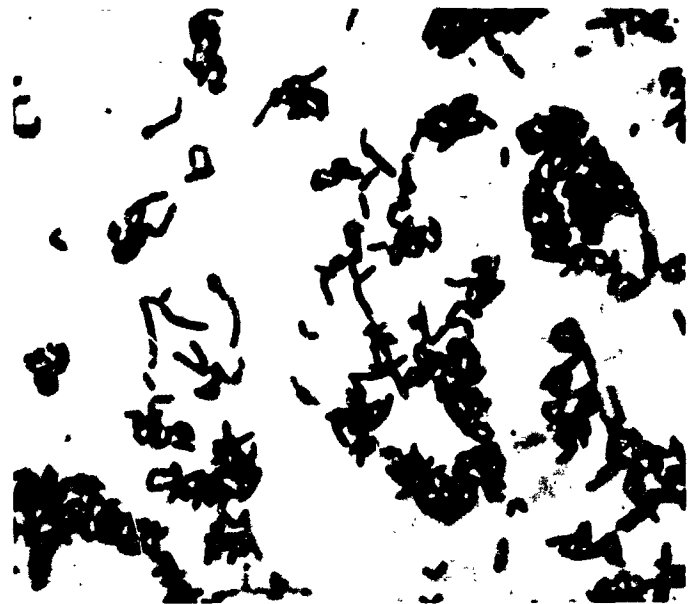


Fig. 17. Photomicrograph ($\times 1250$) of gram-variable bacterial rods cultured from soil 503. Isolate taken from agar plate illustrated in Fig. 15

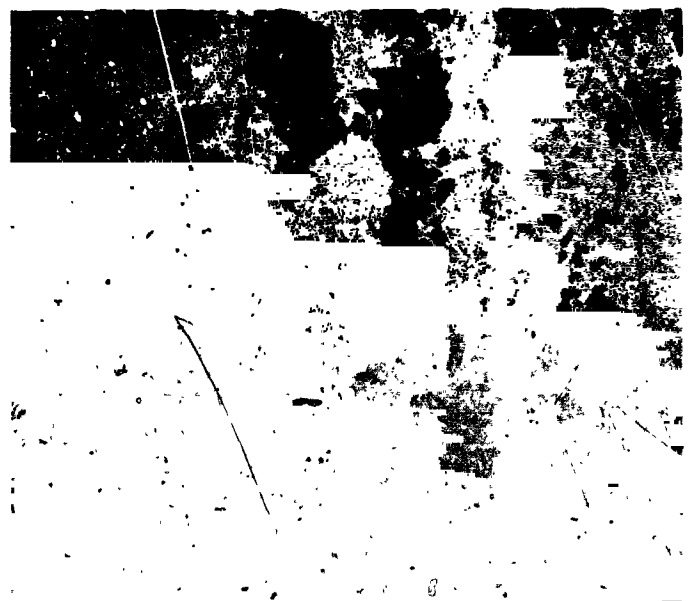


Fig. 18. Photomicrograph ($\times 1250$) of gram-positive and gram-negative bacterial rods cultured in fluid thioglycollate medium from soil 503

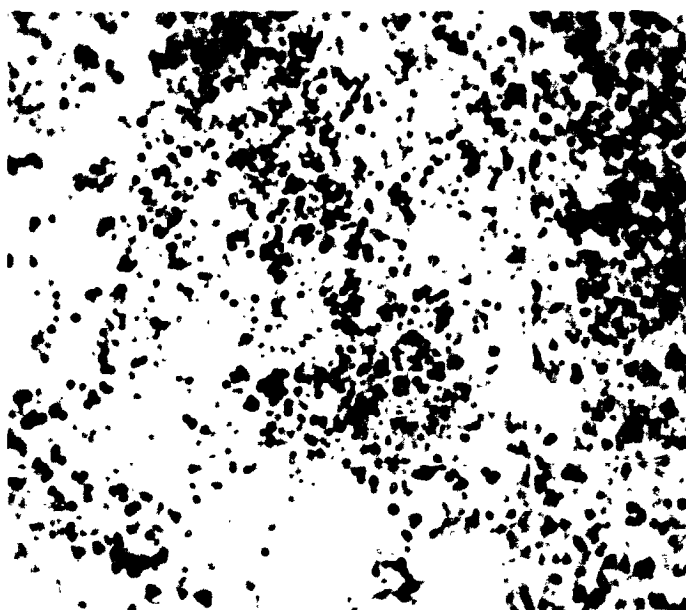


Fig. 19. Photomicrograph ($\times 1250$) of gram-positive and gram-negative bacterial cocci cultured from soil 500 at room temperature. Isolant taken from agar plate illustrated in Fig. 14

4. Concluding Remarks

Four samples from one soil site in McKelvey Valley, Antarctica were studied and their properties compared with values obtained for other desert soils. For most soil properties, there was generally not much variation with depth to hard, icy permafrost. Soil temperatures decreased with depth as the relative humidity and moisture values increased. Water availability, because of below freezing temperatures, was a definite limiting factor for activity of microorganisms.

All of the soil samples showed properties of a saline soil, with significant amounts of detectable Ca^{++} , Na^{+} , SO_4^{--} , NO_3^{-} , and Cl^{-} . There was a noticeable lack of water-soluble K^{+} , Mg^{++} , Mn^{++} , and PO_4^{--} . Determinations for pH, electrical conductivity, and *in situ* Eh indicated that the soil environment would be restrictive to some microorganisms, but favorable for salt-tolerant organisms or halophiles. The cation exchange capacity was quite low.

Although the soil site and general area in McKelvey Valley showed no evidence of human contamination prior to this study, the presence of a skua and one piece of desiccated seal skin indicated that contamination with some organics could have occurred. The winds that blew across the valley usually came from the direction of the sea and brought in some air contaminants, but these microorganisms appeared to be similar to other micro-

organisms (bacteria and fungi) cultured from the surface soil.

Abundance of common groups of microflora in these soils was very low or nil, and much lower than for typical United States desert soils. Some of the highest values were obtained with fluid thioglycollate. The abundances of algae, fungi, and sulfate-reducing microorganisms were among the lowest values encountered in any other desert soils previously analyzed. The actinomycetes, which were the most abundant populations in the extremely arid Chilean Atacama Desert, were not found in the samples of Antarctic soil, but the soil microflora contained a significant proportion of yeasts.

Mesophiles, as well as microorganisms with psychrophilic capabilities, and aerobes, anaerobes, and microaerophiles were present but not abundant, as determined by the culture methods used for this study. These microorganisms usually grew slowly, and were nonpigmented rather than colored colonies. More spreaders were found in the 6-in. surface of soil than in the samples procured from lower depths. There were as many, if not more, microorganisms in the subsurface samples than in the surface sample. This same phenomenon has also been observed for soils from the Atacama Desert and the Sahara Desert.⁶ However, in the samples from Antarctica, it was observed that heat sensitivity could be an important factor in obtaining viable microorganisms.⁷

Additional Antarctic soils from other dry valleys will be analyzed and studied. The abundances of microflora were determined for this study utilizing routine media, and may not have revealed the presence of specialized physiological groups, such as halophiles. These and other microflora will be sought later. Soil extracts of Antarctic materials will also be used in culture media. It is quite obvious that the ecology of Antarctic soil microflora is complex, and that the presence or abundance of soil microflora is limited by factors that are unfavorable for the survival, growth, and reproduction of organisms. The Antarctic dry valleys can provide valuable information prior to the search for extraterrestrial life in other harsh environments. *In situ* investigations, as well as laboratory studies, should not be neglected for the Antarctic continent.

B. Triaxial Jet Hydrogen Flame Ionization Detector, E. M. Bollin

During the investigation of pyrolysis of life-related compounds, it became necessary to design a hydrogen

ionization detector that could operate under conditions not generally encountered in gas chromatography. The flame detector was used to monitor the products from a differential thermal analysis (DTA)-effluent gas analysis (EGA) thermal head assembly. Several commercial flame detectors were evaluated. However, the DTA instrumentation and the manner in which it was used required the development of a specialized detector for convenience and flexibility.

The triaxial jet detector (Fig. 20) developed during this study proved to be very flexible and stable. The detector stabilized within 10 min after ignition, and was used unshielded, without a detector oven, and exposed to a strong fluctuating draft from air-conditioning. Normal gas flows were as follows: carrier, 30 to 90 cc/min; hydrogen, 60 cc/min; and air, 300 to 600 cc/min.

The detector was assembled from a Gelman 1-in.-diameter aluminum closed filter holder, stainless-steel tubing, and Swagelock fittings. The cap of the filter holder was thinned to provide clearance for the collector electrode. This was the only machining necessary. When the larger cap of the Gelman stainless-steel pressure filter funnel is used, no machining other than drilling and tapping is necessary. Modification of the Swagelock fittings noted in Fig. 20 may be achieved by drilling. The Microdet bulk-head connector will accommodate slight differences in collector loop wire size by modification of the spring ferrule tension. The fittings were assembled with Teflon front ferrules during initial adjustment and alignment.

Aside from the obvious ease and simplicity of construction of this detector, there are certain design concepts not immediately obvious that contribute greatly to the function of this device as a stable detector capable of operating well under conditions normally not acceptable, nor required, in standard gas chromatography work.

Since the inception of the device, the need for a laminar flame structure has been recognized by most workers in the field. The rather long vertical tube dimensions, which are required by the Swagelock fittings to assemble the triaxial jet, contribute to a great degree in ensuring that laminar flame conditions prevail. The tube length for each component is in the correct order for its respective sensitivity to nonlaminar conditions: the hydrogen, longest; the sample, intermediate; and air, shortest.

The design of the air jet contributes especially to the insensitivity of the device to fluctuating ambient conditions. The air flow may be increased to nearly blast

conditions without creating flame turbulence or an increase in background noise due to nonlaminar conditions. The flame is insulated from the environment by this rapidly moving annular sheet of collimated air. A sufficient amount of air diffuses toward the flame to support combustion. A short distance beyond the primary reaction zone the air flow pattern becomes divergent. This permits a considerable variation in the hydrogen flow and in the position of the collector ring. For any specific hydrogen flow and collector position, the air flow may be adjusted to maintain the collector just below incandescence. Below this point, fluctuations in flow rates have little effect upon background current.

The separate jet for carrier gas and sample allows a considerable variation in other parameters that normally affect a flame detector. Because of the large area of this jet orifice, rather large variations in carrier flow have very little effect upon the flame size and shape. In normal practice, carrier flow is restricted to the small diameter of the hydrogen jet and combined with the hydrogen flow. Therefore, it is competitive with the hydrogen in determining the configuration of the flame, and the sensitivity of a change in flow can be rather large. Although some doubt was experienced concerning the possible loss of sample by outward diffusion into the air stream, no such effect could be found. The diffusion of the air through the sample to the hydrogen results in molecular diffusion driven by a steep concentration gradient in such a manner as to contain the sample rather completely. The divergent cone of the air jet above the region of primary reaction also further contains the sample within the reaction zone. Another mechanism of sample entrapment is the gas acceleration (which may exceed 1000 g) due to the extreme temperature gradient at the primary reaction front of the flame. This serves very effectively to pull the sample into the reaction zone. When operating the detector just below incandescence of the collector, a small shift in base line is noticeable when the carrier gas flow is stopped completely. However, operation of the detector is not otherwise disturbed.

The triaxial configuration allows further freedom in detector design. Since a much smaller total volume of gas is handled by the hydrogen jet, the constraints of carrier gas irregularities do not have to be compensated by low pressure drop at the jet. Thus, the jet may be made much smaller than normal. This allows a higher velocity of hydrogen within the plane of the orifice while still maintaining a laminar flame, and moves the transport or preheat region slightly farther from the face of the jet. This results in a flame reaction front that tends to be less

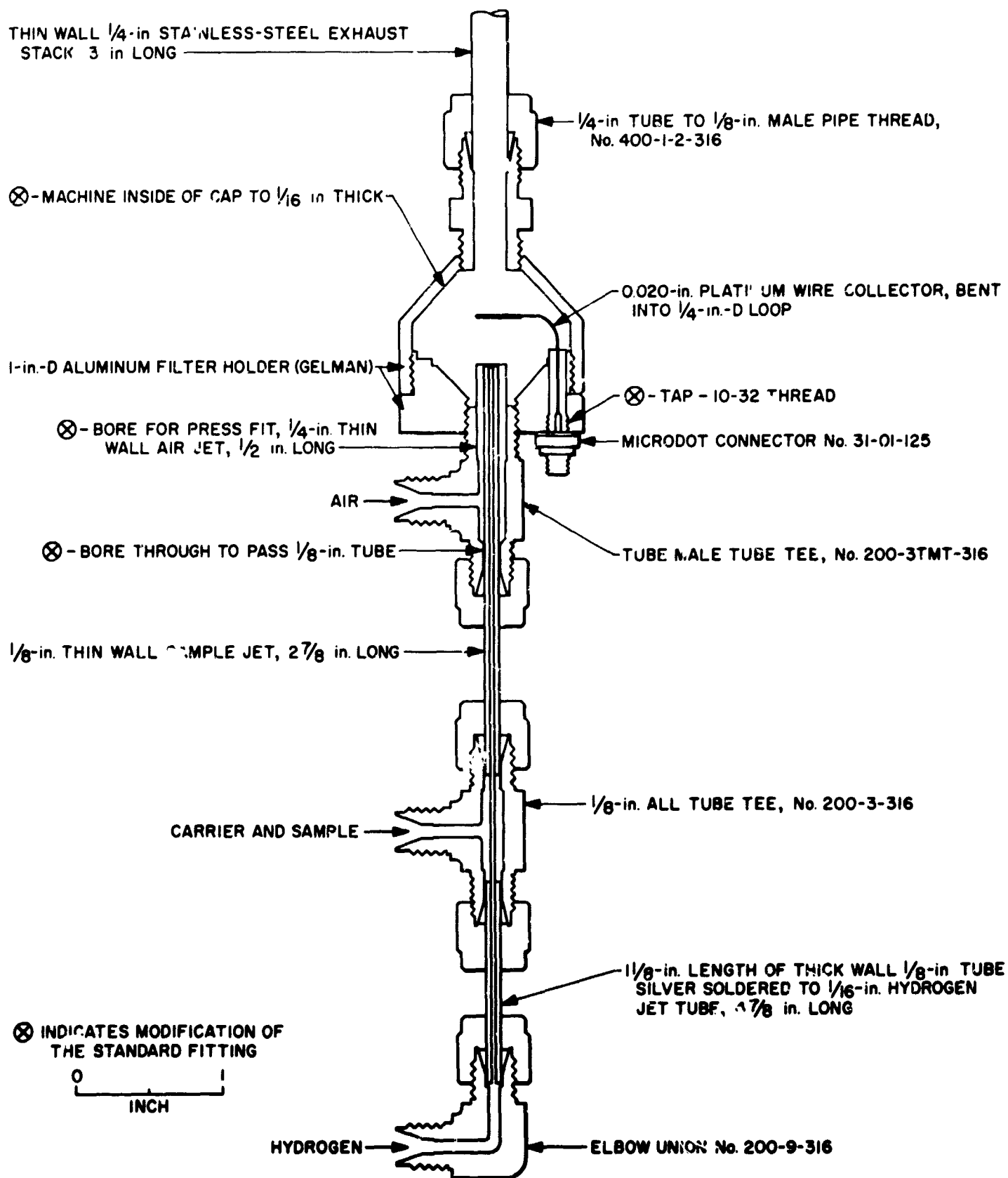


Fig. 20. Triaxial jet hydrogen flame ionization detector

efficient in heating the jet to incandescence, which is an important source of noise. With the smaller orifice, thicker walled tubing with a higher thermal mass, which contributes to the conduction of heat from the jet, may be used.

An additional feature of the triaxial configuration, and one that has not yet been examined, is the rather long path length over which the sample is exposed to ultra-

violet radiation before entering the flame reaction zone. The large area of the sample jet and the complete optical filling of its opening by the primary reaction zone of flame provide an approximate 3-in. length of tubing that is intensely illuminated by ultraviolet radiation. This could possibly excite many molecular species to a high level before entering the reaction zone. How or to what extent this mechanism may contribute to the performance of the detector is not known.

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XIX. Fluid Physics

SPACE SCIENCES DIVISION

N67-29160

A. A Kinetic Collision Model for Gas Mixtures,

M. T. Chahine

1. Introduction

Recent work (Ref. 1) with the Bhatnagar-Gross-Krook (BGK) collision model for one-particle distribution functions (Ref. 2) has raised interest in further extending this theory to multicomponent gas mixtures (Refs. 3-6). In essence, the BGK model equation approximates the Boltzmann collision operator by a relatively simple expression which retains most of the basic characteristic features of the full Boltzmann equation (BE).

In this phase of research, a generalization of the BGK model to gas mixtures has been developed. The resulting kinetic equation: (1) satisfies the laws of conservation of mass, momentum, and energy; and (2) reproduces the same collision frequencies and exchange of momentum and energy as for a gas mixture in which each constituent is composed of hard molecules and is separately in local Maxwellian equilibrium without necessarily being close to equilibrium with the rest of the gas. In the next phase of research, this model will be applied to the study of

the shock-wave flow problem in a binary gas mixture, using an iteration scheme similar to that adopted in Ref. 1.

2. Formulation of the Kinetic Model

To treat the case of mixtures of two particles i and j , we define a distribution function for each species, e.g.,

$$\frac{\partial f_i}{\partial t} = \frac{\delta f_{ii}}{\delta t} + \frac{\delta f_{ij}}{\delta t} \quad (1)$$

where $f_i(\mathbf{V}_i, \mathbf{x}, t)$ is the number of particles of the i th species in the range $d\mathbf{V}_i d\mathbf{x}$ around $\mathbf{V}_i$ and $\mathbf{x}$. Gross and Krook (Ref. 3) have postulated the following simple expression for the collision terms:

$$\frac{\delta f_{ij}}{\delta t} = n_j A_{ij} (F_{ij} - f_i) \quad (2)$$

with

$$F_{ij} = n_i \left(\frac{m_i}{2\pi k T_{ij}} \right)^{3/2} \exp \left[-\frac{m_i}{2k T_{ij}} (\mathbf{V}_i - \mathbf{U}_{ij})^2 \right]$$

The mass density $m_i n_i$, average velocity U_{ii} , and kinetic temperature T_{ii} are the first moments of the distribution function

$$\begin{bmatrix} m_i n_i \\ n_i U_{ii} \\ 3kT_{ii} \end{bmatrix} = \int f_i \begin{bmatrix} m_i \\ V_i \\ m_i (V_i - U_{ii})^2 \end{bmatrix} dV_i \quad (3)$$

To complete this system of equations, we need to determine the expressions A_{ij} , U_{ij} , and T_{ij} .

We chose A_{ij} such that the number of collisions of particles of the i th type with others of the j th type is the same as for a mixture of hard molecules whose constituents are separately in local Maxwellian equilibrium. For the hard molecule case, the number of collisions N_{ij} per unit volume and time can be evaluated for $U_{ii} \times U_{jj} = 0$ to be

$$N_{ij} = \frac{n_i n_j \sigma_{ij}}{(\pi a)^{1/2}} \left[(1 + 2M^2) \frac{\pi^{1/2} \phi(M)}{2M} + e^{-M^2} \right] \quad (4)$$

where

$$a = \left(\frac{2kT_{ii}}{m_i} + \frac{2kT_{jj}}{m_j} \right)^{-1}$$

$$M = a^{1/2} (U_{jj} - U_{ii})$$

= diffusion Mach number

$$\sigma_{ij} = \pi (r_i + r_j)^2$$

$$= \text{collision cross-section for hard molecules of radii } r_i \text{ and } r_j$$

$$\phi(M) = \text{erf}(M)$$

Equating Eq. (4) with the number given by the model equation, which is $n_i n_j A_{ij}$, we obtain

$$A_{ij} = A_{ji} = \frac{\sigma_{ij}}{(\pi a)^{1/2}} \left[(1 + 2M^2) \frac{\pi^{1/2} \phi(M)}{2M} + e^{-M^2} \right] \quad (5)$$

For the self-collision frequency, $M = 0$, and the above expression reduces to

$$A_{ii} = 2\sigma_{ii} \left(\frac{4kT_{ii}}{\pi m_i} \right)^{1/2} \quad (6)$$

3. Conservation Laws and Relaxation Properties

The model equation directly satisfies the conservation of mass for each constituent, as well as for the whole mixture. The momentum and energy are not instantaneously conserved, since they are exchanged from one species to the other by cross collisions.

The time rate of change of the i th component of momentum resulting from collisions with the j th component is

$$\frac{\partial}{\partial t} (m_i n_i U_{ii}) = m_i \int V_i \frac{\partial f_i}{\partial t} dV_i = m_i n_i n_j A_{ij} (U_{ij} - U_{ii}) \quad (7)$$

The conservation of total momentum becomes

$$m_i (U_{ij} - U_{ii}) + m_j (U_{ji} - U_{jj}) = 0 \quad (8)$$

Similarly, the conservation of total energy is

$$\begin{aligned} (T_{ij} - T_{ii}) + (T_{ji} - T_{jj}) + \frac{m_i}{3k} (U_{ij}^2 - U_{ii}^2) \\ + \frac{m_j}{3k} (U_{ji}^2 - U_{jj}^2) = 0 \end{aligned} \quad (9)$$

Eqs. (8) and (9) form a set of four equations with eight unknowns U_{ij} , U_{ji} , T_{ij} , and T_{ji} ; therefore, we need four more independent relationships to complete the system. These additional relationships will be derived from a study of the relaxation properties.

In the case of a mixture of hard molecules where each of the species i and j is in local equilibrium with itself, we can express the time rate of change of momentum and energy as

$$\begin{aligned} \frac{\partial}{\partial t} (m_i n_i U_{ii}) &= \frac{m_i m_j n_i n_j \sigma_{ij}}{(m_i + m_j) (\pi a)^{1/2}} (U_{ij} - U_{ii}) \\ &\times \left[\left(2 + 2M^2 - \frac{1}{2M^2} \right) \frac{\pi^{1/2} \phi(M)}{2M} \right. \\ &\left. + \left(1 + \frac{1}{2M^2} \right) e^{-M^2} \right] \end{aligned} \quad (10)$$

a.d

$$\frac{\partial}{\partial t} \left(\frac{3}{2} n_i k T_{ii} \right) = \frac{4m_i m_j n_i n_j \sigma_{ij}}{(m_i + m_j)^2 (\pi a)^{1/2}} \left\{ k(T_{jj} - T_{ii}) \left[(1 + 2M^2) \frac{\pi^{1/2} \phi(M)}{2M} + e^{-M^2} \right] + \frac{m_j (U_{jj} - U_{ii})}{2} \left[\left(1 + M^2 - \frac{1}{4M^2} \right) \frac{\pi^{1/2} \phi(M)}{2M} + \left(\frac{1}{2} + \frac{1}{4M^2} \right) e^{-M^2} \right] \right\} \quad (11)$$

Equating Eqs. (10) and (11) with their corresponding values for the model, we obtain the additional four relationships. Solution of this system yields

$$U_{ij} = U_{ii} + \alpha \frac{m_j}{(m_i + m_j)} (U_{jj} - U_{ii}) \quad (12)$$

$$T_{ij} = T_{ii} + 2 \frac{m_i m_j}{(m_i + m_j)^2} \left[\frac{4}{3} (T_{jj} - T_{ii}) + \frac{\alpha}{2} (2 - \alpha) \frac{m_j}{3k} (U_{jj} - U_{ii})^2 \right] \quad (13)$$

where

$$\alpha = 1 + \frac{\left(1 - \frac{1}{2M^2} \right) \frac{\pi^{1/2} \phi(M)}{2M} + \frac{1}{2M^2} e^{-M^2}}{(1 + 2M^2) \frac{\pi^{1/2} \phi(M)}{2M} + e^{-M^2}} \quad (14)$$

For $M \rightarrow 0$, $\alpha \approx 4/3$; for $M \rightarrow \pm \infty$, $\alpha \approx 1.0$.

4. Conclusion

The collision model given by Eq. (2) with parameters as defined in Eqs. (3), (5), (6), (12), and (13) forms a closed mathematical system of integro-differential equations which is much simpler than the full BE. The physical implications of the BGK collision operator are rather clear: It assumes that the molecules, say, of the i th constituent emitted into the velocity range dV_i around V_i are the results of self, ii , and cross, ij , collisions. Furthermore, these colliding particles enter the range dV_i with a Maxwellian distribution of velocities centered around average velocities U_{ii} and U_{ij} and with kinetic temperatures T_{ii} and T_{ij} for self collisions and cross collisions, respectively.

B. The Flow Creating a Concentration of Vorticity Over a Stationary Plate, T. Maxworthy

1. Introduction

When examining the interaction of a tornado with the ground, we are interested in the boundary-layer flow created by the tornado and, as it turns out, in the nature of the rotating outer flow itself. In most boundary-layer problems, the latter is assumed to be known; however, in this case, apparently it is not. Several complicated interactions occur: Since the boundary layers formed must have a smaller angular velocity than the outer flow, they cannot support the outer-flow radial pressure gradient, and a radial inflow is formed. As this converging inflow approaches the center of the tornado, it must eventually erupt into an axial flow over some finite radius. This erupting flow must, in turn, interact directly with the outer flow and alter it in some way. As will be shown, the outer flow has an intense, meridional flow in the neighborhood of the intersection between the wall and the center of rotation. This, in turn, interacts with the boundary layer to not only modify it, but to also, in its turn, be modified by the boundary-layer efflux. After many such interactions, this complicated mechanism finally gives the flow which is observed in the laboratory and, hopefully, is present in nature. The many qualitative attempts to understand the phenomena have been reviewed by Fultz (Ref. 7, which also contains an extensive bibliography on the subject). Ingenious solutions to the problem of creating a suitable convergence of vorticity have been demonstrated, and many realistic-looking tornadoes have been produced. Most of the work did not produce any quantitative results, and it is only in the past few years that a few attempts have been made to do so. Most of these have had their origin in trying to understand the "vortex tube," but, because of their special character, they do not seem to have any direct application to our present problem.

The most recent study of the tornado interaction is that of Turner (Ref. 8). He has approximated the convective

driving mechanism of a real tornado by bubbling air, close to the upper surface, into the center of a rotating tank of water. A strong vortex is formed; however, as a model, it suffers from several disadvantages which he well realizes. It is rotating over a rotating surface, which has a strong effect on the nature of the interaction there. Also, it is surrounded by essentially solid-body rotating flow, which has a strong constraining effect on any radial motion. Thus, axial motion is confined to a central upflow and compensating annular downflow just outside of it. Axial flow beyond about two or three core diameters is zero. In a real tornado, it is unclear as to whether the upward mass flow is not redistributed at considerable radial distances, and whether any evidence of downward axial flow just outside of the central core represents only a part of the total upward mass flux right at the center. In fact, in the present case where mass is being introduced with swirl at large radial distances, the boundary-layer flow from large distances, by representing a significant amount of the exit flow, in a very real sense controls the intensity of the vortex. Furthermore, as the present study shows, some doubt must be cast upon the use of the tangential velocity profile (and its associated pressure gradient) as the *only* driving force for the boundary layer. There is even a real question whether boundary-layer theory is ever valid right at the center of rotation. There the flow turns very rapidly, is ejected, and goes through a breakdown process before it forms the core of the vortex, normally seen far above the surface, all within a distance of the order of the boundary-layer thickness!

2. Apparatus and Experimental Technique

The apparatus used, shown in Fig. 1, is similar to many other devices which have been used to look at the "tornado" phenomenon. It consists of a cylindrical, 11-in.-diameter, 6-in.-deep Lucite tank filled with water. A central sink is connected through a pump and flowmeter to three tangential jets arranged around the periphery. (No attempt has yet been made to simulate the thermodynamic processes of a real tornado, and the present arrangement is probably not the optimum one for observing this phenomenon.) To test some simple physical intuitions concerning the flow, another experiment was performed with the same sink and jets in a tank having approximately three times the diameter and the same depth as that shown in Fig. 1.

Two methods were used to observe the flow in both tanks: (1) dye injection into the boundary layer and main flow to obtain qualitative information about the flow pattern, and (2) the "hydrogen bubble" technique to ob-

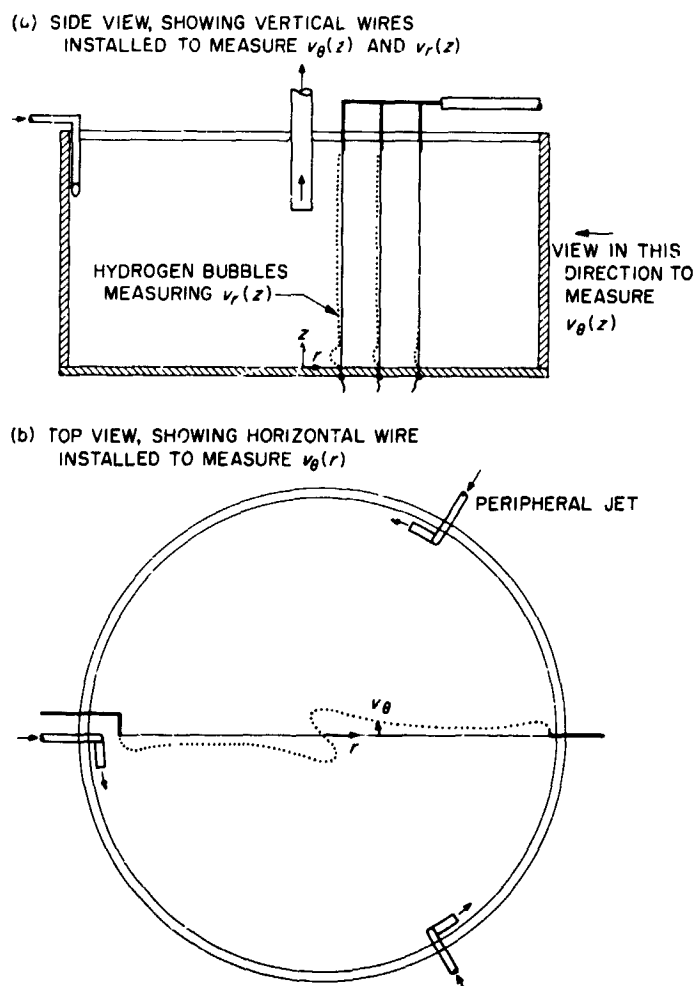


Fig. 1. Apparatus used to investigate the tornado phenomenon

tain quantitative information about the velocity distributions in regions of interest. This has been described in detail by Schraub, *et al.* (Ref. 9). The technique used to measure the radial variation of the tangential velocity, i.e., $v_\theta(r)$ in a cylindrical coordinate system (z, r, θ) , is shown in Fig. 1(b).

In Fig. 1(a), a more complicated geometric situation is shown. Three vertical wires are stretched through the bottom of the tank and are supported, close to the free surface, by 0.040-in.-diameter stainless-steel prongs. By radially viewing these wires and the bubbles produced, we can measure $v_\theta(z)$; by viewing tangentially, we can measure $v_r(z)$ for the three radial locations indicated in the elevation of the apparatus.

3. Results and Discussion

Since most of the experiments were performed in the tank having an 11-in. diameter and a 6-in. depth, only the

exceptions to this case will be noted. Dye injection in various parts of the tank gave the first indication of the nature of the flow field. When dye was introduced through the bottom of the tank into the part of the boundary layer very close to the wall, it moved directly to the center of the tank, was ejected vertically upward, and then spread violently outward to larger radii. After a short axial distance of considerable agitation, a new state was reached, with the dye reappearing at the center of rotation; this state persisted to the exit. Such an ejection and breakup of the dye trace is shown in Fig. 2(a).

If dye was injected further out into the boundary layer, it still moved to the center, but erupted a little before it arrived there; it then wrapped itself around the central stem of Fig. 2(a) into a helix of large pitch angle, at least 45 deg or greater. Anticipating the results of the quantitative velocity measurements to be described, the breakdown, in fact, occurred at the outer limit of the boundary layer. All of the inflowing boundary-layer fluid took part in the breakdown process and was the fluid that eventually made up the central part of the vortex core that was sucked out of the sink tube. This central breakdown¹ has so many of the features of the so-called "vortex break-

¹It appears that this phenomenon has been most convincingly brought to the attention of the scientific community in a film entitled "Secondary Flows in Fluid Mechanics" by E. S. Taylor of the Massachusetts Institute of Technology. Study of several of the very early works on tornado flow reveals that Wilcke (Ref. 13; more recently reported in Ref. 7) was probably the first to observe and sketch the breakdown phenomenon, and that it was rediscovered several times thereafter.

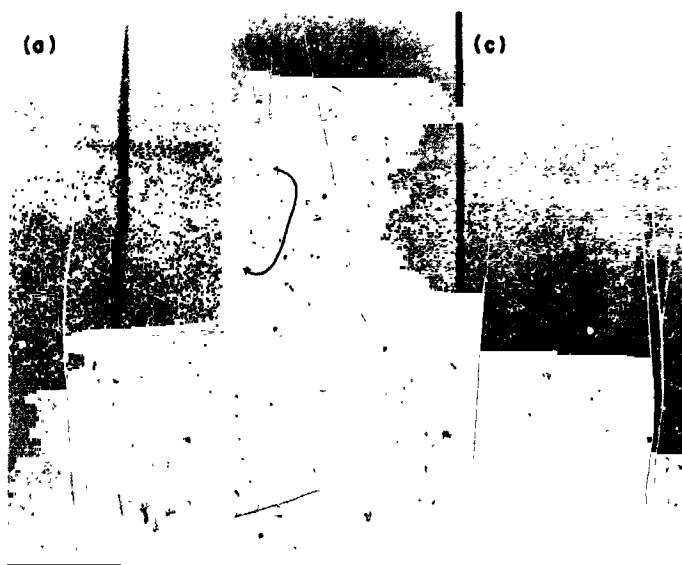


Fig. 2. Vortex breakdown

down" phenomenon [according to Benjamin (Ref. 10) and many others] that it seems certain to be of the same generic type as the disturbances seen by Harvey (Ref. 11), Elle (Ref. 12), and numerous other workers under a variety of circumstances.

As the flow rate (hence, the vortex strength) was increased, the breakdown became more violent (Fig. 2b and 2c) and occurred closer to the wall, since the boundary-layer thickness was decreased, partially in response to the increased velocity of the outer tangential flow. The flow after the breakdown was spread over a larger radius and was mixed with some of the fluid originating from outside the core. The breakdown, if the sink were lowered toward it, could eventually be made to jump inside the tube. If the tube were then raised, the breakdown would move some distance before it would pop out and return to its original position. During this process, the boundary-layer flow and initial supercritical eruption did not change to the accuracy that would enable one to make that determination; the implication here is that, with the tube far removed from the wall, the back pressure is so great that the jump moves as far upstream as it can go, and, as this back resistance is decreased, the breakdown can move downstream to some new position of equilibrium (cf., the behavior of a hydraulic jump as the downstream water height is externally varied).

With the diameter of the sink changed, but at the same high flow rate, the following observations were made: With a 1½-in.-diameter sink, the breakdown occurred and the final diameter of the exit dye column was of the order of the tube diameter; with a ½-in.-diameter tube, the final core flow was reduced in diameter again to the order of the tube diameter; and with a ¼-in.-diameter tube, the core just after the breakdown was still of the order of ½-in. diameter, reducing to ¼-in. as the flow exited through the small pipe. From these tests, it again seems that the flow downstream from the breakdown phenomenon was controlled to some extent by the conditions there; however, the existence of a minimum core size shows that this is not entirely true, and that some dynamic process (presumably viscous forces) becomes important when the velocity gradients tend to become too great.

Having investigated the effects of varying downstream conditions, we turn to the more practically important case of varying the upstream conditions. Because the central core is made up of boundary-layer fluid (probably mixed with fluid from the outside in the case of the more violent breakdowns), it seems probable that a change in the character of the boundary layer will have a considerable effect

on the initial supercritical flow and, hence, on the final subcritical exit flow. These changes can be made in several ways: If a circular wire-screen barrier is placed on the ground around the center of rotation, the boundary layer is forced over and through this resistance. Both angular and radial momentum are changed, and the final vortex flow is considerably weaker than that at the same flow rate but with no barrier. The effect of a simple, circular loop of thick wire, of the same height as the barrier, is not as great, presumably because the angular momentum of the boundary-layer flow is less affected by passing *over* a solid barrier than by passing *through* a stationary, porous screen. However, the most dramatic demonstration of the boundary layer's control over the vortex strength can be made by placing the same jets and sink in a tank having a diameter of 3 ft and a depth of 6 in. At the same flow rate as in the smaller tank, we are introducing about three times the initial angular momentum into the flow; however, since the boundary layer has to flow at least three times farther, the net result is that the vortex strength is considerably reduced, maximum angular velocities being lowered by at least a factor of 5. This great sensitivity to the boundary-layer flow suggests at least one possible means of controlling the growth of a real tornado with the expenditure of a minimum amount of energy. The method can be demonstrated by two more qualitative experiments in widely different pieces of apparatus.

In the present equipment, the introduction of a violent disturbance into the boundary layer, a jet of water from a syringe, destroyed the coherence of an intense vortex for a considerable length of time, although eventually the flow was able to reorganize itself into its original

form. With the "fire-storm" apparatus of Prof. H. W. Emmons at Harvard University, a very large and intense vortex can be created through the induction of an inflow of angular momentum by burning a pool of liquid fuel at the center of a table surrounded by a rotating porous screen. Here again, the character of the vortex can be greatly affected by blowing a puff of air into the boundary layer and upsetting its normal character.

One further dye study gave a qualitative picture of the rest of the flow field outside the boundary layer and the inner region of the concentrated core. For the case of a weak, laminar breakdown, apart from the obvious motion in the boundary layer, several other features of interest were found. The outer region of the concentrated core is made up of fluid that has followed a zig-zag, up-and-down path through the main body of fluid. The up-and-down flows are turned around as they approach the stationary wall and the free surface, where they interacted in some way with the boundary layers on the bottom and the constant pressure surface above. When the intensity of the vortex is increased, the upward and downward flows are also increased in intensity, and the annular region in which they take place decreases in width so that more regions of axial flow occur. Experience in vortex-tube experiments suggests that these regions can become very narrow, and they have been mistaken for evidence of boundary-layer separation. Further discussion on these points will be postponed until more evidence is presented.

Fig. 3 shows a hydrogen-bubble picture of $v_\theta(r)$ at the middle level of the tank and at a known flow rate. This shape persists at all vertical levels (z) above the boundary



Fig. 3. Hydrogen-bubble picture of $v_\theta(r)$

layer to the accuracy of the measurements. This statement is further verified by the shape of the $v_a(z)$ profiles in Fig. 4, which were taken at one radial station. The shape shown in Fig. 3 is characteristic of the $v_a(r)$ profiles for all inlet flow rates. Its most interesting feature is the extensive region of almost constant velocity spanning the annular region between the jets and the sink and a second

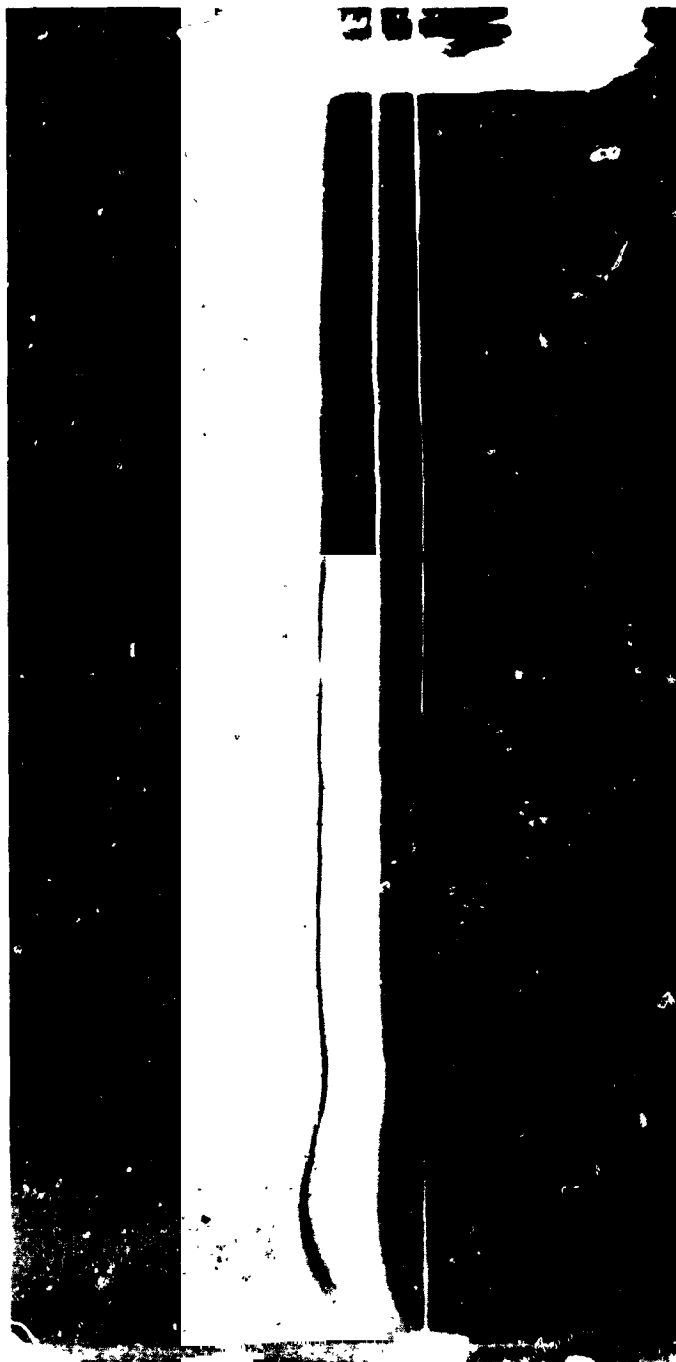


Fig. 4. Hydrogen-bubble picture of $v_a(z)$ at one radial station

maximum in the velocity close to the outer edge of the tank. This is undoubtedly caused by the way we have returned the inlet jet flow to the container; the fluid chooses to return as some form of thick sidewall boundary layer. Since this is only a function of the method, we have chosen to put fluid back into the outer flow and bottom boundary layer; since the flow close to the center is not critically dependent on how this is done, we will ignore further details of this region in what follows.

The most significant calculation we can perform with these profiles is to compute the boundary-layer flow that would be expected with such an outer flow. The most accurate determination of these flows is contained in the extensive work of Mack (Ref. 14) for power-law outer distribution ($v_a \sim r^n$) over a disc of finite radius, and one assumes that the centrifugal pressure gradients of such flows are the driving mechanisms for the boundary layer. Unfortunately, our profiles are not of this simple form, and we are forced to use some more indirect calculation scheme. Rott and Lewellen (Ref. 15) have developed an approximation which depends only on the local values of v_a and r . They assume that the mass flow Q in the boundary layer is a function of the local circulation, $\Gamma = v_a r$, times a universal function of r , which automatically accounts for the boundary-layer history up to the point in question. Thus,

$$\frac{Q}{R_0 (\Gamma_0 v)^{1/2}} = \frac{\Gamma}{\Gamma_0} \left\{ 1.26 \left[1 - \left(\frac{r}{R_0} \right)^{4/3} \right]^{3/4} \right\} \equiv \frac{\Gamma}{\Gamma_0} F \left(\frac{r}{R_0} \right) \quad (1)$$

where R_0 is the outer radius of the disc and Γ_0 is the circulation at that radius. When $\Gamma = \Gamma_0 = \text{constant}$, i.e., a potential vortex, the universal function agrees very closely with the series solution of Mack, and it is possible to replace $F(r/R_0)$ with his more precise calculation. Since $\Gamma_0 = 0$ at R_0 for our flow, we have actually chosen to use the maximum value of Γ to represent Γ_0 , which occurs inside the boundary region on the vertical cylindrical wall. Then, R_0 is the radius at this maximum value of Γ_0 . Calculations based on Eq. (1) and the experimental measurements of $v_a(r)$ are shown in Fig. 5, superposed on Mack's calculated curves. Also shown are experimental measurements of Q based on the $v_a(z)$ profiles found using three vertical hydrogen-bubble wires (Fig. 6). The discrepancies are too startling to be ignored. There are three possible explanations for these differences:

- (1) The theory requires the boundary layer to start with zero thickness and zero mass flow at $r = R_0$, whereas in reality it may have a considerable starting thickness

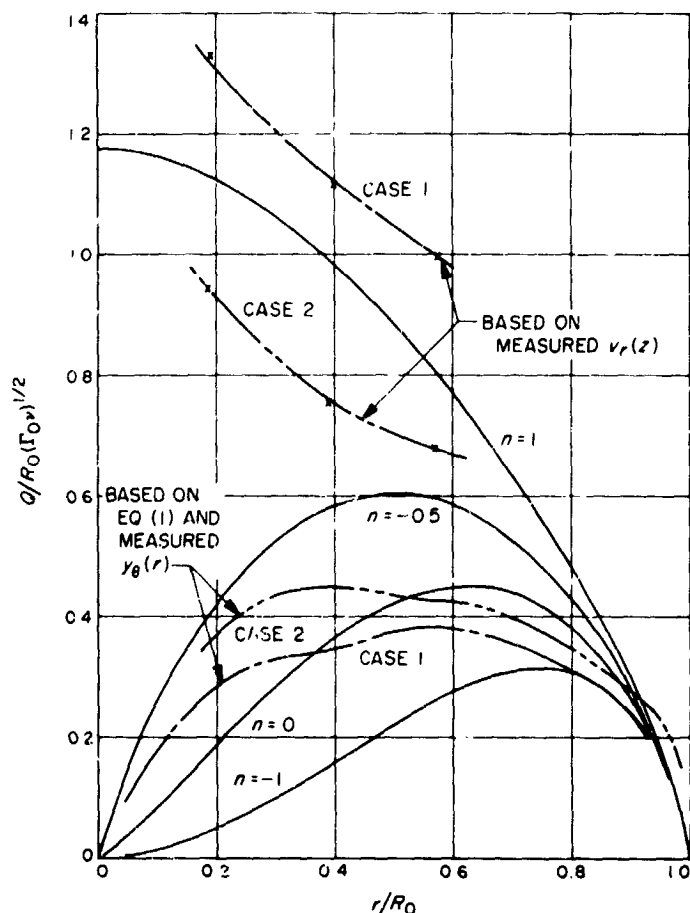


Fig. 5. $Q/R_0 (\Gamma_0 v)^{1/2}$ vs r/R_0

and mass flow. However, this probably does not completely explain the trend of the mass flow with decreasing r . Theory always has Q decreasing in the region of interest (i.e., $r < 7.5$ cm), whereas the experiment shows that Q invariably increases as we approach the concentrated vortex.

(2) This means that more fluid is entering the boundary layer than boundary-layer calculations can allow for, which, in turn, implies that the pressure gradient being imposed on the layer can no longer be solely given by a balance of centrifugal acceleration and pressure gradient in the outer flow. To see that this in fact could be so, we look at the radial equation of motion in the outer flow:

$$-\frac{1}{\rho} \frac{\partial p}{\partial r} = -\frac{v_z^2}{r} + \left(-\frac{v_r^2}{r} + v_z \frac{\partial v_r}{\partial z} - v_r \frac{\partial v_z}{\partial z} \right) \quad (2)$$

All of the extra terms, i.e., those in parentheses above, are known to exist, but experimentally it is hard to determine

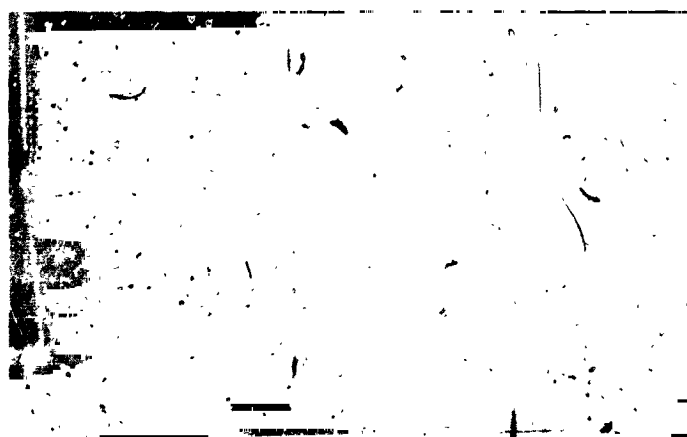


Fig. 6. Hydrogen-bubble picture of $v_r(z)$ at three radial stations

their possible order of magnitude. Observations of the central breakdown region indicate that it has very large axial and radial velocity gradients; e.g., the axial velocity on the centerline changes from a large value, associated with the boundary-layer eruption, to zero after the jump, in a distance of the order of a boundary-layer thickness. Therefore, the terms of Eq. (2), other than v_z^2/r , could combine to increase the outer-flow pressure gradient towards the value for a free vortex, as is required in Case 1.

(3) Since a considerable axial flow is imposed upon the boundary layer, it is also necessary to reexamine the assumptions that have been incorporated into the boundary-layer concept. In the axial equation of motion, it is usually assumed that Δp , the pressure difference across the layer, is small. However, in the present case, we may anticipate that this may no longer be so, since terms like $v_z \Delta v_z$, where Δv_z is the axial velocity difference across the layer, may now become of considerable importance within the layer. Similarly, terms in the equations of motion which are ignored in boundary-layer theory, e.g.,

$$v \left[\frac{\partial^2}{\partial r^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \right) \right], v_\theta, v_r, v_z$$

may well become overwhelmingly important as $r \rightarrow 0$ for any solution which is not of the similarity type. This means, of course, that conventional boundary-layer theory is no longer valid for these flows, a statement which requires further careful measurements before it can be taken very seriously. These measurements will be reported in future issues of the SPS, Vol. IV.

4. Conclusions

The major results and conclusions can be summarized as follows:

- (1) The tangential velocity profile of the swirling flow of a concentrated laboratory vortex has been measured.
- (2) Boundary-layer flow calculations based on these profiles do not agree with direct measurements of the radial mass flux in the boundary layer.
- (3) This discrepancy can be corrected by allowing for an intense local meridional flow impressed on the boundary layer, in the neighborhood of the axis, which increases the layer's transport of matter.
- (4) In turn, as this boundary-layer material approaches the center of rotation, it erupts to produce this same intense meridional circulation, a "vortex breakdown" similar to the type that described other circumstances (e.g., swirling flow in tubes and flow over delta wings at angle of attack).
- (5) The strength of the vortex is dependent on the history of the boundary layer, thus pointing to possible methods of controlling tornadoes by disturbing the boundary layer at the ground and destroying the tornado's main source of angular momentum.

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XX. Physics

SPACE SCIENCES DIVISION

N67-29161

A. A Spectrometer for Simultaneous Observation of the Spectra of Two Nuclei of Different Magnetogyric Ratios, D. D. Elleman, S. L. Manatt, and C. D. Pearce

To perform field-sweep heteronuclear double-resonance experiments, one must consider the difference in relative sweep distances (as measured in field units) introduced by the difference of gyromagnetic ratios of the sets of nuclei being studied. Simultaneous observation of the two nuclei would greatly facilitate this type of experiment (SPS 37-41, Vol. IV, pp. 180, 181). We describe herein some modifications to our standard nuclear magnetic resonance (NMR) probes that make this technique feasible.

Ideally, a heteronuclear double-resonance experiment requires a multiple-frequency synthesizer spectrometer with nuclear field stabilization. Therefore, a single spectral line can be continuously observed while one or more types of nuclei in the sample molecule are irradiated by changing the other relevant frequencies. Experiments of this type, previously referred to as INDOR (Ref. 1) experiments, can give significant information on the nuclear-

spin energy level schemes and on relaxation phenomenon. It is possible to realize some of the advantages of the ideal spectrometer mentioned above in simple field-sweep double-resonance experiments without the extra complication of nuclear field stabilization. The requisite components and modifications to perform such experiments are: (1) two Varian 4311 RF units of the appropriate frequencies, (2) frequency synchronization between the two RF units, (3) double tuning of the Varian 4331A probe transmitter circuit, (4) double tuning of the Varian 4331A probe receiver circuit, and (5) a convenient means for simultaneously displaying the two nuclear signals when the field is swept.

The basis of our system is a Varian HR-60. In one disposition, the Varian 4311 units were at frequencies of 24.3, 56.4, and 60.0 Mc for a number of ^{31}P -, ^{19}F -, and ^1H -containing molecules which we have been studying. The master oscillator crystals were removed from these units and replaced by two Hewlett-Packard 5100A frequency synthesizers. The synthesizers were driven from the same Hewlett-Packard 5100A driver unit. The receiver circuit of a 60-Mc Varian 4331A probe was modified as shown in Fig. 1(a) so as to give a good signal-to-noise

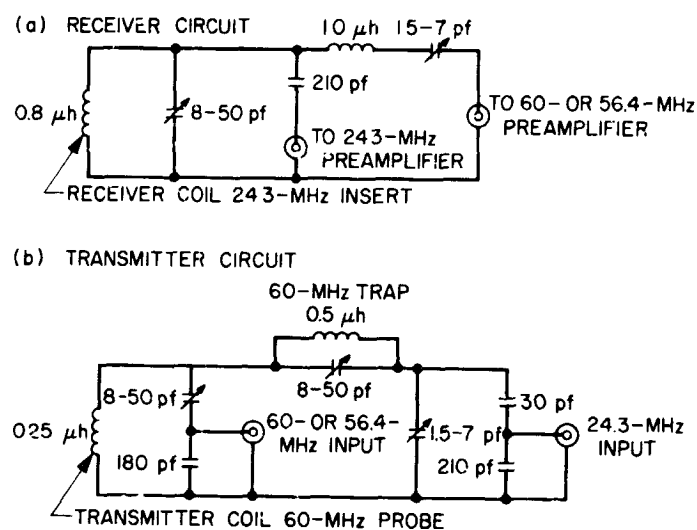


Fig. 1. Receiver and transmitter circuits of Varian 4331A probe modified for dual-frequency operation

value for 24.3 Mc and a somewhat smaller signal-to-noise value at 56.4 Mc and 60.0 Mc. We estimate that the signal-to-noise value at 24.3 Mc was down a factor of only 2 to 3 from that of a standard 24.3-Mc probe. A 24.3-Mc receiver insert was used in the 60-Mc probe so some of the signal-to-noise loss at the proton and fluorine frequencies was gained back from the lower frequency insert, which has

about four to five times more turns of wire than 60-Mc inserts. The transmitter circuit of the 60-Mc probe was modified as shown in Fig. 1(b) so as to resonate at 24.3 and either 56.4 or 60 Mc. The additional probe components were housed in two aluminum boxes attached to the probe (Fig. 2). To obtain more RF power in certain decoupling experiments, the output of either Varian 4311 unit was coupled to a Boonton 230A power amplifier, whose output was fed to the probe transmitter circuit. Usually, in simple field-sweep experiments, it was possible to record spectra directly from the output of the Varian 4311 RF phase detectors; however, for certain experiments it was necessary to lock the field through a synchronous detector locked on the tetramethylsilane proton line.

Some simultaneously recorded ^{31}P and ^1H spectra of trimethylphosphite, $(\text{CH}_3\text{O})_3\text{P}$, are shown in Fig. 3(a,b,c). Positive and negative signal enhancements of greater than 10 are evident in Fig. 3(b) and 3(c), respectively. In Fig. 3(d), the ^{31}P frequency-sweep spectrum of this compound was recorded while the field was locked on a tetramethylsilane line and one of the proton lines was irradiated with an audio modulation appropriate to give the most pronounced intensity perturbations of the least intense lines. These are barely visible in Fig. 3(a). The ^{31}P center-band frequency was swept by feeding a negative ramp from a

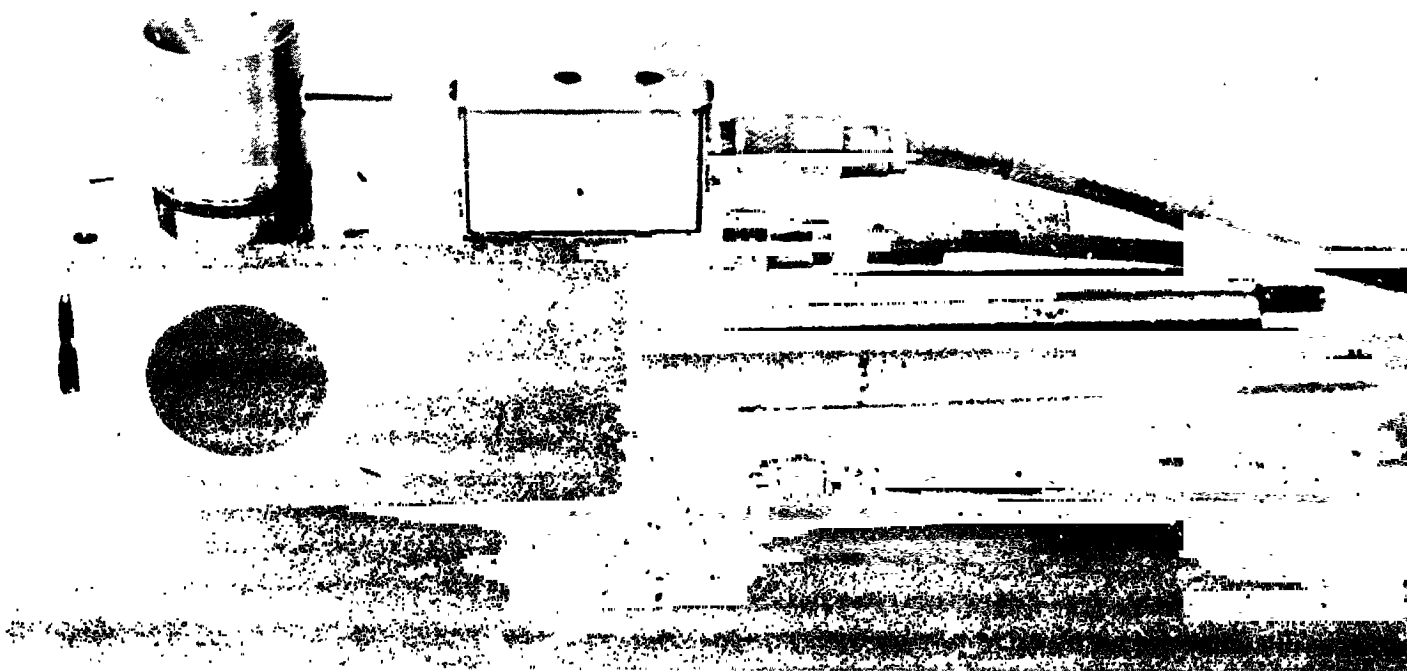


Fig. 2. Modified Varian 4331A probe

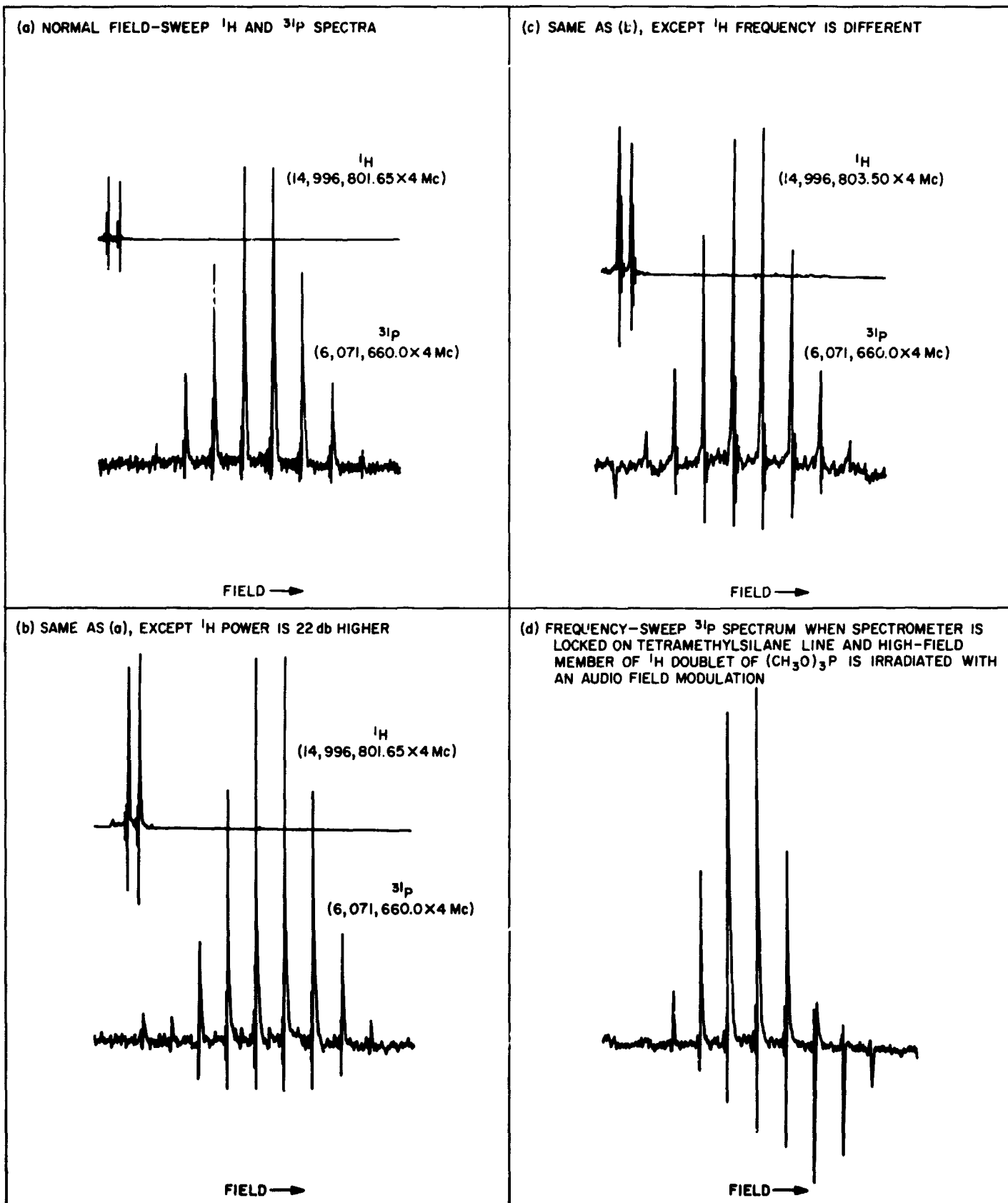


Fig. 3. NMR spectra of $(\text{CH}_3\text{O})_3\text{P}$ (neat)

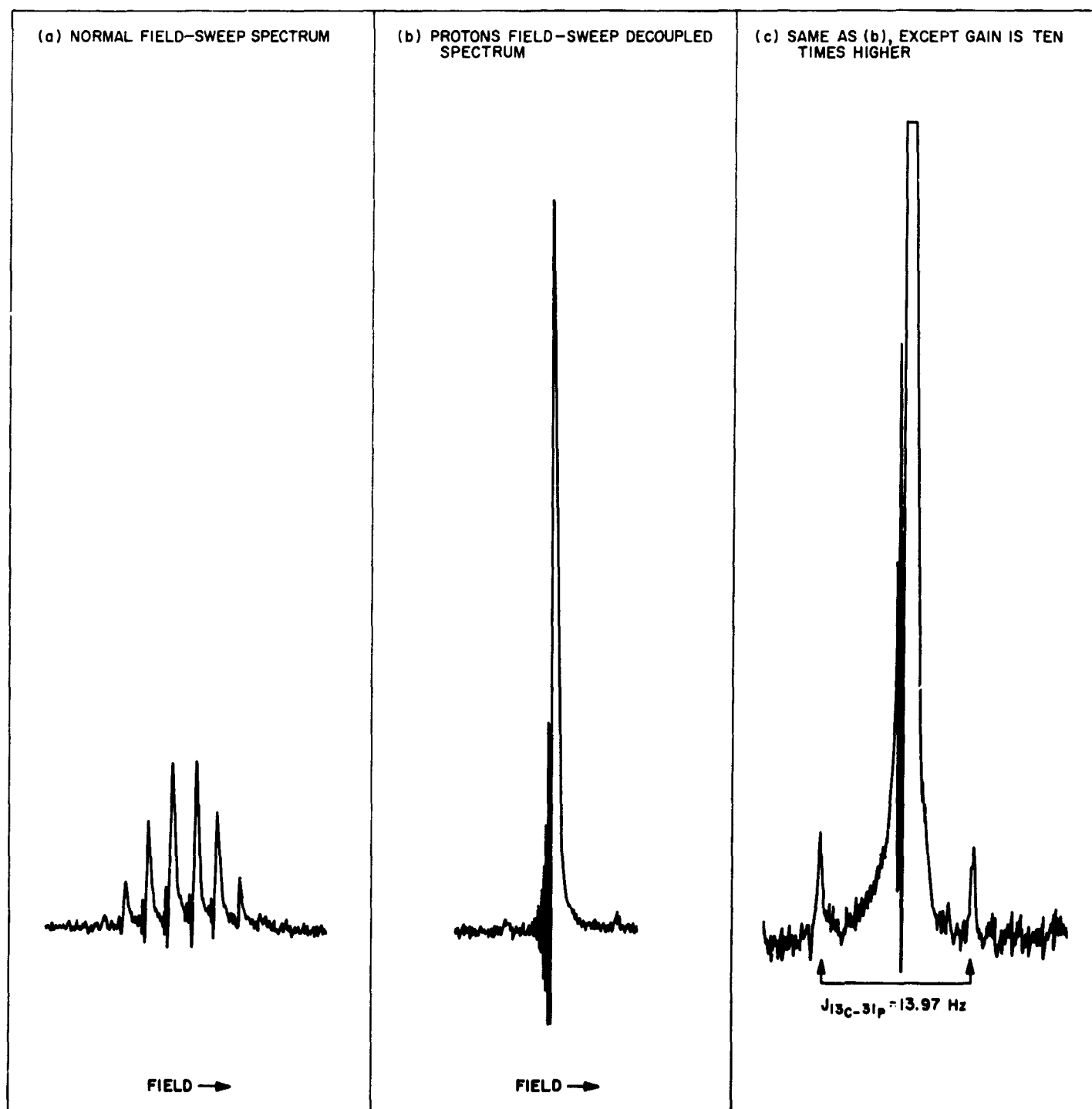


Fig. 4. ^{31}P spectra of $(\text{CH}_3)_3\text{P}$ (neat)

motor-driven 10-turn pot to the search oscillator of the appropriate frequency synthesizer. On completely decoupling the protons in trimethylphosphite, the ^{31}P signal intensity increased by a factor of 1.5 from that expected for just collapse of the multiplet structure of Fig. 3(a).

Fig. 4(a) shows the field-sweep ^{31}P spectrum of trimethylphosphine, $(\text{CH}_3)_3\text{P}$; Fig. 4(b) shows the field-sweep completely decoupled spectrum. Note the two weak signals on either side of the main signal. Fig. 4(c) shows the same region at higher gain. The doublet structure we ascribe to a directly bonded ^{13}C - ^{31}P coupling of -13.97 ± 0.04 Hz. This assignment is based on two facts: At progressively lower levels of ^1H irradiation, the doublet signals broaden and finally disappear, even though the protons in the molecules $(^{12}\text{CH}_3)_3^{31}\text{P}$ are still completely decoupled. Furthermore, the doublet is not centered about the decoupled ^{31}P signals from the $(^{12}\text{CH}_3)_3^{31}\text{P}$ species because of a ^{31}P isotopic chemical shift of 0.98 ± 0.04 Hz of the $(^{13}\text{CH}_3)(^{12}\text{CH}_3)_2\text{P}$ species. The negative sign is given this ^{13}C - ^{31}P coupling relative to the ^{13}C - H coupling because, if the ^1H frequency is increased some 40 Hz, it is observed in field-sweep spectra that the high-field line becomes slightly sharpened and the low-field line becomes significantly broadened. The reverse effect is observed when the ^1H frequency is decreased.

B. Low-Temperature Chromatographic Separation of Some Permanent Gases, J. King, Jr.

1. Introduction

The permanent gases argon (Ar), oxygen (O_2), nitrogen (N_2), and methane (CH_4) have been separated on an alumina (Al_2O_3) column at low temperatures. These gases were chosen to obtain experimental evidence in support of the electrostatic theory of physical adsorption (Ref. 2). The fact that they must also be included in any chromatographic atmospheric analysis gives added impetus to the investigations.

In the electrostatic theory, the dominant attractive energy between the gases and the surface was proposed to be (Ref. 2)

$$\Phi_{\text{att}} = -\frac{\alpha}{2} E_z^2 \quad (1)$$

where α is the polarizability of the adsorbed gas, and E_z is the electric-field intensity normal to the surface. There is a unique relationship between the molecular polarizabilities of the chosen gases which allows them to be used

as tests for Eq. (1). The diatomic molecules, O_2 and N_2 , have two distinct polarizabilities: $\alpha_{\parallel}$ along the internuclear axis, and $\alpha_{\perp}$ normal to the axis. CH_4 has an isotropic polarizability, and its behavior on the surface should be similar to that of a rare gas (e.g., Ar). When O_2 and N_2 are freely rotating, the effective polarizability is an average of $\alpha_{\parallel}$ and $\alpha_{\perp}$ and is defined as (Ref. 3, p. 949).

$$\alpha_{\text{av}} = \frac{1}{3} (\alpha_{\parallel} + 2\alpha_{\perp}) \quad (2)$$

The polarizabilities of the permanent gases are given below, where the molecular diameter σ is included to show that the molecules are of comparable sizes:

Item	Ar	O_2	N_2	CH_4
$\alpha_{\parallel}, \text{\AA}^3$	1.63	2.35	2.38	2.60
$\alpha_{\perp}, \text{\AA}^3$	1.63	1.21	1.45	2.60
$\alpha_{\text{av}}, \text{\AA}^3$	1.63	1.60	1.76	2.60
$\sigma, \text{\AA}$	3.40	3.47	3.70	3.82

If Eq. (1) does represent the major attractive force, then O_2 and N_2 , when freely rotating on the surface, should behave like Ar since their effective polarizabilities are similar. Conversely, if their rotational motion is hindered, their behavior should deviate from that of Ar . Since N_2 has a large quadrupole moment (Ref. 3, p. 1028), its rotational motion should be affected in a strong electric field. The electric-field quadrupole interaction will tend to align the molecule's internuclear axis with the field. O_2 , whose quadrupole moment is rather small (Ref. 3, p. 1028), should be less affected. Under conditions where the interaction is so strong that N_2 ceases to rotate, its behavior on the surface should approach that of CH_4 . To observe these effects, the experiments were performed at low temperatures, where the molecules have less kinetic energy.

2. Experimental

Some innovations were made in the ordinary gas chromatography setup. Since the activity of the Al_2O_3 had to be closely controlled, it was necessary to have ultra-dry carrier gas. Any water, even in trace quantities, greatly reduces the activity. Therefore, to eliminate all traces of water, the carrier gas was passed through a molecular sieve trap at liquid-nitrogen temperature prior to sample injection. Temperature variation and control were obtained by automatically maintaining an isopentane

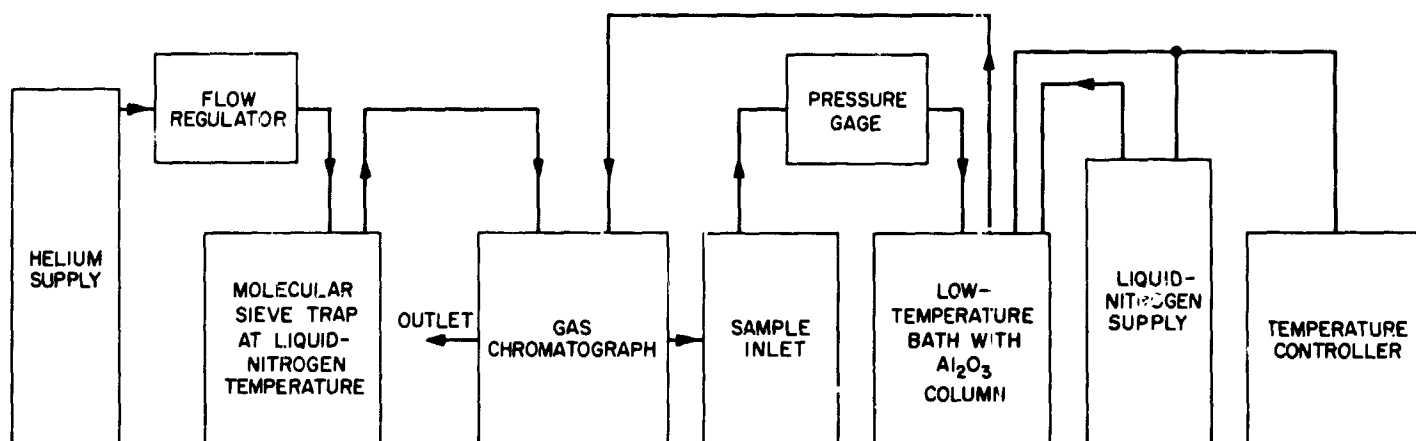


Fig. 5. Block diagram of experimental apparatus

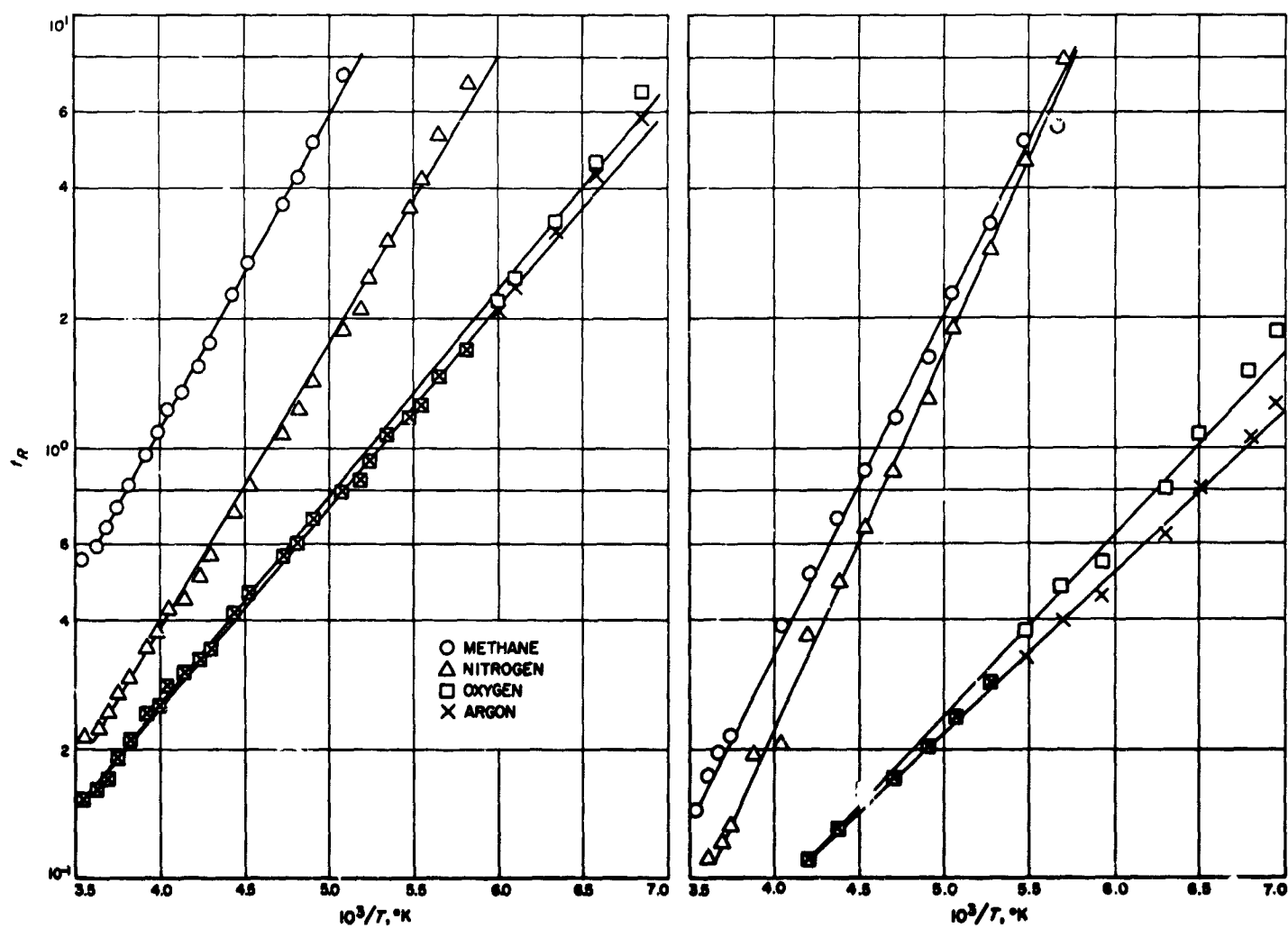


Fig. 6. Chromatographic separations on Al_2O_3 column activated at both 350 and 900°C for 100 hr

bath at the desired temperature. The isopentane was contained in a low-temperature dewar. Inside the dewar was a large copper cylinder with copper tubing wound on its outside. The tubing was connected to a liquid-nitrogen source, which was controlled by a sensing element immersed in the isopentane and an automatic temperature controller. Magnetic stirring was obtained by use of a small propeller attached to the bottom of the copper cylinder and a rotating magnet outside of the dewar. A block diagram of the experimental setup is shown in Fig. 5.

The column was made by coiling 8 ft of quartz tubing to fit into the low-temperature dewar. The column was packed with Alcon Type F-1 activated Al_2O_3 (~150 mesh) by applying compressed air at the inlet and a vacuum at the outlet and vibrating the column by means of a drill motor. The weight of the material in the column was 70.2 g.

The experimental procedure consisted of: (1) activating the column at 350°C for 100 hr in a tubular heater, (2) placing the column in the low-temperature bath, (3) performing the separations over the temperature range (4) replacing the column in the heater and activating at 900°C for 100 hr, and (5) repeating the separations over the temperature range with the newly activated column. All of these functions were performed without discontinuing the flow of the helium carrier gas through the column. The same flow rate was maintained at each temperature by using a flow regulator and systematically readjusting the flow when it deviated by more than 1% from $100\text{ cm}^3/\text{min}$.

3. Results and Discussion

A semi-log plot of the retention times, t_R , of the gases as a function of the reciprocal temperature is presented in Fig. 6. The times plotted have been corrected for temperature and pressure effects, as well as for the dead space in the column. The latter quantity is obtained by passing through the column some gas, e.g., neon (Ne), which is not adsorbed on the column. Thus (Ref. 4),

$$t_R = \frac{3}{2} (t_{\text{obs}} - t_{\text{Ne}}) \frac{T(P_i/P_o)^2 - 1}{T_o(P_i/P_o)^3 - 1} \quad (3)$$

where t_{obs} is the observed time; t_{Ne} is the time it took Ne to traverse the column; T is the column temperature; T_o is room temperature; and P_i and P_o are the inlet and outlet pressures, respectively.

The resulting straight lines in Fig. 6 are indicative of linear adsorption isotherms and allow comparisons to be made between the two conditions. Nitrogen is seen to behave quite differently on the two columns. On the highly active column (heated to 900°C), the retention of N_2 approaches that of CH_4 at low temperatures; on the less active column, the difference in the retention times of N_2 and CH_4 is approximately constant, and the retention time of N_2 approaches that of O_2 and Ar at the high temperatures.

The observed behavior supports the basic assumptions of the electrostatic theory of physical adsorption. The data also suggest the existence of at least two distinct adsorption sites on the Al_2O_3 surface. On the material heated to 350°C , water blocked the most active surface sites, and the quadrupole-field interaction was not strong enough to perceptively hinder the rotation of N_2 . However, when the material is heated to 900°C , water is driven from the most active sites, and the fields over these sites are strong enough to orientate N_2 with its axis perpendicular to the surface. The existence of these sites on the Al_2O_3 surface had been previously suggested (Ref. 2).

4. Conclusion

The results of the chromatographic separations of Ar, O_2 , N_2 , and CH_4 have been used to support, at least qualitatively, some of the assumptions of the electrostatic theory of physical adsorption. A quantitative treatment of the data is under way to obtain information about barriers to rotation, molecular orientation on the surface, and the distances of the adsorbed molecules from the Al_2O_3 surface.

C. An Explanation of the Gibbs' Phenomenon,

M. M. Saffran

The Gibbs' phenomenon is discussed in nearly every text on mathematical physics and on Fourier series. The phenomenon occurs when one attempts to represent a function that has a jump discontinuity by a Fourier series; the series representation "overshoots" the actual function in the neighborhood of the discontinuity. In this article the Gibbs' phenomenon is shown to be a consequence of two well-known facts: (1) The ordinary eigenfunctions of a linear differential operator form a complete set, and (2) the "torn" or discontinuous eigenfunctions have lower eigenvalues than do the ordinary eigenfunction. The latter fact is supported by the common observation that a wine glass "rings" at a lower frequency when it is cracked.

Consider now the eigenvalue problem defined by the functional of $\psi(r)$

$$E(\psi) = \frac{\int d\tau [|\nabla\psi|^2 + V(r)|\psi|^2]}{\int d\tau |\psi|^2} \quad (1)$$

with the region of integration within some finite closed surface. On this surface, $\psi(r)$ is required to satisfy specified boundary conditions. The eigenfunctions are defined as the functions ψ_n that are the extrema of Eq. (1). The ordinary eigenfunctions are the extrema sought among the class of continuous functions. The "torn" eigenfunctions are the extrema with respect to a different class of functions. A function of this class is discontinuous on some surface which divides the original region in two, but is continuous elsewhere. As is well-known, the torn eigenfunctions have derivatives which vanish at the jump surface when taken normal to the surface. As has already been mentioned, the n th torn eigenfunction has an eigenvalue lower than that of the n th ordinary eigenfunction.

The set of ordinary eigenfunctions, ψ_n , is complete; i.e., if ϕ is a function that obeys the same restrictions as ψ (namely, that it is a continuous function obeying the same boundary conditions as ψ), then

$$S(\phi) = \int d\tau |\phi|^2 = \sum_{n=0}^{\infty} |C_n|^2 \quad (2)$$

$$\mathcal{E}(\phi) = E(\phi) S(\phi) = \sum_{n=0}^{\infty} \lambda_n |C_n|^2 \quad (3)$$

Here, C_n denotes $\int d\tau \phi \psi_n^*$, and λ_n is the eigenvalue of ψ_n (Ref. 5). Though the proof is not included here, it is not difficult to show that, even if ϕ is continuous except at a surface where it jumps, Eq. (2) is true. (The proof is similar to that on p. 157 of Ref. 5.) Thus, if

$$\tilde{\phi} = \sum C_n \psi_n$$

then

$$S(\phi - \tilde{\phi}) = 0 \quad (4)$$

Therefore, the two functions ϕ and $\tilde{\phi}$ are the same in the mean-square. Now, suppose that ϕ is a torn eigenfunction ϕ_n with the eigenvalue μ_n , and μ_0 is the lowest eigenvalue.

As we will show later.

$$E(\phi_0 - \tilde{\phi}_0) \neq 0 \quad (5)$$

If we assume $V(r)$ has an upper bound, then, using Eq. (4), we can show

$$\int d\tau V(r) |\phi_0 - \tilde{\phi}_0|^2 = 0$$

Thus, we conclude from Eq. (5) that

$$\int d\tau |\nabla(\phi_0 - \tilde{\phi}_0)|^2 \neq 0 \quad (6)$$

If we define

$$[\tilde{\phi}_0]_n \equiv \sum_{k=0}^n C_k \psi_k \quad (7)$$

then Eq. (5) shows

$$\lim_{n \rightarrow \infty} S(\phi_0 - [\tilde{\phi}_0]_n) \rightarrow 0 \quad (8)$$

and Eq. (6) shows

$$\lim_{n \rightarrow \infty} \int d\tau |\nabla(\phi_0 - [\tilde{\phi}_0]_n)|^2 \neq 0 \quad (9)$$

Thus, while the sequence $[\tilde{\phi}_0]_n$, $n = 0, 1, \dots, \infty$, converges to ϕ_0 in the mean-square, it cannot converge to ϕ_0 point-by-point because of Eq. (6). To see this, let us suppose that $[\tilde{\phi}_0]_n$ converges to a continuous function $[\tilde{\phi}_0]_\infty = \phi_0$. But, if this is true, then

$$\nabla([\tilde{\phi}_0]_\infty - \phi_0) = 0 \quad (10)$$

and Eq. (6) cannot be true. Thus, $[\tilde{\phi}_0]_n$ cannot converge to ϕ_0 everywhere, and, if it does converge everywhere, it cannot converge to a continuous function everywhere.

Eq. (5), used in this argument, will now be shown to be true. We evaluate

$$\mathcal{E}(\phi_0 - [\tilde{\phi}_0]_n) \quad (11)$$

as a sum of two integrals, each integral being taken on each side of the jump surface of ϕ_n . The integration is performed by parts, and the resulting surface integrals

vanish since the normal derivatives of ϕ_0 vanish on the jump surface. We find

$$\begin{aligned} \mathcal{E}(\phi_0 - [\tilde{\phi}_0]_n) &= \mu_0 S(\phi_0) - \mu_0 S(\phi_0, [\tilde{\phi}_0]_n) \\ &\quad - \mu_0 S([\tilde{\phi}_0]_n, \phi_0) + \mathcal{E}([\tilde{\phi}_0]_n) \end{aligned} \quad (12)$$

$$S(\psi, \phi) \equiv \int d\tau \psi^* \phi$$

which can be written as

$$\mu_0 S(\phi_0 - [\tilde{\phi}_0]_n) + \mathcal{E}([\tilde{\phi}_0]_n) - \mu_0 S([\phi_0]_n) \quad (13)$$

If we note that

$$\mathcal{E}([\tilde{\phi}_0]_n) > \lambda_0 S([\tilde{\phi}_0]_n) \quad (14)$$

we see that

$$\lim \mathcal{E}(\phi_0 - [\tilde{\phi}_0]_n) > (\lambda_0 - \mu_0) \lim S([\tilde{\phi}_0]_n) > 0 \quad (15)$$

The next step is to show that any torn eigenfunction ϕ_m yields a Gibbs' phenomenon. To show this, we consider the function

$$\bar{\phi}_m = \phi_m - \sum_{\lambda_n \leq \mu_m} \langle \phi_m | \psi_n \rangle \psi_n \quad (16)$$

This function is ϕ_m orthogonalized to all ψ_n with eigenvalues less than μ_m . If we now evaluate

$$\mathcal{E}([\tilde{\phi}_m]_k - \bar{\phi}_m) \quad (17)$$

we find that

$$\begin{aligned} \mathcal{E}([\phi_m]_k - \bar{\phi}_m) &= \mathcal{E}([\tilde{\phi}_m]_k - \phi_m) \\ &= \mathcal{E}([\tilde{\phi}_m]_k) - \mu_m S([\tilde{\phi}_m]_k, \phi_m) \end{aligned} \quad (18)$$

However, it is clear that

$$\mathcal{E}([\tilde{\phi}_m]_k) > \lambda_m S([\tilde{\phi}_m]_k) \quad (19)$$

But, since $\lambda_m > \mu_m$, we finally see that

$$\mathcal{E}([\tilde{\phi}_m]_k - \phi_m) > 0 \quad (20)$$

As demonstrated above, this is all we need to show that a function, here ϕ_m , exhibits a Gibbs' phenomenon.

We can now demonstrate that any torn function ψ manifests a Gibbs' phenomenon. All we need show is that

$$\mathcal{E}(\tilde{\psi}) > \mathcal{E}(\psi) \quad (21)$$

The remaining steps are the same as those in the demonstration that ϕ_0 manifests a Gibbs' phenomenon. To prove Eq. (21), we expand ψ in the torn eigenfunctions appropriate to the tear in ψ . We denote the expansion up to the n th torn eigenfunction as $[\hat{\psi}]_n$. It is well-known that $\mathcal{E}(\hat{\psi}) = \mathcal{E}(\psi)$. Since we have shown above that, for any torn eigenfunction,

$$\mathcal{E}(\tilde{\phi}_n) > \mathcal{E}(\phi_n) \quad (22)$$

clearly then this is also true for any linear combination of the torn eigenfunctions. Thus,

$$\mathcal{E}([\tilde{\psi}]_k) > \mathcal{E}([\hat{\psi}]_n) \quad (23)$$

and so

$$\mathcal{E}([\tilde{\psi}]_k) > \mathcal{E}(\hat{\psi}) \quad (24)$$

But, it is easy to show that

$$\mathcal{E}(\tilde{\psi}) \geq \mathcal{E}([\tilde{\psi}]_k) \quad (25)$$

Therefore, since $\mathcal{E}(\psi) = \mathcal{E}(\hat{\psi})$, we have what we set out to prove:

$$\mathcal{E}(\tilde{\psi}) > \mathcal{E}(\psi) \quad (26)$$

In this article, we have shown that the Gibbs' phenomenon is a result of trying to expand a mode of low frequency (or energy) in a linear combination of modes, each having a higher frequency (or energy) than the mode we are attempting to represent. This is most clear when the mode is the lowest torn eigenfunction, which has a frequency lower than any ordinary mode. The Gibbs' phenomenon is a manifestation of how the expansion accommodates two conflicting requirements: (1) that the expansion for the torn mode converge in the mean-square to the torn mode itself, and (2) that the expansion have a higher mean frequency than the mode it represents. The accommodation is achieved by the expansion developing a discontinuity which leads to a discontinuous gradient. It is this discontinuity in gradient that keeps the mean frequency of the expansion higher than the frequency of the mode it represents, while simultaneously allowing the expansion to converge to this mode in the mean-square.

D. Relativistic Collapse to a Schwarzschild Sphere,¹

H. D. Wahlquist and F. B. Estabrook

1. Introduction

The interior Schwarzschild solution describing a static (rigid, non-rotating) sphere of perfect fluid at constant density is the simplest analytic metric known for a finite gravitating body. It has not usually been noted that the field equations allow an arbitrary function of time to be introduced into this metric without destroying the properties of simplicity, rigidity, and constant density, but resulting in a time-dependent pressure field (see, however, Ref. 6). We have exploited this time-dependence to construct idealized models of spherically symmetric, non-static processes, such as accretion or relativistic collapse onto a central body. The models are of sufficient simplicity to permit an exact analytical solution of the entire gravitational problem, making comparison of the predictions of general relativity with Newtonian and post-Newtonian theory for the dynamics of the models unambiguous, even in extreme relativistic regimes.

The complete solutions are obtained by piecing together three different, well-known, space-time metrics across surfaces of discontinuity. All junction conditions required by the Lichnerowicz postulate of admissible coordinates (Ref. 7) are satisfied. The metrics, however, are expressed in "non-admissible" comoving coordinates, and the fitting is actually accomplished using these coordinates and the tetrad or dyadic formulation of the junction conditions.²

The three metrics employed are: (I) the interior Schwarzschild with constant density ρ_s but time-dependent pressure, (II) the Oppenheimer-Snyder solutions (Ref. 8) for the spherically symmetric motion of incoherent matter, and (III) the exterior Schwarzschild. Fig. 7 is a schematic space-time diagram of the simplest combination of these. It represents a contracting spherical dust cloud (region II, outer boundary Σ_1) which is condensing and accreting on the surface Σ_3 of a growing Schwarzschild sphere (region I, below dashed line) to build the final configuration, a static Schwarzschild sphere of radius r_m (region I, above dashed line, boundary Σ_2). On Σ_3 , the matter undergoes a phase transformation from the incoherent state of zero pressure and varying density

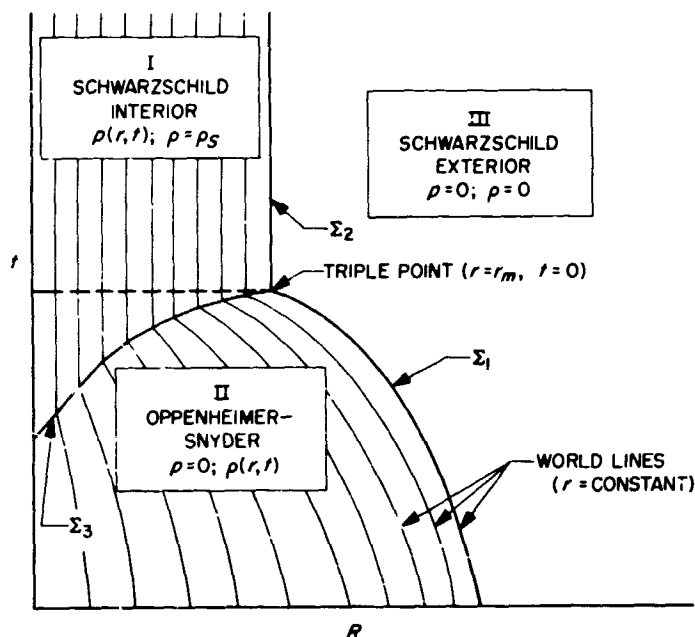


Fig. 7. Contraction of dust cloud to Schwarzschild sphere

to the interior state of variable pressure and constant density ρ_s . Some other possible mosaics of these metrics are shown in Fig. 8; their analytical descriptions can also be constructed using the dyadic methods.

The general dyadic equations for timelike congruences (Refs. 9 and 10) are specialized for the present case by imposing several conditions. The matter is to be non-rotating ($\Omega = 0$), and we shall adopt non-rotating axes ($\omega = 0$). Spherical symmetry is imposed by representing the only distinguished spatial direction at every point with a unit, radial, 3-vector $\hat{u}$ and expressing all vectors and dyadics in terms of it. The remaining kinematical variables of the matter then take the forms

$$\left. \begin{aligned} \mathbf{a} &= a\hat{u} \\ \mathbf{s} &= \frac{1}{3}\theta\mathbf{I} + \sigma(\mathbf{I} - 3\hat{u}\hat{u}) \end{aligned} \right\} \quad (1)$$

where a is the absolute acceleration, θ is the expansion, and σ is the shear. Similarly, the electric and magnetic components of the Weyl tensor, represented by two symmetric traceless dyadics, must have the forms

$$\left. \begin{aligned} \mathbf{A} &= \alpha(\mathbf{I} - 3\hat{u}\hat{u}) \\ \mathbf{B} &= \beta(\mathbf{I} - 3\hat{u}\hat{u}) \end{aligned} \right\} \quad (2)$$

¹A paper on this subject, including a more detailed derivation of results than that presented here, has been accepted for publication in *The Physical Review*.

²Estabrook, F. B., and Wahlquist, H. D., *Tetrad Formulation of Junction Conditions in General Relativity* (in press); cf. SPS 37-41, Vol. IV, p. 189.

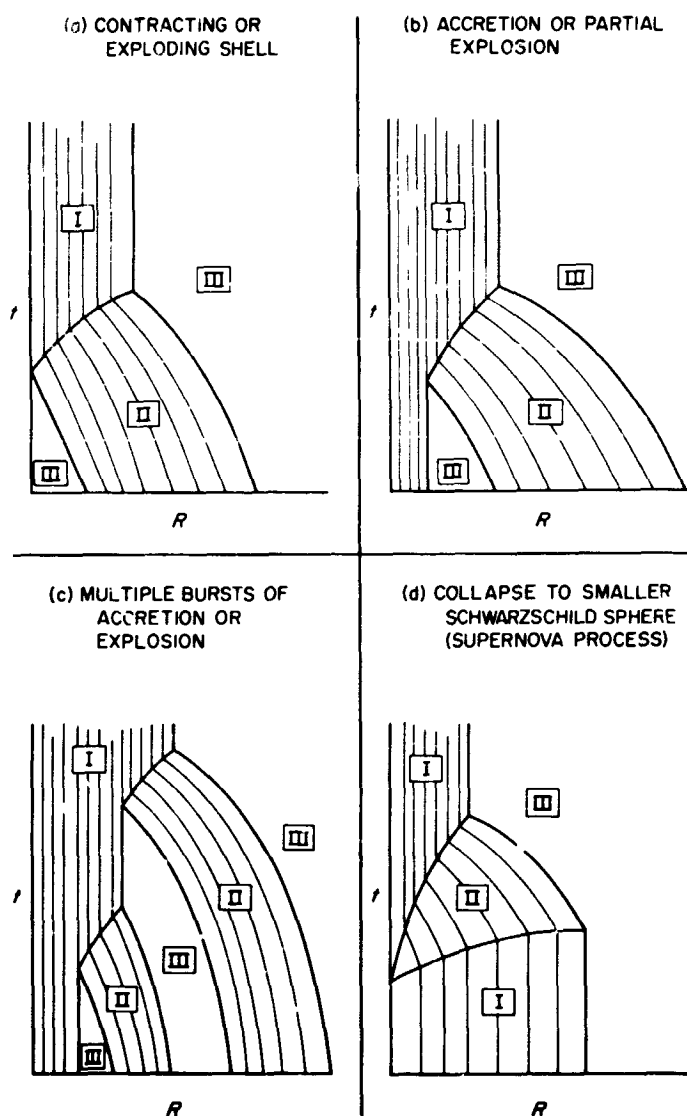


Fig. 8. Other processes employing the same metrics as Fig. 7

It can be shown from the equations that, in fact, in the present case $\beta = 0$. Finally, for a comoving frame of reference, the momentum density vanishes ($t = 0$), while the remaining components of the Einstein tensor will include the local proper energy density ρ and the stress dyadic for a perfect fluid

$$T = -pI \quad (3)$$

Each of the three regions used in these models is characterized by different further specializations. In region I, we want to describe a rigid fluid with constant density ($\theta = \sigma = \dot{\rho} = \rho' = 0$), and, since the space-time of the interior Schwarzschild solution is conformally flat, we also have $\alpha = 0$. For the incoherent matter of region II, we put $p = a = 0$. In region III, which being empty offers no

physically preferred timelike congruence, we adopt the timelike isometry for our reference frame so that, in addition to $p = \rho = 0$, we have $\theta = \sigma = 0$, and all time derivatives vanish.

If we now introduce comoving spatial coordinates (r, ψ, χ) , the spherical metric can be written in the form

$$ds^2 = -\frac{1}{\phi^2} dt^2 + \frac{1}{\delta^2} dr^2 + R^2 (d\psi^2 + \sin^2 \psi d\chi^2) \quad (4)$$

where R denotes the usual radial curvature coordinate and ϕ , δ , and R are functions of r and t only. Inserting the appropriate assumptions, the dyadic differential equations can be solved for the metric coefficients ϕ , δ , and R for each of the three regions. In region I, the solution is unique up to an arbitrary function of time only, $F(t)$; in region II, three arbitrary functions of r only, $f(r)$, $g(r)$, and $h(r)$, remain; in region III, the solution is unique.

Having obtained the metrics and physical quantities for each region expressed in terms of intrinsic comoving coordinates, the remaining task is to join them together properly at the three surfaces of discontinuity Σ_1 , Σ_2 , and Σ_3 . This problem is usually formulated theoretically in terms of admissible coordinates (Ref. 7), for which the metric tensor itself and its first derivatives are *everywhere* continuous. When such coordinates can be discovered, the matching problem is quite simple. In general, of course, the comoving coordinates used here will not satisfy the requirements of admissibility. To proceed, however, by searching for a set of admissible coordinates valid for all three regions leads to considerable difficulties. Not the least of these difficulties arise from the facts that the metrics of regions I and II are not explicit, but involve several functions as yet arbitrary, and that the forms of the boundary surfaces themselves are not yet completely specified. In practice, this problem can be handled in a quite straightforward manner by turning to a tetrad or dyadic formulation of the junction conditions which can be specially adapted to the matching of metrics expressed in comoving coordinates. The general formalism of this approach has been developed (see Footnote 2), and it has been shown that the continuity conditions used below are equivalent to, and in fact guarantee, the existence of admissible coordinates without explicitly employing them.

When the 29 general dyadic junction conditions of the general formalism for motion normal to spatial boundaries are written for spherical symmetry and comoving

frames of reference, the following set of 7 continuity conditions results:

$$\frac{\cot \psi}{R} \quad (5a)$$

$$\gamma \left[\eta + v \left(\frac{1}{3} \theta + \sigma \right) \right] \quad (5b)$$

$$\gamma \left[v \eta + \frac{1}{3} \theta + \sigma \right] \quad (5c)$$

$$\gamma \left[\gamma^2 v v' + \gamma^2 \dot{v} + a + v \left(\frac{1}{3} \theta - 2\sigma \right) \right] \quad (5d)$$

$$\gamma^2 v (p + \rho) \quad (5e)$$

$$\gamma^2 [v^2 \rho + p] \quad (5f)$$

$$\frac{1}{3} \rho + \alpha \quad (5g)$$

Each of these expressions must be continuous across the boundary. The quantity v is the proper, radial 3-velocity of the moving spherical 2-surface, as observed from the comoving frame of reference on each side, and $\gamma = (1 - v^2)^{-1/2}$. We assume throughout that the angular coordinates ψ and χ are propagated continuously across all boundaries, so the first expression simply requires that R also be everywhere continuous. Conditions (5a)–(5d) ensure that the intrinsic first and second fundamental forms of the boundary are unique. Conditions (5e) and (5f) are the usual relativistic Rankine–Hugoniot relationships in a symmetric form, and Condition (5g) requires continuity of a certain combination of curvature components involving both Einstein and Weyl tensors.

These conditions are now applied to each of the three surfaces of discontinuity to determine the arbitrary functions $F(t)$, $f(r)$, $g(r)$, and $h(r)$. However, when completed, we find that we are still left with two functions of r on the boundary Σ_3 which remain arbitrary. In other words, for each choice of two such functions and the

two parameters ρ_s and r_m , we obtain a unique solution for a model of the type depicted in Fig. 7. The physical significance of this is perhaps most clearly seen by interpreting the functions as: (1) the amount of matter crossing Σ_3 and accreting in region I per unit time, and (2) the velocity of impact of this matter at Σ_3 . Since we impose no thermodynamic constraints at Σ_3 other than the local conservation of energy–momentum given by the Rankine–Hugoniot relationships, both of these quantities can be specified. Recalling the definition of the velocities appearing in the junction conditions, the two quantities can be written as $\rho_s v_I$ and

$$(v_I - v_{II}) / (1 - v_I v_{II}),$$

respectively: so, it is convenient to adopt as arbitrary functions the two velocities

$$u(r) \equiv v_I \quad (6)$$

the velocity of the surface of the growing sphere, and

$$v(r) \equiv \frac{v_I - v_{II}}{1 - v_I v_{II}} \quad (7)$$

the velocity of impact of the dust, both relative to the static matter of region I.

2. Metrics of Fig. 7

We shall now present a summary of the metrics and physical quantities for the type of models depicted in Fig. 7. The solutions are specified by the two velocity functions $u(r)$ and $v(r)$ on Σ_3 , which are arbitrary to within some broad constraints discussed later, and it is convenient to define

$$u_m = u(r_m), \quad v_m = v(r_m) \quad (8)$$

The equations as written are equally valid for either contracting or expanding models.

The space–time metrics and physical quantities in the three regions of Fig. 7 are as follows:

Region I.

$$ds^2 = -\frac{1}{4} \left[3F(t_1) - \left(1 - \frac{2}{3} \rho_s r^2 \right)^{1/2} \right]^2 dt^2 + \left[1 - \frac{2}{3} \rho_s r^2 \right]^{-1} dr^2 + r^2 d\Omega^2 \quad (9)$$

with

$$F(t_1) = \left(1 - \frac{2}{3} \rho_s r^2\right)^{1/2} \left[\frac{1 - u(r)v(r)}{1 - 3u(r)v(r)} \right] \Big|_{r=r_{\Sigma_3}(t_1)}, \quad \left. \begin{array}{l} t_1 < 0 \\ t_1 > 0 \end{array} \right\} \quad (10)$$

$$= \left(1 - \frac{2}{3} \rho_s r_m^2\right)^{1/2},$$

The inequalities here would reverse for an expanding motion. The function of r in the first expression for F is converted to a function of t_1 by means of the equation of the boundary Σ_3 :

$$t_1 = - \int_r^{r_m} \frac{[1 - 3u(x)v(x)]}{u(x) \left[1 - \frac{2}{3} \rho_s x^2\right]} dx \quad (11)$$

Letting $d\Sigma_3$ represent an interval on the surface Σ_3 , we find

$$d\Sigma_3^2 = - \frac{(1 - u^2)}{u^2 \left(1 - \frac{2}{3} \rho_s r^2\right)} dr^2 + r^2 d\Omega^2 \quad (12)$$

The equation for the boundary Σ_2 is simply $r = r_m$. Its intrinsic metric is

$$d\Sigma_2^2 = - \left(1 - \frac{2}{3} \rho_s r_m^2\right) dt_1^2 + r_m^2 d\Omega^2 \quad (13)$$

The pressure field in region I is given by

$$p(r, t_1) = \rho_s \left\{ \frac{\left(1 - \frac{2}{3} \rho_s r^2\right)^{1/2} - F(t_1)}{3F(t_1) - \left(1 - \frac{2}{3} \rho_s r^2\right)^{1/2}} \right\} \quad (14)$$

and, on the boundaries,

$$p_{x_3} = - \rho_s uv, \quad p_{x_2} = 0 \quad (15)$$

Region II.

$$ds^2 = - dt_{II}^2 + \frac{(1 - v^2)}{\left(1 - \frac{2}{3} \rho_s r^2\right)} \left(\frac{\partial R}{\partial r}\right)^2 dr^2 + R^2 d\Omega^2 \quad (16)$$

with $R(r, t_{II})$ given implicitly by

$$t_{II} = \frac{v}{|v|} \int_r^{r_m} \left[\frac{v^2(r) - \frac{2}{3} \rho_s r^2}{1 - v^2(r)} + \frac{2}{3} \rho_s \frac{r^3}{x} \right]^{1/2} dx - \int_r^{r_m} \frac{[1 - u(x)v(x)] dx}{u(x) [1 - v^2(x)]^{1/2} \left(1 - \frac{2}{3} \rho_s x^2\right)^{1/2}} \quad (17)$$

and

$$\frac{\partial R}{\partial r} = \left(\frac{v^2 - \frac{2}{3} \rho_s r^2}{1 - v^2} + \frac{2}{3} \rho_s \frac{r^3}{R} \right)^{1/2} \left\{ \frac{(u - v)}{|v| u (1 - v^2)^{1/2} \left(1 - \frac{2}{3} \rho_s r^2 \right)^{1/2}} \right. \\ \left. + \int_r^R \left(\frac{v^2 - \frac{2}{3} \rho_s r^2}{1 - v^2} + \frac{2}{3} \rho_s \frac{r^3}{x} \right)^{-3/2} \left[\rho_s \frac{r^2}{x} + \frac{1}{2} \frac{d}{dr} \left(\frac{v^2 - \frac{2}{3} \rho_s r^2}{1 - v^2} \right) \right] dx \right\} \quad (18)$$

The equations for the boundaries of region II are: for Σ_3 , $r = R$, or, from Eq. (17),

$$t_{II} = - \int_r^{r_m} \frac{[1 - u(x)v(x)]}{u(x)[1 - v^2(x)]^{1/2} \left(1 - \frac{2}{3} \rho_s x^2 \right)^{1/2}} dx \quad (19)$$

giving Eq. (12) again for $d\Sigma_3^2$; and for Σ_1 , $r = r_m$, so that

$$d\Sigma_1^2 = -dt_{II}^2 + R^2(r_m, t_{II}) d\Omega^2 \quad (20)$$

or, again using Eq. (17),

$$d\Sigma_1^2 = - \left[\frac{v_m^2 - \frac{2}{3} \rho_s r_m^2}{1 - v_m^2} + \frac{2}{3} \rho_s \frac{r_m^3}{R} \right]^{-1} dR^2 + R^2 d\Omega^2 \quad (21)$$

The proper density in region II is given by

$$\rho(r, t_{II}) = \rho_s \frac{r^2}{R^2} \left(\frac{\partial R}{\partial r} \right)^{-1} \quad (22)$$

and, on Σ_3 ,

$$\rho_{\Sigma_3} = \rho_s \frac{u(1 - v^2)}{(u - v)} \quad (23)$$

Region III.

$$ds^2 = - \left(1 - 2 \frac{M}{R} \right) dt_{II}^2 + \left(1 - 2 \frac{M}{R} \right)^{-1} dR^2 + R^2 d\Omega^2 \quad (24)$$

with

$$M = \frac{1}{3} \rho_s r_m^3 \quad (25)$$

The boundary equations for this region are: for Σ_2 , $R = r_m$, giving Eq. (13) for $d\Sigma_2^+$ (with $t_I = t_{III}$ on Σ_2); and for Σ_1 ,

$$t_{III} = \frac{v}{|v|} \frac{\left(1 - \frac{2}{3} \rho_s r_m^2\right)^{1/2}}{(1 - v_m^2)^{1/2}} \int_{r_m}^R \left[\frac{v_m^2 - \frac{2}{3} \rho_s r_m^2}{1 - v_m^2} + \frac{2}{3} \rho_s \frac{r_m^3}{x} \right]^{-1/2} \left[1 - \frac{2}{3} \rho_s \frac{r_m^3}{x} \right]^{-1} dx \quad (26)$$

giving Eq. (21) again for $d\Sigma_1^+$.

Certain constraints must be imposed on the parameters and velocities if these solutions are to be complete and everywhere regular with no further boundaries. First, to describe the situation of Fig. 7 rather than that of Fig. 8(a), we must choose $v(0) = 0$. Second, to ensure that $0 \leq p(r, t_I) < \infty$, we must have

$$0 \leq (-uv) < \frac{\left[3 \left(1 - \frac{2}{3} \rho_s r^2 \right)^{1/2} - 1 \right]}{3 \left[1 - \left(1 - \frac{2}{3} \rho_s r^2 \right)^{1/2} \right]} \quad (27)$$

From this, it follows that

$$\frac{2}{3} \rho_s r_m^2 < \frac{8}{9} \quad (28)$$

which is the usual limit for the interior Schwarzschild solution. Next, from Eq. (22), we must have $\partial R / \partial r > 0$; thus, from the first factor of Eq. (18),

$$\frac{2}{3} \rho_s r^2 \leq v^2 < 1 \quad (29)$$

The inequality on the left-hand side above may be re-written in an obvious notation as an energy constraint

$$\frac{1}{2} v^2 - \frac{M(r)}{r} \geq 0 \quad (30)$$

required to ensure that the matter of region II comes from (or goes to) infinite distance. The second factor of Eq. (18) must also be prevented from vanishing, which it might do if

$$\frac{d}{dr} \left[\frac{v^2 - \frac{2}{3} \rho_s r^2}{1 - v^2} \right] \quad (31)$$

became sufficiently negative (all other terms of the factor being positive). Such behavior of the energy on Σ_s leads

to shells of matter overtaking others (intersection of the world lines in region II), violating the assumption of incoherent matter in region II.

It may be noted from Eq. (10) that, in general, a discontinuity of the pressure in region I occurs at the surface $t_I = 0$ (the dashed line of Fig. 7). Physically, this would appear as a shock wave of infinite speed, resulting from the sudden cessation of accretion at $r = r_m$. The discontinuity is removed by setting $u_m = 0$, which corresponds simply to vanishing density at the outer boundary of the collapsing cloud, as is evident from Eq. (23). Letting $u(r) = 0$ anywhere, however, gives rise to some difficulties with the coordinate r in region II and on Σ_3 at such points. These can be handled by using instead a new coordinate $\bar{r}$, defined, for instance, by

$$d\bar{r} = (\rho_{\Sigma_3})^{-1} dr$$

For arbitrary choices of the functions $u(r)$ and $v(r)$, satisfying the foregoing constraints and $u_m = 0$, the models may still have the acausal feature that the surfaces of constant pressure in region I are spacelike. This is a result of the incompressibility of the matter in region I and is not surprising. For such fluids, in the words of Sommerfeld (Ref. 11), the pressure takes on the rather unphysical character of a "Lagrangian multiplier . . . a reaction against the condition of incompressibility . . . without energetic consequence." Nevertheless, here its contribution to the gravitational field is fully included.

E. Particular Solutions of a System of Nonlinear First-Order Ordinary Differential Equations,

J. S. Zmuidzinas

Let E_p be the Euclidean space of all p -vectors $x = (x^1, \dots, x^p)$, $-\infty < x^i < \infty$. Consider the system of p ordinary differential equations

$$\dot{x}^i = X^i(x) \quad (1)$$

where

$$\dot{x}' \equiv dx'/dt$$

and the x' are given functions of $x = x(t)$. The parameter t is supposed to lie in the interval $[0, \infty)$. It is assumed that the x' are not explicit functions of t , i.e., that $\partial X'/\partial t = 0$.

Particular solutions of Eq. (1) can be obtained from the knowledge of a set of invariant relations (IRs) defined as follows: Let $f_0(x)$ be some function of the x' which is differentiable a sufficient number of times. Introduce the operator

$$\nabla \equiv X' \partial_i = d/dt$$

where

$$\partial_i = \partial/\partial x'$$

and put

$$f_n(x) = \nabla^n f_0(x), \quad n = 0, 1, 2, \dots$$

Suppose it happens that each $f_m, m > n$, vanishes identically if we set $f_0(x) = f_1(x) = \dots = f_n(x) = 0$. Then, we say that the set $\{f_k(x) = 0, k = 0, 1, 2, \dots, n\}$ is a set of IRs.

It is easy to find a characterization of an f_{n+1} vanishing when $f_k(x) = 0, k = 0, \dots, n$. It is sufficient to demand that f_{n+1} be linear in the f_k :

$$f_{n+1}(x) = \sum_{k=0}^n a_k(x) f_k(x) \quad (2)$$

where the a_k are some functions of x nonsingular at the "point" $(f_0(x), \dots, f_n(x)) = 0$ (a variety in E_p). If f_{n+1} is given by Eq. (2), then all $f_m, m > n + 1$, are also of this form. To see this, we use

$$\begin{aligned} \nabla f_k &= f_{k+1}, & k &= 0, 1, 2, \dots, n-1 \\ &= \sum_{i=0}^n a_i f_i, & k &= n \end{aligned}$$

in deducing that

$$\begin{aligned} f_{n+2} &= \nabla f_{n+1} \\ &= \nabla \left(\sum_{k=0}^n a_k f_k \right) \\ &= \sum_{k=0}^n (\nabla a_k \cdot f_k + a_k \cdot \nabla f_k) \\ &= \sum_{k=0}^n (\nabla a_k + a_{k-1} + a_n a_k) f_k \end{aligned}$$

where $a_{-1} \equiv 0$. Similar arguments establish our assertion for all f_m with $m > n + 2$.

Starting with an arbitrary f_0 , there need not exist a finite integer n for which Eq. (2) is true. On the other hand, there may exist many different functions $f_0^{(1)}, f_0^{(2)}, \dots$ for which Eq. (2) holds, with various integers $n^{(1)}, n^{(2)}, \dots$. The $f_0^{(i)}$ with the smallest $n^{(i)}$ are of particular interest.

To illustrate the utility of IRs, consider the system

$$\dot{x} = x^2 + y^2 \quad (3)$$

$$\dot{y} = 2xy \quad (4)$$

Suppose we try

$$f_0(x, y) = x + y \quad (5)$$

Then,

$$\begin{aligned} f_1(x, y) &= \{(x^2 + y^2) \partial_x + 2xy \partial_y\} f_0(x, y) \\ &= (x + y)^2 \\ &= [f_0(x, y)]^2 \end{aligned}$$

Thus, $f_0(x, y) \equiv x + y = 0$ is an IR. Imposing $f_0 = 0$ on our system of differential equations, we obtain

$$\begin{aligned} \dot{x} &= 2x^2 \\ \dot{y} &= -2y^2 \end{aligned}$$

The solutions are elementary:

$$\begin{aligned} x &= \frac{1}{2}(c - t)^{-1} \\ y &= -\frac{1}{2}(c - t)^{-1} \end{aligned}$$

Here, c is an arbitrary constant. These solutions are special cases of the more general solution obtained by adding equations for x and y :

$$\begin{aligned}\dot{f}_0 &= f_0^2 \\ f_0 &\equiv x + y = (c - t)^{-1}\end{aligned}$$

We may use $f_0 = x + y \neq 0$ to eliminate, say, y from Eq. (3), thus obtaining

$$\dot{x} = 2x^2 - 2xf_0 + f_0^2 \quad (6)$$

Putting

$$x = f_0 u$$

substituting into Eq. (6), and using $\dot{f}_0 = f_0^2$, we obtain

$$\dot{u} = f_0(2u^2 - 3u + 1) \quad (7)$$

The solution of this equation follows by quadrature:

$$u = \frac{c' - f_0/2}{c' - f_0}$$

Thus, we have found, with the help of $f_1 = \dot{f}_0 = f_0^2$, the family of solutions

$$\begin{aligned}x &= f_0 u \\ y &= f_0 - x = f_0(1 - u)\end{aligned}$$

Let us now consider a more general system of equations, still homogeneous:

$$\begin{aligned}\dot{x} &= ax^2 + bxy + cy^2 \equiv X \\ \dot{y} &= dx^2 + exy + fy^2 \equiv Y\end{aligned} \quad (8)$$

The homogeneity of X and Y suggests looking for homogeneous IRs. The simplest choice for f_0 is the linear expression

$$f_0 = x + \alpha y \quad (9)$$

We compute $f_1 = \nabla f_0$ and eliminate x from f_1 by means of Eq. (9). The result is

$$\begin{aligned}f_1 &= [d\alpha^3 + (a - e)\alpha^2 + (f - b)\alpha + c]y^2 \\ &\quad - [2d\alpha^2 + (2a - e)\alpha - b]yf_0 \\ &\quad + (d\alpha + a)f_0^2\end{aligned} \quad (10)$$

In order that $f_1 = 0$ when $f_0 = 0$, we must require that the coefficient of y^2 vanish. This requirement yields the following cubic equation for the parameter α :

$$d\alpha^3 + (a - e)\alpha^2 + (f - b)\alpha + c = 0 \quad (11)$$

The three solutions $\alpha_{1,2,3}$ of this equation yield the desired f_0 's:

$$f_0^{(i)}(x, y) = x + \alpha_i y, \quad i = 1, 2, 3$$

For the special case $a = c = 1$, $b = d = f = 0$, and $e = 2$, we find $\alpha = \pm 1$ and so

$$\begin{aligned}f_0^{(1)} &= x + y \\ f_0^{(2)} &= x - y\end{aligned}$$

Here, $f^{(1)}$ is just Eq. (5), while $f^{(2)}$ leads to new particular solutions.

While it is true that $f_0 = 0$ entails $f_1 = 0$, it is not necessarily true that $f_0 = f_0(t)$ yields f_1 independent of y , unless the coefficient of yf_0 in Eq. (10) happens to vanish. The necessary conditions for this coefficient to vanish are found to be

$$e\alpha^2 + (b - 2f)\alpha - 2c = 0, \quad \alpha \neq 0$$

or

$$b = 0, \quad \alpha = 0$$

In general, neither of these conditions will be satisfied.

Let us now choose a homogeneous quadratic expression for f_0 :

$$f_0 = (x + \alpha y)(x + \beta y) \quad (12)$$

Then,

$$f_1 = (X + \alpha Y)(x + \beta y) + (x + \alpha y)(X + \beta Y)$$

If $f_0 = 0$, then either $x + \alpha y = 0$ or $x + \beta y = 0$. Suppose the former is the case. Then,

$$f_1 = [X + \alpha Y]_{x = -\alpha y} (\beta - \alpha) y$$

vanishes if $\beta = \alpha$ (the trivial case) or if

$$[X + \alpha Y]_{x = -\alpha y} = 0$$

But, in the latter case, α must satisfy Eq. (11), and so we are back at the linear case already discussed. Since Eq. (12) is symmetric in α and β , the same conclusion obtains if $x + \beta y = 0$. We see then that choosing Eq. (12) does not lead to new solutions. Obviously, the same is true of higher-degree homogeneous polynomial expressions for f_0 .

Our results for Eq. (8) may be generalized in several ways. First, suppose we have the system

$$\begin{aligned}\dot{x} &= a_0 x^n - a_1 x^{n-1} y + a_2 x^{n-2} y^2 + \cdots + (-)^n a_n y^n \\ \dot{y} &= b_0 x^n - b_1 x^{n-1} y + b_2 x^{n-2} y^2 + \cdots + (-)^n b_n y^n\end{aligned}$$

Then, it is easy to show that

$$f_0 = x + \alpha y = 0$$

is an IR, provided α satisfies

$$\begin{aligned}b_0 \alpha^{n+1} + (a_0 + b_1) \alpha^n + (a_1 + b_2) \alpha^{n-1} \\ + \cdots + (a_{n-1} + b_n) \alpha + a_n = 0\end{aligned}$$

Next, we generalize our results to an arbitrary number of variables $x^i, i = 1, 2, \dots, p$. We take as our system

$$\begin{aligned}\dot{x}^i &= a_{j_1 \dots j_n}^i x^{j_1} \cdots x^{j_n} = X^i \\ (i, j_1, \dots, j_n &= 1, 2, \dots, p)\end{aligned}\quad (13)$$

The homogeneity of the X^i suggests the ansatz

$$x^i = \alpha^i z^\nu, \quad (14)$$

where α^i and ν are nonzero constants. Substituting this into Eq. (13), we find

$$\dot{z} = a_{j_1 \dots j_n}^i \frac{\alpha^{j_1} \cdots \alpha^{j_n}}{\nu \alpha^i} z^{(n-1)\nu+1} \quad (\text{no sum on } i)$$

For consistency, we demand that the coefficient of $z^{(n-1)\nu+1}$ be independent of i :

$$-\lambda \equiv a_{j_1 \dots j_n}^1 \frac{\alpha^{j_1} \cdots \alpha^{j_n}}{\nu \alpha^1} = \cdots = a_{j_1 \dots j_n}^p \frac{\alpha^{j_1} \cdots \alpha^{j_n}}{\nu \alpha^p} \quad (15)$$

Then,

$$\dot{z} = -\lambda z^{(n-1)\nu+1}$$

Hence,

$$\begin{aligned}z(t) &= [(n-1)\nu(c + \lambda t)]^{-(n-1)\nu-1}, & n \neq 1 \\ &= c'e^{-\lambda t}, & n = 1\end{aligned}\quad (16)$$

The $n-1$ equations for the $n-1$ ratios $\bar{\alpha}^j = \alpha^j/\alpha^1$, say, are coupled algebraic of order $n+1$ in the $\bar{\alpha}$'s.

Define the function

$$g(x) = \beta_i x^i \quad (17)$$

where the β_i are so far arbitrary. With Eq. (14), this becomes

$$\begin{aligned}g &= \rho z^\nu \\ \rho &\equiv \alpha^i \beta_i\end{aligned}$$

We also have

$$X^i = \dot{x}^i = \nu \alpha^i z^{\nu-1} \dot{z} = -\nu \lambda \alpha^i z^\nu$$

Therefore,

$$\begin{aligned}\nabla g &= \beta_i X^i \\ &= -\nu \lambda \rho z^{\nu} \\ &= -\nu \lambda z^{(n-1)\nu} g\end{aligned}$$

Clearly, $g = 0$ implies $\nabla g = 0$. Thus, we may identify g with f_0 :

$$f_0(x) = \beta_i x^i = \rho z^\nu$$

Taking $f_0 = 0$ means $\rho = 0$, since the other possibility $z \equiv 0$ is trivial. The $f_0 = 0$ solutions are thus those with $x^i = \alpha^i z^\nu$, where z is given by Eq. (16). The β_i are no longer arbitrary, but constrained by $\rho = \alpha^i \beta_i = 0$. A more

general class of solutions is obtained by taking $\rho \neq 0$. Then, $f_0 = \rho z^\nu$ or

$$\beta_i x^i(t) = \rho [z(t)]^\nu$$

Suppose $\beta_p \neq 0$ (relabel if necessary). Then, we may solve for x^p and eliminate it from our original system of Eq. (13) in favor of $x^1, \dots, x^{p-1}$ and the known function z , thus reducing the number of unknown functions to $p-1$. Making the further substitution $x^i = z^\nu y^i$, $i = 1, 2, \dots, p-1$, we can cast these equations in a form similar to Eq. (7), which may, in certain cases, be amenable to solution or further reduction.

Work on further generalizations to nonhomogeneous X^i is in progress and will be reported later.

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XXI. Communications Systems Research

TELECOMMUNICATIONS DIVISION

N67-29162

A. Block Coding and Synchronization Study: Subcarrier Tracking Methods and Communication System Design, W. C. Lindsey

1. Introduction

Various communication systems, e.g., binary PSK, transmit information in the form $s(t) = (2)^{1/2} A_m(t) \sin(\omega_c t + \theta)$. In order that the received signal be demodulated coherently, it is necessary to determine or estimate the phase θ and frequency of the subcarrier $(2)^{1/2} \sin(\omega_c t + \theta)$ with as little error as possible. If the signal $s(t)$ contains a residual component of sufficient strength at the subcarrier frequency, this component could be tracked with a narrow-band phase-locked loop and used to provide the desired reference signal. On the other hand, the power contained in the residual component represents power which does not convey any information other than the frequency and phase of the subcarrier. Thus, it represents power not available for the transmission of data, and, in practice, it is always of interest to investigate techniques which conserve and save power.

Several practical methods are available which rely upon the transmission of a reference signal. For example, the phase reference may be transmitted along with a PSK signal, and in order to maintain proper phase-synchronization, the phase-keyed signal and the reference signal must be close to each other in frequency and in time such that any channel fluctuations along the propaga-

tion path affect both signals the same way. For completeness, several methods of great practical importance are discussed below.

First, we have differential phase-shift keying (DPSK). In a DPSK system the PSK signal serves as the data signal and the reference signal. The phase of the signal received during one signaling interval serves as a reference for the next keying interval. The Kineplex (Ref. 1) is an example of a system which has been mechanized.

Second, we have the so-called adjacent tone reference PSK system (AT-PSK). The reference signal for this system is transmitted at an adjacent frequency simultaneously with the keyed signal. At the receiver, the phase of the reference is adjusted to compensate for the frequency difference between the reference signal and the phase-keyed signal. A practical system which employs this principle is illustrated by the DEFT system (Ref. 2).

Third is a system referred to as the quadrature reference PSK system (Q-PSK). In this system, the phase of one quadrature component is modulated with the data stream while the phase of the in-phase component remains unkeyed. The Kathryn system is an example of this technique (Ref. 3).

Finally, the so-called decision-directed measurement PSK (DDM-PSK) technique is employed. This system

reconstructs a reference signal by estimating the modulation itself and using this estimate to eliminate it from the received signal. The decision directed system is, in essence, a generalization of the DPSK system, which uses the previous signaling interval. A computer simulation of this type of system has been carried out by Proakis, Drouilhet, and Price (Ref. 4). More recently Bussgang and Leiter (Ref. 5) derived the performance of a communication system in which a reference signal is transmitted at a frequency adjacent to the phase-keyed tone. Also, Bussgang and Leiter report results pertinent to the problem of the joint occurrence of two character errors on a multiple phase-keyed signal.

A number of methods have been proposed for generating a reference subcarrier from the received signal even when the residual subcarrier component is not available. This report analyzes and compares two methods of great practical interest in deep-space work. The results of the analysis are used to establish the performance of phase-coherent communication systems which utilize such subcarrier tracking methods. The first, the squaring-loop method, has been analyzed in a number of papers, Refs. 6-9. The second method, originally proposed by Costas, is the Costas-loop (Ref. 10). This article establishes the performance of these two subcarrier tracking methods using the Fokker-Planck apparatus as opposed to using linear tracking theory. The results are then used in predicting the performance of uncoded and block-coded communication systems. The theory developed is useful in the design and testing of subcarrier tracking loops and data detectors.

2. The Squaring-Loop Method

Of main concern here will be that of establishing a coherent subcarrier reference for demodulation of 180-deg PSK modulation. The mechanization of a typical squaring loop is illustrated in Fig. 1. The received signal $y(t)$ is bandpass filtered, squared to remove the modulation $m(t)$, and the resultant double frequency term is tracked by means of a conventional phase-locked loop (PLL) whose noise bandwidth is ω_L cycles. When the output frequency

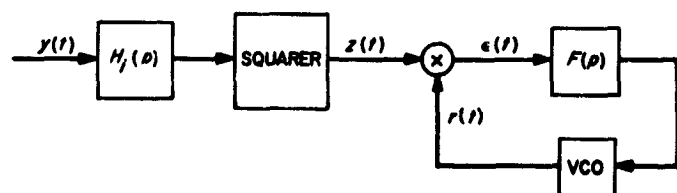


Fig. 1. The squaring loop

of the PLL is divided by two, a coherent reference signal is available for demodulation purposes.

In deciding upon a method of determining the performance of the squaring loop, a significant parameter is the bandwidth of the bandpass filter whose transfer function is denoted by $H_i(p)$, and p is the Heaviside operator. In fact, if the input is contaminated by white noise of spectral density N_0 , ω cycle single-sided, and if the bandwidth of the filter is so large that the correlation time τ_i of its output noise is much smaller than the time constant $1/\omega_L$ of the phase-locked loop,¹ the squaring-loop may be analyzed by using the mathematical techniques available from the theory of Markov processes—in particular, the Fokker-Planck equation, Ref. 11.

As the bandwidth B_i of the bandpass filter is narrowed, the correlation time τ_i of the output noise increases and may become equal or even greater than the time constant $1/\omega_L$ of the phase-locked loop. The cases when $\tau_i \geq 1/\omega_L$ are no less important in practice than the other extreme, when $\tau_i < 1/\omega_L$. However, the latter case is considered here for a constant frequency signal, and we neglect any spurious noise which may be generated due to imperfect system oscillators. Such fluctuations may be included with no great mathematical difficulty.

Let the observed data $y(t)$ be denoted by

$$y(t) = (2)^{1/2} A m(t) \sin(\omega_0 t + \theta) + n(t) \quad (1)$$

where $m(t)$ is the signal envelope, i.e., the modulation, and let

$$n(t) = n_1(t) \cos(\omega_0 t + \theta) + n_2(t) \sin(\omega_0 t + \theta) \quad (2)$$

be a realization of narrowband noise process, where $n_1(t)$ and $n_2(t)$ are sample functions of joint stationary Gaussian processes. We assume that the correlation time τ_n of the noise is small in comparison with the time constant of the PLL, i.e., $\tau_n \ll 1/\omega_L$.

Assuming a perfect square-law characteristic, the output process $z(t)$ is related to the input process $y(t)$ through

$$z(t) = [y(t) H_i(p)]^2 \quad (3)$$

¹Correlation time of the random process $\{x(t)\}$ is defined by the relation $\tau = \int_0^\infty |R_x(\tau)| d\tau$, where $R_x(\tau)$ is the normalized correlation function of the process. The parameter τ gives some idea of the size of the time interval over which correlation extends between values of the process $x(t)$.

where $H_i(p)$ is the transfer function of the bandpass filter and $p = d/dt$ is the Heaviside operator.

On substituting for $y(t)$ into Eq. (3) and taking only the terms around $2\omega_0$ yields, in operator form,

$$z(t) = H_i(p) \left\{ \left[-A^2 m^2(t) + \frac{n_1^2(t)}{2} - \frac{n_2^2(t)}{2} - (2)^{1/2} A m(t) n_2(t) \right] \cos(2\omega_0 t + 2\theta) + [(2)^{1/2} A m(t) n_1(t) + n_1(t) n_2(t)] \sin(2\omega_0 t + 2\theta) \right\} \quad (4)$$

The output of the multiplier is $\epsilon(t) = K_m z(t) r(t)$, where K_m is the multiplier constant. A convenient representation for $r(t)$ is denoted by

$$r(t) = (2)^{1/2} \sin[2\omega_0 t + 2\hat{\theta}] \quad (5)$$

If one takes only those terms in the base band frequency region, the product $r(t) z(t)$ becomes

$$z(t) r(t) = \frac{K_m H_i(p)}{2} \left\{ \left[A^2 m^2(t) - \frac{n_1^2(t)}{2} - \frac{n_2^2(t)}{2} + (2)^{1/2} A m(t) n_2(t) \right] \sin[2(\theta - \hat{\theta})] + [(2)^{1/2} A m(t) n_1(t) + n_1(t) n_2(t)] \cos[2(\theta - \hat{\theta})] \right\} \quad (6)$$

The phase $\hat{\theta}(t)$ of the voltage control oscillator (VCO) output is related to its input through

$$\hat{\theta}(t) = \frac{K_{VCO}}{p} z(t) r(t) F(p) \quad (7)$$

where K_{VCO} is the VCO gain constant in rad/sec-v. Neglecting any doppler present (this will be small in practice) on $\theta(t)$ we have from Eqs. (6) and (7) the following stochastic differential equation of operation of a squaring loop, viz.,

$$p\phi + \frac{K_m K_{VCO} A^2 m^2(t) H_i^2(p) F(p) \sin 2\phi}{2} = u(t, \phi) \quad (8)$$

where $\phi = \theta - \hat{\theta}$ and

$$u(t, \phi) = \frac{K_{VCO} K_m F(p) H_i^2(p)}{2} \left\{ \left[\frac{n_1^2(t)}{2} - \frac{n_2^2(t)}{2} - (2)^{1/2} A m(t) n_2(t) \right] \sin 2\phi - [(2)^{1/2} A m(t) n_1(t) + n_1(t) n_2(t)] \cos 2\phi \right\} \quad (9)$$

If we let $\Phi = 2\phi$, $K = K_{VCO} K_m$, assume that over the bandwidth of significant interest that the filter $H_i(p) = 1$, and consider a first-order PLL, i.e., $F(p) = 1$, we have

$$\dot{\Phi} + K A^2 m^2(t) \sin \Phi = K \left\{ \left[\frac{n_1^2(t)}{2} - \frac{n_2^2(t)}{2} - (2)^{1/2} A m(t) n_2(t) \right] \sin \Phi - [(2)^{1/2} A m(t) n_1(t) + n_1(t) n_2(t)] \cos \Phi \right\} \quad (10)$$

We may now determine the probability density of Φ , using the Fokker-Planck method. The equation of operation is of the form $\dot{\Phi} = F[\Phi, u(\Phi, t)]$ for which the corresponding Fokker-Planck equation (Ref. 11) is, in the stationary case,

$$\frac{1}{2} \frac{\partial^2}{\partial \Phi^2} [K_2(\Phi) p(\Phi)] - \frac{\partial}{\partial \Phi} [K_1(\Phi) p(\Phi)] = 0 \quad (11)$$

where

$$K_1(\Phi) = \overline{F[\Phi, u(\Phi, t)]}$$

and

$$K_2(\Phi) = \int_{-\infty}^{\infty} \overline{\{F[\Phi, u(\Phi, t)] F[\Phi, t + \tau]\}} d\tau - K_1^2(\Phi)$$

and the bar denotes statistical averaging over the ensemble. If we make the assumptions that Φ is a slowly varying process, $m(t) = \pm 1$, we find from Eqs. (10) and (11) that

$$K_1(\Phi) = KA^2 \sin \Phi$$

and

$$K_2(\Phi) = K^2 \sigma^2 \int_{-\infty}^{\infty} [\sigma^2 R_{n_1}^2(\tau) + 2A^2 R_{n_1}(\tau)] d\tau \quad (12)$$

where σ^2 and $R_{n_1}(\tau)$ are, respectively, the variance and the envelope of the correlation function of the noise component in Eq. (2). They correspond to the variance and correlation function of the independent processes $n_1(t)$ and $n_2(t)$ in Eq. (2). Substitution of Eq. (12) into the partial differential equation given in Eq. (11) and using the boundary conditions

$$\int_{-\pi}^{\pi} p(\Phi) d\Phi = 1$$

$$p(\Phi + 2\pi) = p(\Phi) \quad (13)$$

we have as a solution to Eq. (11)

$$p(\Phi) = \frac{\exp \left[\frac{AK}{K_2} \cos \Phi \right]}{2\pi I_0(AK/K_2)} \quad |\Phi| < \pi \quad (14)$$

where $I_0(x)$ is the modified Bessel function of zero order and of argument x . If we introduce the change of variable $\Phi = 2\phi$ and make use of the Jacobian of the transformation, we find that

$$p(\phi) = \frac{\exp [(AK/K_2) \cos 2\phi]}{\pi I_0(AK/K_2)}; \quad |\phi| < \pi/2 \quad (15)$$

As a first example of our results, assume that the normalized correlation function of the envelope of the input noise process possesses a Markov-type power spectrum with variance $\sigma^2 = N_0 B_i$, i.e.,

$$R_{n_1}(\tau) = \exp [-2B_i |\tau|] = R_{n_2}(\tau) \quad (16)$$

where B_i is the one-sided bandwidth of the noise $n_1(t)$ or $n_2(t)$. Physically, Eq. (16) represents a noise source that has been generated by passing white noise through an RC filter which possesses a 3-db frequency of $B_i/2\pi$ Hz. Thus, $K_2(\phi)$ in Eq. (12) becomes

$$K_2 = 2K^2 \sigma^2 \left[\frac{\sigma^2}{4B_i} + \frac{A^2}{B_i} \right] \quad (17)$$

and the solution in Eq. (15) is given by

$$p(\phi) = \frac{\exp [D \cos 2\phi]}{\pi I_0(D)}; \quad |\phi| \leq \pi/2 \quad (18)$$

where

$$D = \frac{x}{8} \left[\frac{1}{1 + \frac{32}{xy}} \right] \quad (19)$$

and

$$x = \frac{2A^2}{N_0 \omega_L}; \quad y = \frac{B_i}{\omega_L}$$

Here ω_L is taken to be the bandwidth of the loop, as defined from the linear PLL theory, i.e.,

$$\omega_L = 2b_L = \frac{1}{2\pi j} \int_{-\infty}^{\infty} |H(s)|^2 ds = A^2 K/4 \quad (20)$$

where $H(s)$ is the closed-loop transfer function of the loop in linearized form. The square of the signal amplitude A is present because of the squaring operation.

As a second example, assume that

$$R_{n_1}(\tau) = R_{n_2}(\tau) = \frac{\sin \pi B_1 \tau}{\pi B_1 \tau} \quad (21)$$

then it is easy to show that

$$D = \frac{\pi}{4} \left[\frac{1}{1 + 1/xy} \right] \quad (22)$$

where

$$x = \frac{2A^2}{N_0 w_L}; \quad y = \frac{w_L}{B_1}$$

Other presquaring filters may be easily evaluated. The two examples given represent results for the limiting cases of the class of Butterworth-type spectra.

If one assumes that ϕ is small, then the distribution of ϕ becomes Gaussian with variance

$$\sigma_\phi^2 = \frac{1}{4} \sigma_\Phi^2 = \left\{ x \left[\frac{1}{1 + 1/xy} \right] \right\}^{-1} \quad (23)$$

This result agrees with that obtained using linear PLL theory (Refs. 1-4). The variance of the phase-error Φ , as determined from Eq. (14), is

$$\sigma_\Phi^2 = \frac{\pi^2}{3} + 4 \sum_{k=1}^{\infty} \frac{(-1)^k I_k(D)}{k^2 I_0(D)}$$

where $I_k(D)$ is the modified Bessel function of order k and argument D . For large D , σ_Φ^2 approaches $1/D$, as it should.

3. The Costas-Loop

In the Costas-loop shown in Fig. 2, the phase of the data subcarrier is extracted from the suppressed carrier signal $s(t)$ plus noise $n(t)$ by multiplying the input voltages of the two phase detectors (multipliers) with that produced from the output of the VCO and a 90-deg phase shift of that voltage, filtering the results and using this

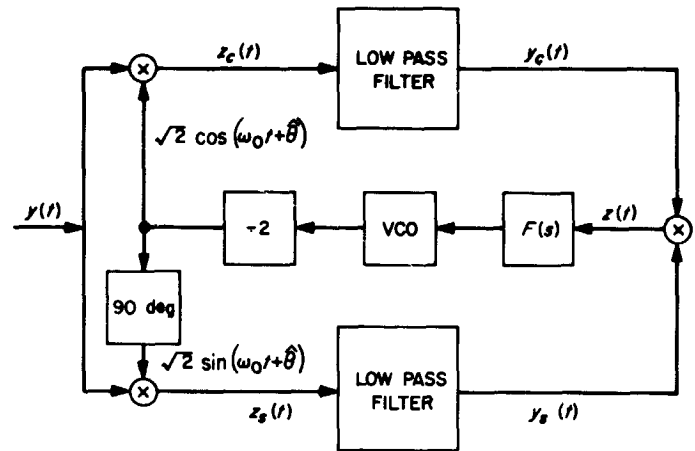


Fig. 2. The Costas-loop

signal to control the phase and frequency of the loop's VCO output.

If we denote the output of the upper loop multiplier by $z_c(t)$ and the output of the lower loop multiplier by $z_s(t)$ (see Fig. 2) then the output $z_c(t)$ is

$$z_c(t) = y(t) \cdot (2)^{1/2} \cos(\omega_0 t + \hat{\theta}) \quad (24)$$

while the output of the low-pass filter becomes

$$y_c(t) = \left[Am(t) + \frac{n_2(t)}{(2)^{1/2}} \right] \sin \phi + \frac{n_1(t)}{(2)^{1/2}} \cos \phi \quad (25)$$

when Eq. (1) is substituted into Eq. (24) and all double frequency terms are neglected. Similarly, the output $y_s(t)$ is given by

$$y_s(t) = \left[Am(t) + \frac{n_2(t)}{(2)^{1/2}} \right] \cos \phi - \frac{n_1(t)}{(2)^{1/2}} \sin \phi \quad (26)$$

The control voltage $z(t) = y_c(t) y_s(t)$ becomes

$$\begin{aligned} z(t) = & \frac{1}{2} \left(Am(t) + \frac{n_2(t)}{(2)^{1/2}} \right)^2 \sin 2\phi \\ & + \frac{n_1(t)}{(2)^{1/2}} \left(Am(t) + \frac{n_2(t)}{(2)^{1/2}} \right) \left(\frac{1 + \cos 2\phi}{2} \right) \\ & - \frac{n_1(t)}{(2)^{1/2}} \left(Am(t) + \frac{n_2(t)}{(2)^{1/2}} \right) \left(\frac{1 - \cos 2\phi}{2} \right) \\ & - \frac{n_1^2(t)}{4} \sin 2\phi \end{aligned} \quad (27)$$

Now

$$\hat{\theta} = K_{vco} K_m F(p) \cdot z(t) \quad (28)$$

and if we omit all dc terms, the stochastic differential equation which governs the behavior of the Costas-loop in the presence of noise reduces to

$$\begin{aligned} \Phi + KA^2 F(p) m^2(t) \sin \Phi = KF(p) \left\{ \left[\frac{n_1^2(t)}{2} - \frac{n_2^2(t)}{2} \right. \right. \\ \left. \left. - (2)^{1/2} A m(t) n_2(t) \right] \sin \Phi - [(2)^{1/2} A n_1(t) m(t) + n_1(t) n_2(t)] \cos \Phi \right\} \end{aligned} \quad (29)$$

If, in the previous case, we ignore the effects of the filter $H_i(p)$, then the stochastic differential equation obtained for the squaring-loop method and the stochastic differential equation for the Costas-loop are identical. Thus, the solution for $p(\Phi)$ is identical, and the noise behavior of the two circuits is the same. This, of course, assumes that the low-pass filter transfer functions can be obtained by simply translating, by f_0 Hz, the bandpass filter function of the squaring-loop.

From this it may be concluded that the two approaches to subcarrier tracking yield equivalent results when the filters in the Costas-loop are the low-pass equivalents of the bandpass filter in the squaring-loop. The choice of which loop to use cannot be determined on theoretical grounds, and consequently, must be determined from an engineering hardware point of view, i.e., the relative ease with which the corresponding filters can be constructed. Both methods of subcarrier tracking exhibit the usual 180-deg phase ambiguity inherent in all systems that attempt to recover the subcarrier phase from a modulated signal, i.e., changing the sign of the received signal leaves the sign of the recovered subcarrier unaltered.

An obvious question coming to mind is to ask for the presquaring filter which maximizes the signal-to-noise ratio (SNR) at the output of the phase-locked loop. This problem has been solved, and the optimum filter, for the case where the modulating spectrum is narrow with respect to the carrier frequency, has been shown (SPS 37-37, Vol. IV, p. 290) to be given by

$$H_i(p) = k \left(\frac{S_s(p)}{S_s(p) + N_0/2} \right)^{1/2} \quad (30)$$

where k is an arbitrary positive constant and $S_s(p)$ is the power spectrum of modulated signal $s(t)$. For large signal-

to-noise conditions the optimum presquaring filter given by Eq. (30) becomes $H_i(s) \sim k$ while for small signal-to-noise conditions the optimum filter becomes

$$H_i(p) \sim \left(\frac{2S_s(p)}{N_0} \right)^{1/2} \quad (31)$$

This says that for small signal-to-noise conditions the optimum filter is matched to the signaling spectrum. Arbitrarily setting $k = 1$ says that the optimum filter for large SNR is an ideal bandpass filter for which the performance has been assessed. On the other hand, for small SNR the performance of the two loops may be assessed once the spectrum of the modulated signal $s(t)$ is defined. It is our conjecture that the improvement over an ideal bandpass filter is negligibly small in the SNR region where such synchronization techniques are useful in practice. In the next section we show that squaring-loops or Costas-loops are most useful in data detection systems where the ratio of data rate R to the tracking loop bandwidth w_L is large, i.e., high data rate systems.

Various other approaches to the problem of estimating the subcarrier phase when no residual component is present at the subcarrier frequency are available, and in some cases have been analyzed. Layland (Ref. 9) and Proakis (Ref. 4) analyze methods which essentially estimate the modulation itself. This estimate is used in an attempt to eliminate the modulation from the subcarrier. This, therefore, provides an unmodulated sinusoid which can be tracked by a PLL.

4. Performance of Correlation Receivers

Consider the situation where $\{m(t)\}$ represents the set of signals $(x_k(t), k = 1, \dots, N)$. For the present we

assume that each signal in the set occurs with equal probability, contains equal energies, exists for a time duration of T seconds and is orthogonal, i.e.,

$$\int_0^T x_k(t) x_j(t) dt = \delta_{jk} \quad (32)$$

where $\delta_{jk} = 1$ for $j = k$ and $\delta_{jk} = 0$ for $j \neq k$. In the presence of white Gaussian noise the optimum receiver, i.e., the one which minimizes the error probability, computes

$$C_k = \int_0^T y(t) x_k(t) dt \quad (33)$$

for all $k = 1, \dots, N$ and makes its decision in favor of that signal which yields the largest C_k .

Of particular interest here is the case where the set of signals $\{x_k(t)\}$ are code words taken from an orthogonal code dictionary containing $N = 2^n$ code words, i.e., the signals are sequences of $+$ and $-$ 1's. In this case the time duration T becomes the product of the number of bits per code word times the time duration per bit, i.e., $T = nT_b$. If one assumes that word sync and symbol sync² (i.e., the instants in time where one word begins and another ends and the instants in time where the modulation may change states) are known exactly and that either the squaring-loop method or Costas-loop is used to provide subcarrier sync, the conditional probability that the data detector will err may be shown to be given by (Refs. 13 and 14)

$$P_E(\phi) = 1 - P_c(\phi) = 1 - \int_{-\infty}^{\infty} \frac{\exp(-y^2/2)}{(2\pi)^{1/2}} dy \left[\int_{-\infty}^{y + (2Rn)^{1/2} \cos \phi} \frac{1}{(2\pi)^{1/2}} \exp\left(-\frac{x^2}{2}\right) dx \right]^{2^n - 1} \quad (34)$$

where $R = A^2 T_b / N_0$. The average word error probability is obtained from Eq. (34) by averaging over the distribution of $p(\phi)$, i.e.,

$$P_E = \int_{-\pi/2}^{\pi/2} p(\phi) P_E(\phi) d\phi \quad (35)$$

Thus, from Eqs. (18), (34) and (35) we have³

$$P_E = 1 - \int_{-\pi/2}^{\pi/2} \frac{\exp(D \cos 2\phi)}{\pi I_0(D)} d\phi \int_{-\infty}^{\infty} \frac{\exp(-y^2/2)}{(2\pi)^{1/2}} dy \left[\int_{-\infty}^{y + (2nR)^{1/2} \cos \phi} \frac{\exp(-x^2/2)}{(2\pi)^{1/2}} dx \right]^{2^n - 1} \quad (36)$$

where

$$D = \frac{\delta y R}{4} \left[\frac{1}{1 + \frac{1}{\delta y R}} \right] \quad (37)$$

if an ideal bandpass filter precedes the squaring-loop or

$$D = \frac{\delta y R}{8} \left[\frac{1}{1 + \frac{32}{\delta y R}} \right] \quad (38)$$

if an RC filter centered around ω_n precedes the squaring-loop. The parameters R , δ , y and $\mathcal{R}$ are defined by

$$R = \frac{A^2 T_b}{N_0}; \quad \sigma = \frac{2\mathcal{R}}{\omega_L}; \quad y = \frac{\omega_L}{B_i}; \quad \mathcal{R} = \frac{1}{T_b} \quad (39)$$

We point out that the parameter $\mathcal{R}$ is the data rate of the system.

²This assumption is not too restrictive since jitter on the phase of the subcarrier is more deleterious on system performance than jitter about those instants in time with which the modulation may change states. This, of course, is a consequence of coherent detection.

³In some cases the bit-error probability is of interest. The ratio of the bit-error probability to the word error probability is $2^{n-1}/2^n - 1$ (Ref. 14).

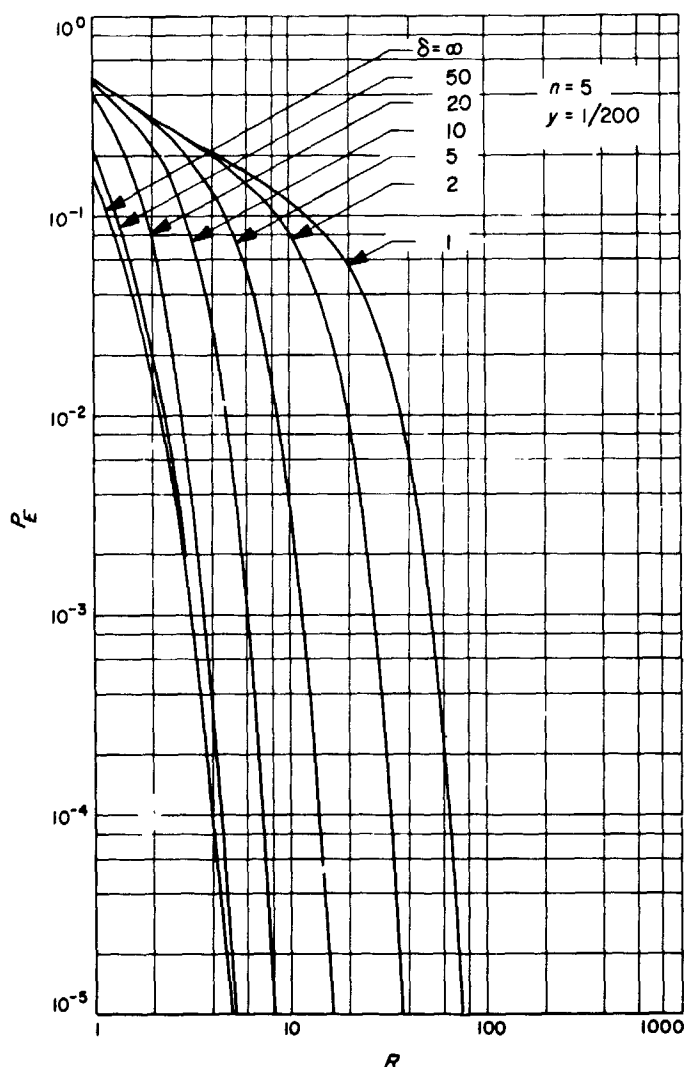


Fig. 3. Word error probability P_E vs R ($n = 5$)

Eq. (36) has been integrated numerically using an IBM 704 computer for $n = 5-8$. The results of the numerical integration are illustrated in Figs. 3-6 for the situation where an ideal bandpass filter precedes the PLL. The parameter γ was set at $1/2000$, since this is typical of what might be encountered in practice. It is clear from these figures that obtaining subcarrier sync by the method outlined here is most beneficial in systems where $\delta = 2R/\omega_L \gg 1$, i.e., high data rate systems.

One may proceed to develop the performance of such a system for biorthogonal codes. However, if one recalls that for $n \geq 5$, the performance of systems which utilize orthogonal code dictionaries is approximately equal to systems which employ biorthogonal code dictionaries; then the results presented in Figs. 3-6 may be used in carrying out a particular design where biorthogonal codes

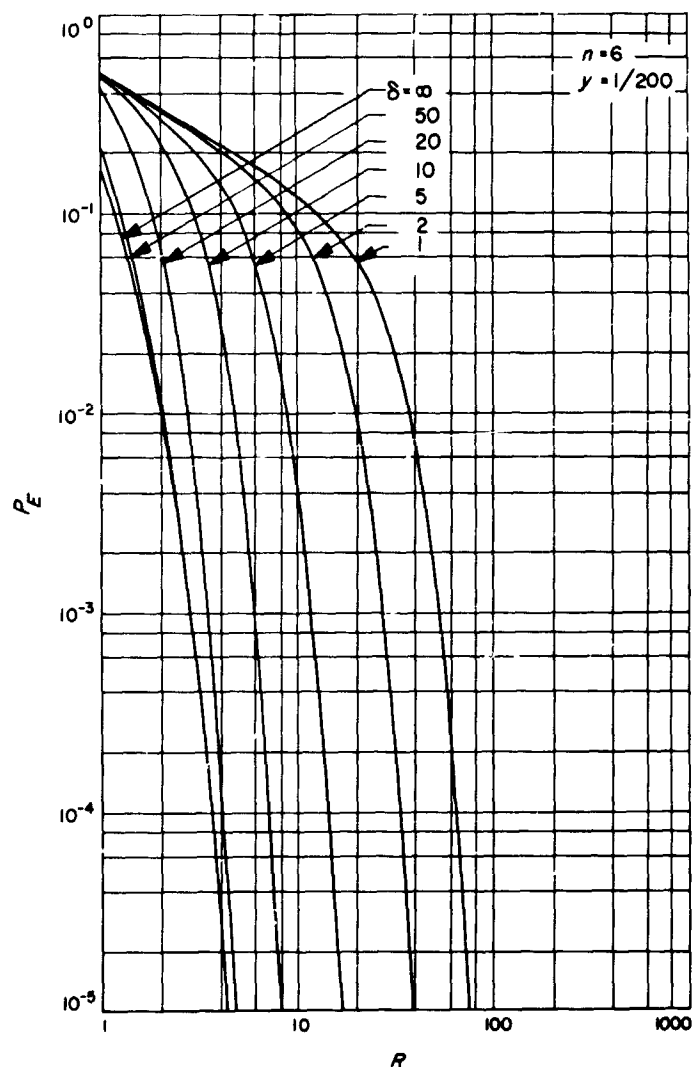


Fig. 4. Word error probability P_E vs R ($n = 6$)

are used. The performance of a block-coded system which uses biorthogonal codes is given by

$$P_E = 1 - \int_{-\pi/2}^{\pi/2} P_c(\phi) \frac{\exp[D \cos 2\phi]}{\pi I_0(D)} d\phi \quad (40)$$

where

$$P_c(\phi) = \int_{-(2nR)^{1/2} \cos \phi}^{\infty} \frac{\exp(-y^2/2)}{(2\pi)^{1/2}} \times \left[\int_{-y + (2nR)^{1/2} \cos \phi}^{y + (2nR)^{1/2} \cos \phi} \frac{\exp(-x^2/2)}{(2\pi)^{1/2}} dx \right]^{2^{n-1}-1} dy \quad (41)$$

For $n \geq 5$, numerical integration of Eq. (38) on the IBM 7090 produces results, for all practical purposes, equivalent to those shown in Figs. 3-6.

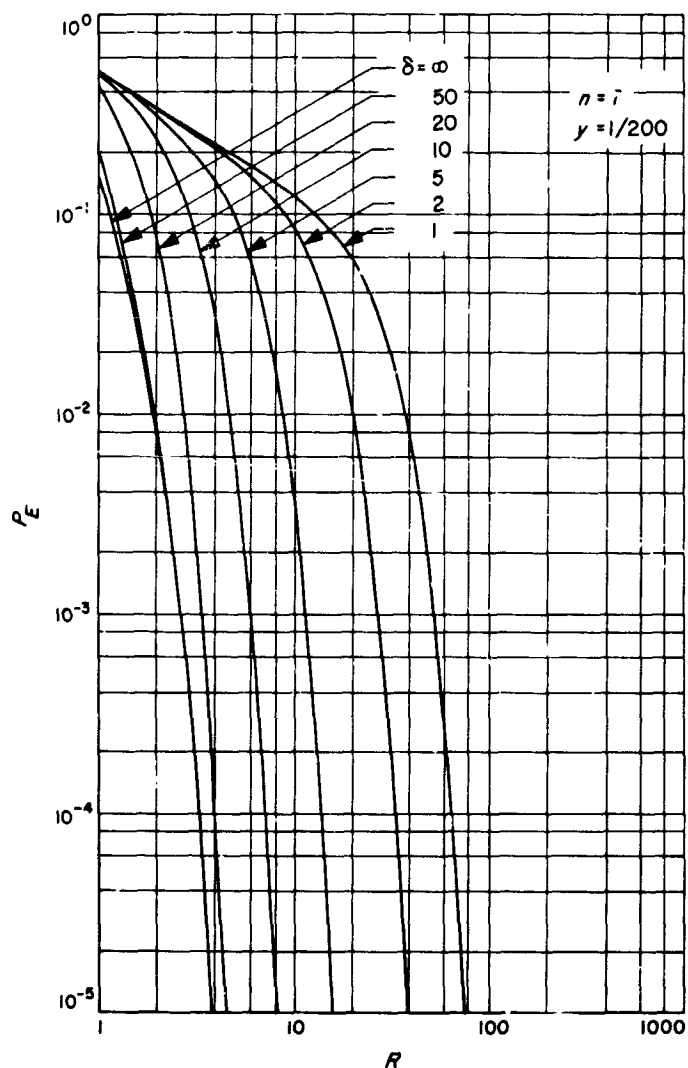


Fig. 5. Word error probability P_E vs R ($n = 7$)

Finally, it is of interest to understand how the value of $y = B_i/\omega_L$ affects the performance of a particular design. This trend is best illustrated, for various values of y , in Figs. 7-9 for uncoded telemetry systems, and in Figs. 4, 10 and 11 for block-coded telemetry systems. The results given in Figs. 7-9 were obtained by numerical integration of Eq. (40) with $n = 1$, while the results given in Figs. 4, 10 and 11 are, for all practical purposes, valid for biorthogonal codes, even though they were computed from Eq. (34). This is due to the fact, mentioned earlier, that for $n \geq 5$ the performance of telemetry systems which employ orthogonal codes is approximately equivalent to that of telemetry systems which employ biorthogonal codes (Refs. 13 and 14). An obvious conclusion, which may be reached here, is that for a fixed δ and R system, performance improves as the ratio $y = B_i/\omega_L$ becomes larger. This result is comprehensible from a physical point of view.

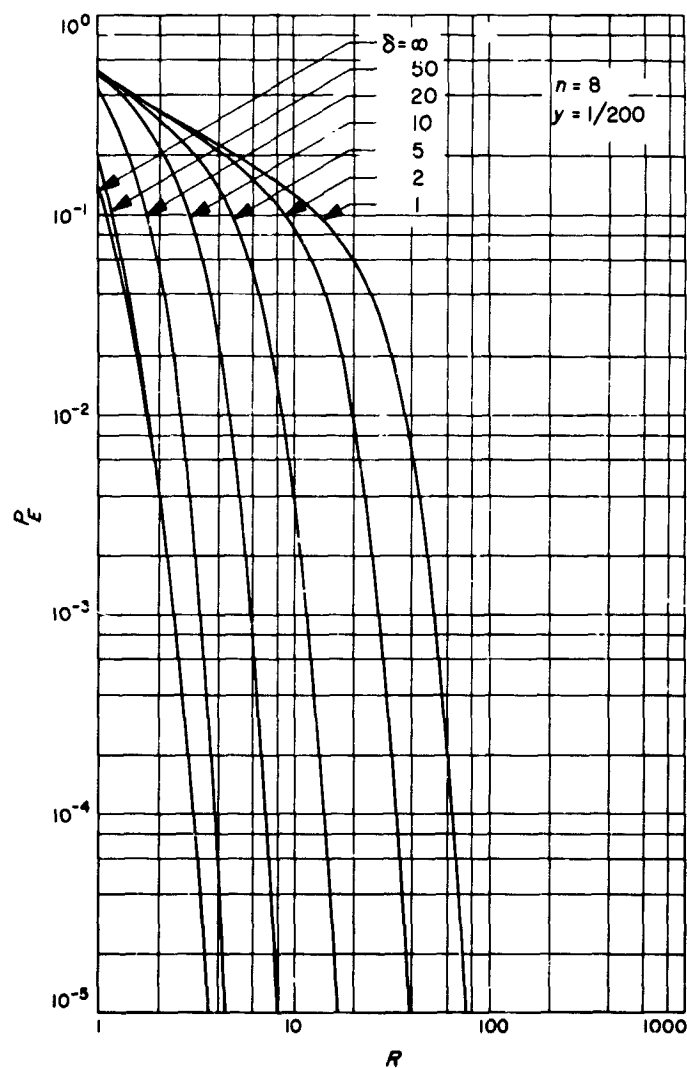


Fig. 6. Word error probability P_E vs R ($n = 8$)

5. Conclusion

A model probability distribution for the phase-error exhibited by the squaring-loop or Costas-loop has been derived using the Fokker-Planck equation. The parameters of this distribution are evaluated in terms of the covariance function of the input noise and, in particular, for two specific noise spectra. The model distribution is then used to assess the degradation in performance of a coded or uncoded telemetry system which tracks the phase of the subcarrier, using this method. If the phase of a suppressed carrier signal is derived from the modulated data subcarrier by means of a Costas-loop or a squaring-loop, the critical design parameter, which indicates the usefulness of such tracking loops in the demodulation process, is the ratio of the data-rate to the bandwidth of the loop. In the case of coded systems, this implies high-data rates for error rates less than 10^{-2} .

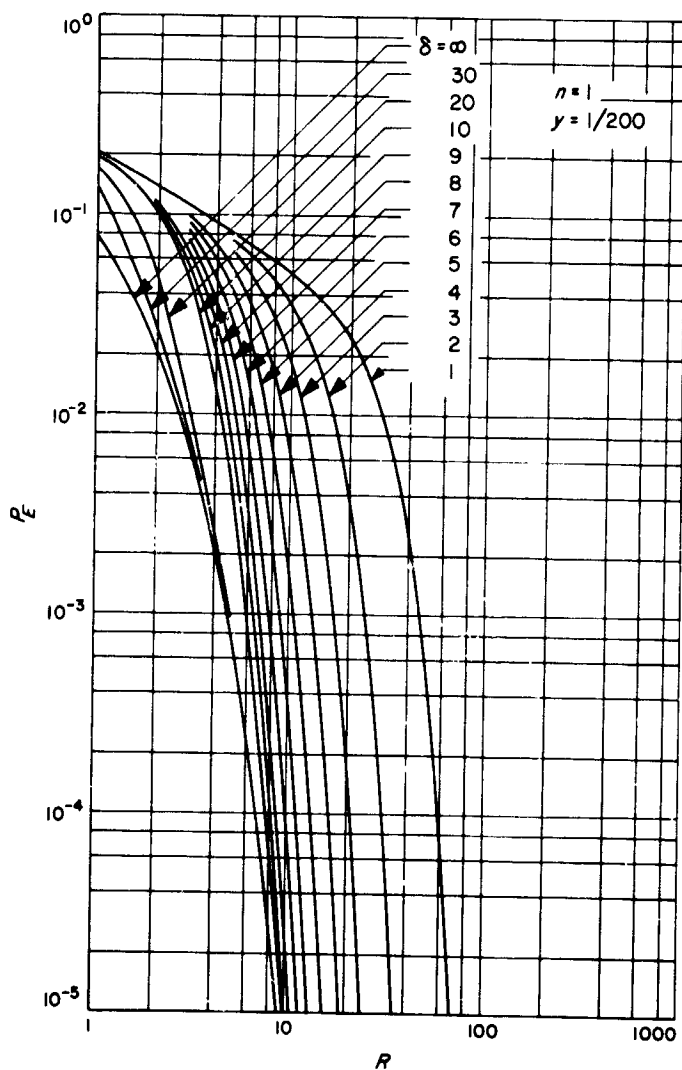


Fig. 7. Bit error probability P_E vs R ($n = 1, \gamma = 1/200$)

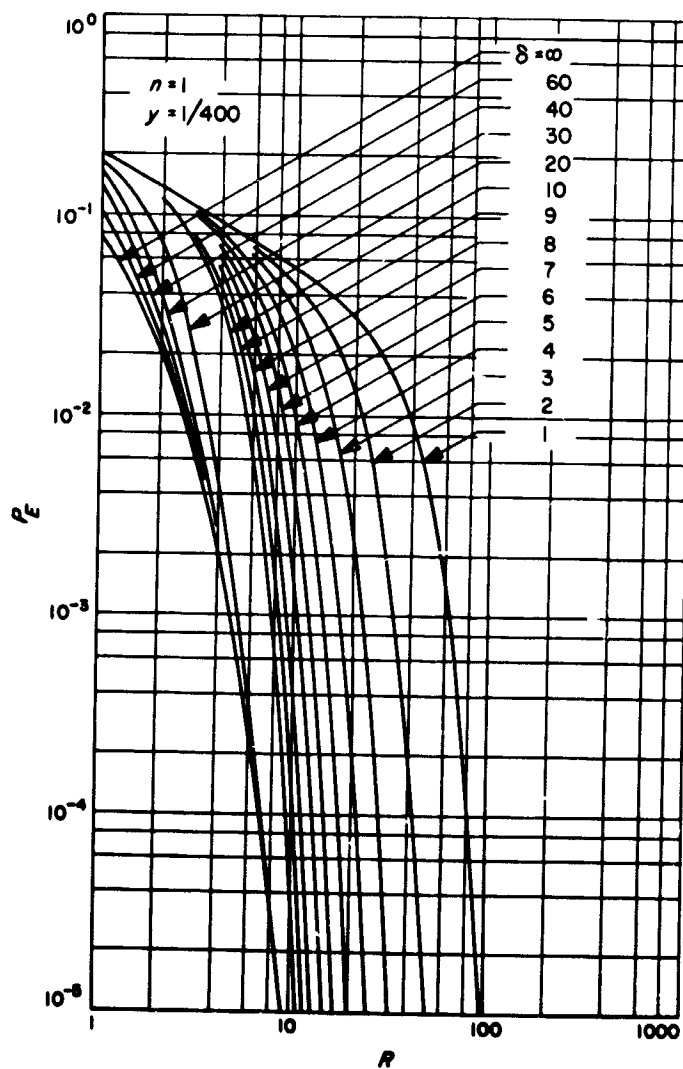


Fig. 8. Bit error probability P_E vs R ($n = 1, \gamma = 1/400$)

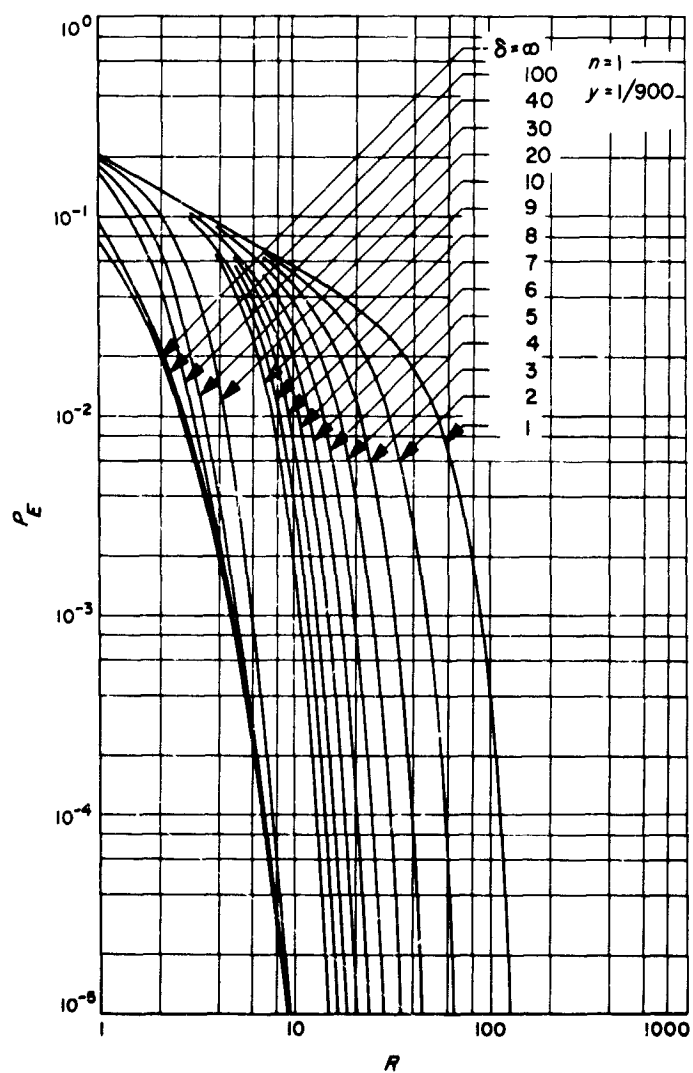


Fig. 9. Bit error probability P_E vs R ($n = 1, \gamma = 1/900$)

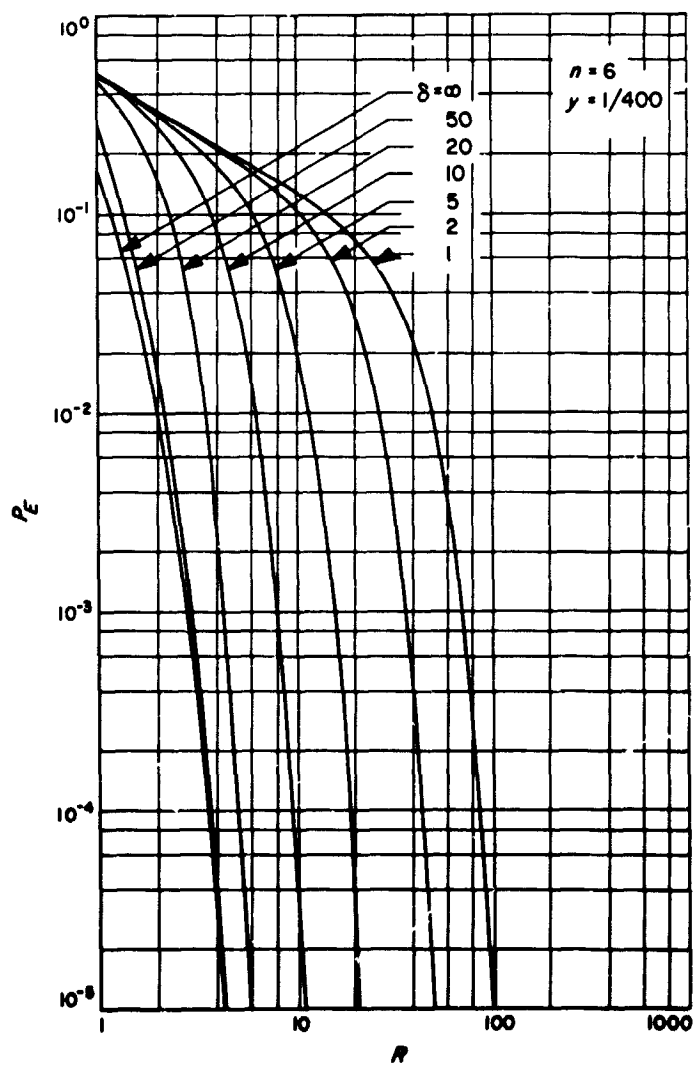


Fig. 10. Word error probability P_E vs R ($n = 6, \gamma = 1/400$)

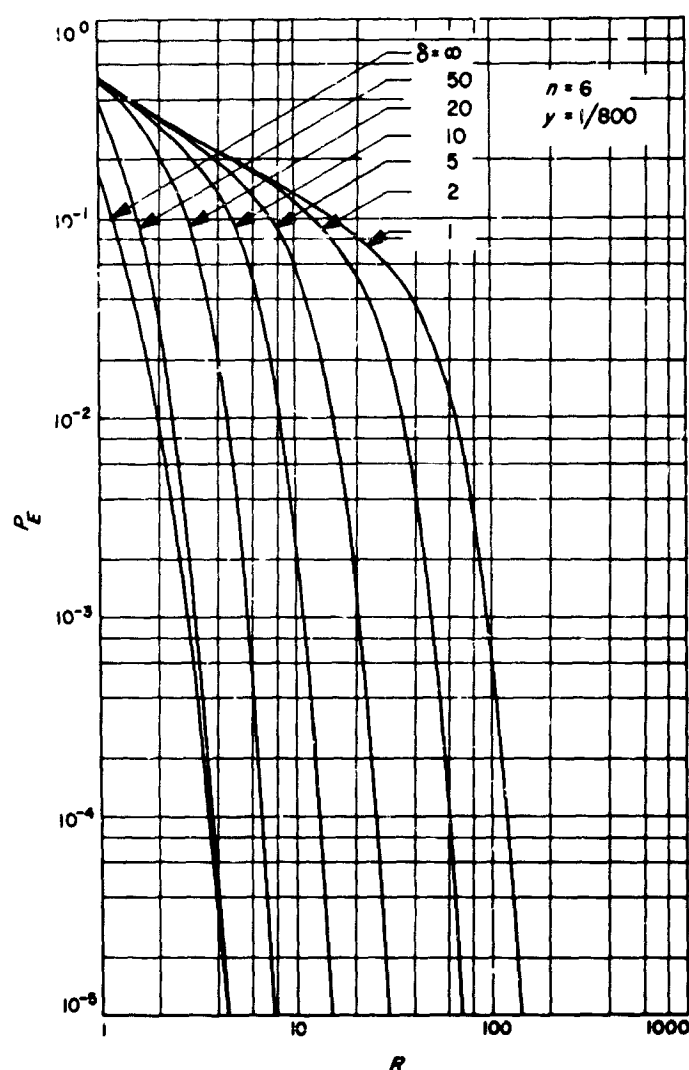


Fig. 11. Word error probability P_E vs R ($n = 6$, $\gamma = 1/800$)

B. Block Coding and Synchronization Study: Power Allocation and Noisy Reference Losses in Phase Coherent Communication Systems, W. C. Lindsey

1. Introduction

The problem of power allocation in deep-space telemetry and command systems is becoming more important in the design and engineering of communication systems. The reason, of course, is that it is necessary to be able to predict accurately, prior to launch, the actual behavior and performance of the system at various times after launch. If this can be accomplished with precision, engineering tradeoffs may be recognized, and the cost of the mission may be minimized with respect to mission yield. This is becoming particularly evident in the operation of

a complex communication network, such as the deep space network.

In this article we present a method whereby the total transmitter power may be optimally allocated in a single-channel, phase-coherent communication system of the type discussed in Ref. 14. The novelty of the method lies in the fact that it is simple and can be carried out without the aid of a general purpose digital computer. The method takes into consideration the radio-frequency (RF) carrier phase-jitter due to a noisy RF reference. The power is allocated on the basis of minimizing the probability that the data detector will err in making its decision. Other results are given which enable one to determine the losses due to noisy demodulation references in one-way and two-way systems. The symbols used in our calculations are defined in Table 1.

2. Basic System Model

In order to shorten the subsequent derivation, we draw heavily upon previous results and the notation given in Refs. 14-16. The basic form of a two-way coherent communication link is depicted in Fig. 12. Briefly,⁴ the reference transmitter phase modulates the RF carrier, say

⁴Reference 14 gives a more detailed description of the overall system.

Table 1. Definition of symbols

Symbol	Definition
$n = 1$	One-way communications
$n = 2$	Two-way communications
P_a	Total average transmitted power
T_{ba}	Time duration per bit
R_a	System data rate
N_{0a}	Noise spectral density (single-sided)
ϕ_a	System phase-error
$F_a(s)$	Carrier tracking loop filter
w_{La}	Bandwidth of carrier tracking loop
r_a	Second-order loop parameter ratio $(P)^{1/2} K \tau_c^2 / \tau_i$
$H_a(s)$	Closed loop transfer function of the carrier tracking loop assuming linear PLL theory
δ_a	Ratio of system data rate to carrier tracking loop bandwidth
R_a	Total signal power-to-noise spectral density ratio times the duration per bit
G	Static phase gain
β	Ratio of carrier tracking loop bandwidth in the vehicle system to that in the reference system
$P_E(n)$	System error probability
$P_{E0}(n)$	Minimum system error probability
R_{d1}	SNR in data (one-way link)
R_{d2}	SNR in data (two-way link)
α	SNR in carrier tracking loop

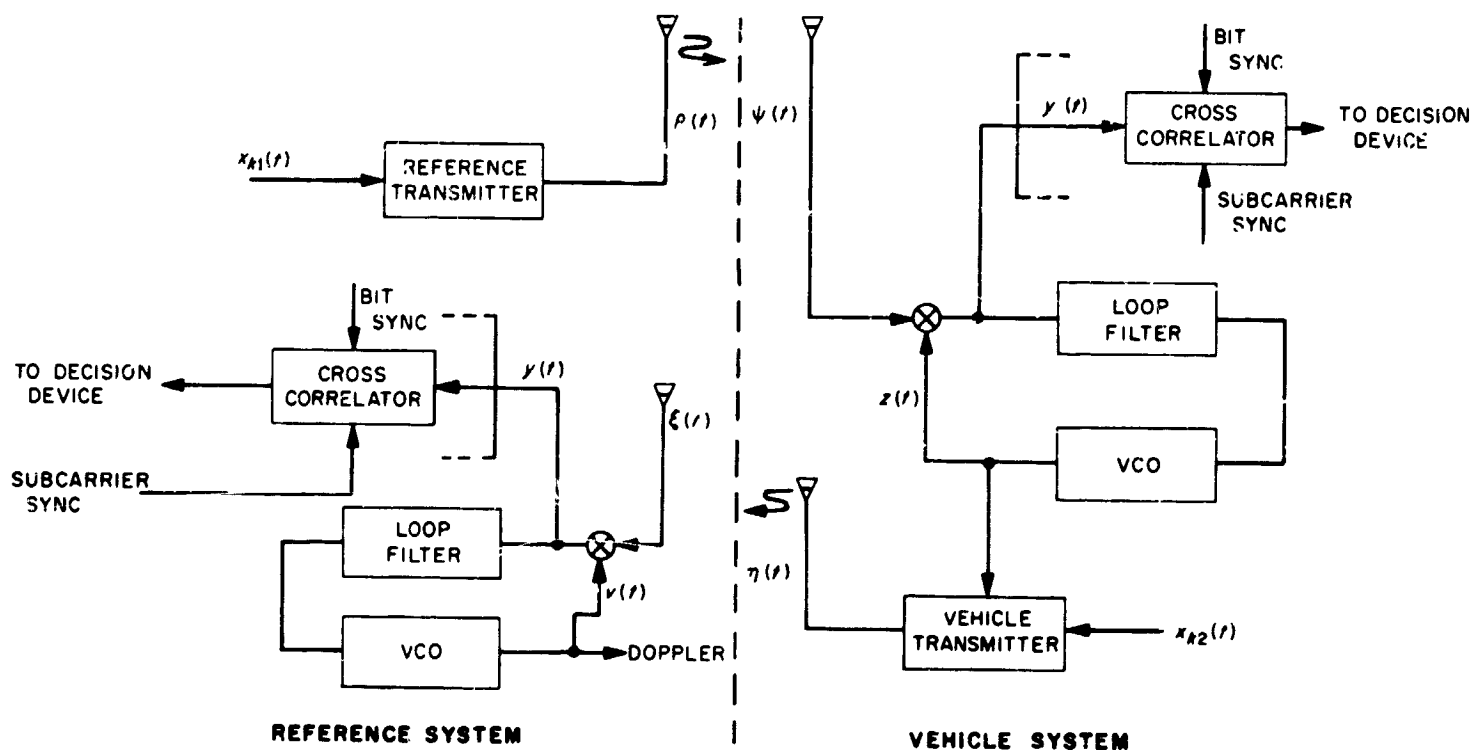


Fig. 12. Two-way communication link

$c(t)$, with one of two equally probable, equienergy signals $x_k(t)$, $k = 1, 2$ of time duration T_{b1} sec. This signal is conveniently denoted by

$$\rho(t) = (2P_1)^{1/2} \sin [\omega t + (\cos^{-1} m_1) x_{k1}(t)] \quad (1)$$

where P_1 is the total radiated power, and m_1 is the modulation factor associated with the reference system. The channel introduces an arbitrary (but unknown) phase shift in the transmitted waveform and further disturbs $\rho(t)$ with additive white Gaussian noise $n_1(t)$ of single-sided spectral density of N_{01} w/Hz. Thus, we observe in the vehicle the following signal:

$$\psi(t) = (2P_1)^{1/2} \sin [\omega_1 t + (\cos^{-1} m_1) x_{k1}(t) + \theta_1] + n_1(t) \quad (2)$$

The vehicle tracks the carrier component in $\psi(t)$ by means of a narrow band phase-locked loop (PLL). The output $z(t)$ of the voltage control oscillator (VCO) of this tracking filter is used as a coherent reference in demodulating $\psi(t)$. The vehicle reference waveform $z(t)$ is conveniently taken to be

$$z(t) = (2)^{1/2} \cos (\omega_1 t + \hat{\theta}_1) \quad (3)$$

where $\hat{\theta}_1$ is the PLL estimate of θ_1 in the presence of noise. After neglecting the double frequency components and assuming that the data biphas modulates a square wave subcarrier, it may be shown (Ref. 14) that multiplication of $\psi(t)$ with $z(t)$ produces

$$y(t) = [(1 - m_1^2) P_1]^{1/2} x_{k1}(t) \cos \phi_1 + n'_1(t) \quad (4)$$

where $n'_1(t)$ is white Gaussian noise of single-sided spectral density of N_{01} w/Hz, and $\phi_1 = \theta_1 - \hat{\theta}_1$ is the vehicle subsystem phase error. We assume that this phase error is constant for at least T_{b1} sec. Also, we point out (Ref. 14) that m_1 represents the square root of the ratio of the power remaining in the carrier component to the total power radiated, i.e., $m_1 = (P_{c1}/P_1)^{1/2}$.

The decision in the vehicle is made in favor of that signal which gives rise to the largest cross correlation, i.e., the vehicle demodulator computes

$$q = \int_0^{T_{b1}} y(t) [x_{11}(t) - x_{21}(t)] dt \quad (5)$$

and compares the result with zero. If $q > 0$, x_{11} is announced, and if $q < 0$, x_{21} is announced.

In the reverse direction, i.e., transmission of the data back to the reference subsystem, the output of the vehicle's VCO is used as a carrier for transmission of one of two equienergy, equiprobable waveforms $x_{k2}(t)$, $k = 1, 2$ of time duration T_{b2} sec. In this case, the output of the vehicle is conveniently represented by the following waveform

$$\eta(t) = (2P_2)^{1/2} \sin[\omega_2 t + (\cos^{-1} m_2) x_{k2}(t) + \hat{\theta}_1] \quad (6)$$

Here m_2 is the modulation factor which represents the square root of the ratio of the power in the carrier to the total power radiated, i.e., $m_2 = (P_{c2}/P_2)^{1/2}$, (Ref. 14).

It is clear that using the vehicle VCO output as a carrier reference introduces into the down-link an additional component of noise; however, incorporating this measurement into the system allows one to perform, with extreme accuracy, a two-way doppler measurement. Thus, we postulate a mathematical model of the system so as to include this up-link jitter component, hence, the two-way doppler measurement. However, as we shall see, adjustment of certain parameters will immediately alleviate this up-link RF jitter. The down-link channel (assumed to be statistically independent from the up-link channel) further perturbs $\eta(t)$ by inserting an unknown phase shift θ_2 and additive white Gaussian noise $n_2(t)$ of single-sided spectral density of N_{02} w/Hz. Thus, the reference receiver observes

$$\xi(t) = (2P_2)^{1/2} \sin[\omega_2 t + (\cos^{-1} m_2) x_{k2}(t) + \hat{\theta}_1 + \theta_2] + n_2(t) \quad (7)$$

The ground receiver tracks the carrier component in $\xi(t)$ for the purpose of measuring the doppler and demodulating the data. We denote the output of the reference VCO by

$$v(t) = (2)^{1/2} \cos(\omega_2 t + \hat{\theta}_2) \quad (8)$$

where $\hat{\theta}_2$ is the estimate of phase of the observed carrier component. Multiplying $\xi(t)$ by $v(t)$ and neglecting the double-frequency components, one may show (Ref. 14) that

$$y(t) = ((1 - m_2^2) P_2)^{1/2} x_{k2}(t) \cos \phi_2 + n'_2(t) \quad (9)$$

where $\phi_2 = \theta_2 + \hat{\theta}_1 - \hat{\theta}_2$ is the reference system phase error and is assumed constant for the duration T_{b2} of the

signals $x_{k2}(t)$. Again $n'_2(t)$ is easily shown to be white with a single-sided spectral density of N_{02} w/Hz.

To recapitulate, we see that the design engineer has at his disposal several communication parameters. For the up-link we have the total power radiated P_1 , the single-sided noise spectral density N_{01} , up-link data rate $\mathcal{R}_1 = T_{b1}^{-1}$, vehicle carrier tracking loop bandwidth w_{L1} , and modulation index $m_1 = (P_{c1}/P_1)^{1/2}$. The corresponding down-link parameters are P_2 , N_{02} , $\mathcal{R}_2 = T_{b2}^{-1}$, w_{L2} , and $m_2 = (P_{c2}/P_2)^{1/2}$. In the subsections that follow, we relate these parameters together and determine that value of m_n ($n = 1, 2$) which minimizes the probability of error $P_e(n)$ ($n = 1, 2$), say $P_{e_n}(n)$, for a fixed data rate-to-carrier tracking loop bandwidth ratio, say $\mathcal{R}_n/w_{Ln}$ ($n = 1, 2$). Also the losses, in signal-to-noise ratios (SNR), due to noisy demodulation references are determined.

3. System Phase-Error Distribution

The probability distribution for the subsystem phase error is of great importance in specifying the performance of the two-way link. In fact, the distribution of this phase-error has been previously characterized (Ref. 14), and its probability density function is conveniently represented by

$$P(\phi_2) = \frac{I_0(|\alpha_1 + \alpha_2 \exp(j\phi_2)|)}{2\pi I_0(\alpha_1) I_0(\alpha_2)}, \quad |\phi_2| < \pi \quad (10)$$

where $I_0(x)$ is the imaginary Bessel of zero order and argument x , and α_1 and α_2 are related to the up- and down-link parameters. The validity of using this distribution as a model for the phase-error distribution has been given in Refs. 14 and 15.

In passing we point out that the loop filter, which is of greatest interest in practice, is of the proportional-plus-integral control type,⁵ i.e.,

$$F_n(s) = \frac{1 + \tau_{2n}s}{1 + \tau_{1n}s}; \quad n = 1, 2 \quad (11)$$

If one relates the basic parameter of the carrier tracking loops, i.e., the loop bandwidth w_{Ln} , $n = 1, 2$, to the time

⁵The subscript $n = 1$ refers to parameters, filters, etc. in the vehicle system or one-way operation; while $n = 2$ refers to parameters, filters, etc. in the reference system.

constants in $F_n(s)$, we have

$$w_{Ln} = 2B_{Ln} = \frac{r_n + 1}{2\tau_{Ln}}; \quad r_n = \frac{(P_n)^{1/2} K_n \tau_{Ln}}{\tau_{1n}} \quad (12)$$

where K_n is the equivalent simple-loop gain which depends upon the VCO constant and the multiplier constant, Ref. 16, p. 30. The loop bandwidth B_{Ln} is defined as

$$w_{Ln} = 2B_{Ln} = \frac{1}{2\pi j} \int_{-\infty}^{\infty} |H_n(s)|^2 ds; \quad n = 1, 2 \quad (13)$$

where

$$H_n(s) = \frac{1 + \left(\frac{r_n + 1}{4B_{Ln}}\right)s}{1 + \left(\frac{r_n + 1}{4B_{Ln}}\right)s + \frac{1}{r_n} \left(\frac{r_n + 1}{4B_{Ln}}\right)^2 s^2}; \quad n = 1, 2 \quad (14)$$

and $H_n(s)$, $n = 1, 2$ is the closed loop transfer function of the carrier tracking loops in linearized form. The transfer function of the ground receiver's carrier tracking loop is given by Eq. (14) with $n = 2$, and that of the spacecraft's carrier tracking loop is given by Eq. (14) with $n = 1$.

The parameters α_1 and α_2 , which serve to characterize $p(\phi_2)$, are given by

$$\alpha_1 = \frac{2m_1^2 P_1}{N_{01} w_{L1}} \cdot \frac{1}{K(r_1, r_2, \beta)} \quad \alpha_2 = \frac{2m_2^2 P_2}{N_{02} w_{L2}} = m_2^2 \delta_2 R_2 \quad (15)$$

where

$$\delta_2 = \frac{R_2}{w_{L2}} = \frac{1}{T_{b2} w_{L2}} \quad R_2 = \frac{2P_2 T_{b2}}{N_{02}} \quad (16)$$

defined by

$$K(r_1, r_2, \beta) = \frac{r_1 \beta G^2}{(r_1 + 1) r_2} \times \left[\frac{r_2 + r_1 r_2 \beta (1 + \beta) + \beta^2 (1 + \beta) + r_1 \beta^3}{r_2/r_1 + r_2 \beta + \beta^2 (r_2 + r_1 - 2) + r_1 \beta^3 + r_1 \beta^4/r_2} \right] \quad (17)$$

with

$$\beta = \frac{B_{L1}(r_2 + 1)}{B_{L2}(r_1 + 1)}$$

and G is the static phase gain of the spacecraft receiver, which is determined by the ratio of the input frequency to the output frequency at the spacecraft. In practice, the values of β and G are chosen such that $K(r_1, r_2, \beta)$ is approximately unity; hence, α_1 is approximately the SNR existing in the carrier tracking loop bandwidth.

4. Losses Due to Noisy Demodulation References in One-Way and Two-Way Systems

Before proceeding with the problem of power allocation, we determine the RF losses due to a noisy carrier reference in one-way and two-way telemetry links. For two-way links, the significant contributing factors in this loss are the noise in the ground receiver's demodulation reference $\hat{\theta}_2$ and the phase modulation existing on the vehicles carrier produced by the up-link additive noise.

a. Losses in one-way links. One-way telemetry reception, i.e., reception when the vehicle is operating with an auxiliary oscillator as a frequency reference, has been presented and discussed in Refs. 17 and 18. For the sake of completeness, we include a graph (Fig. 13) which enables the design engineer to evaluate such losses when the reference phase-error is constant over a bit period. These results are obtained by numerically integrating the expression, which specifies the bit-error probability $P_E(1)$ as a function of the SNR in the data, say R_{d1} , and the SNR in the reference system's carrier tracking loop. This bit-error probability has been shown, Ref. 14, to be given by

$$P_E(1) = \lim_{\alpha_2 \rightarrow \infty} \int_{-\pi}^{\pi} p(\phi_2) \operatorname{Erfc}[(2R_{d1})^{1/2} \cos \phi_2] d\phi_2 \quad (18)$$

$w_{L2} \rightarrow 0$

where

$$\operatorname{Erfc}(x) = \frac{1}{(2\pi)^{1/2}} \int_x^{\infty} \exp(-y^2/2) dy \quad (19)$$

Eq. (18) has been plotted in Fig. 14 for various values of the parameters $R_{d1} = (1 - m_1^2) R_1$ and α . Here $\alpha = 2P_{c1}/N_{01} w_{L1} = 2m_1^2 P_1/N_{01} w_{L1}$ denotes the SNR in the carrier tracking loop, and R_{d1} is the SNR in the data channel. The above results indicate the importance of establishing the proper SNR in the carrier tracking loop. If this is not done, a significant loss over the theoretical performance is quite pronounced.

b. Losses in two-way links. For two-way systems, the probability that the data detector will err in its decision

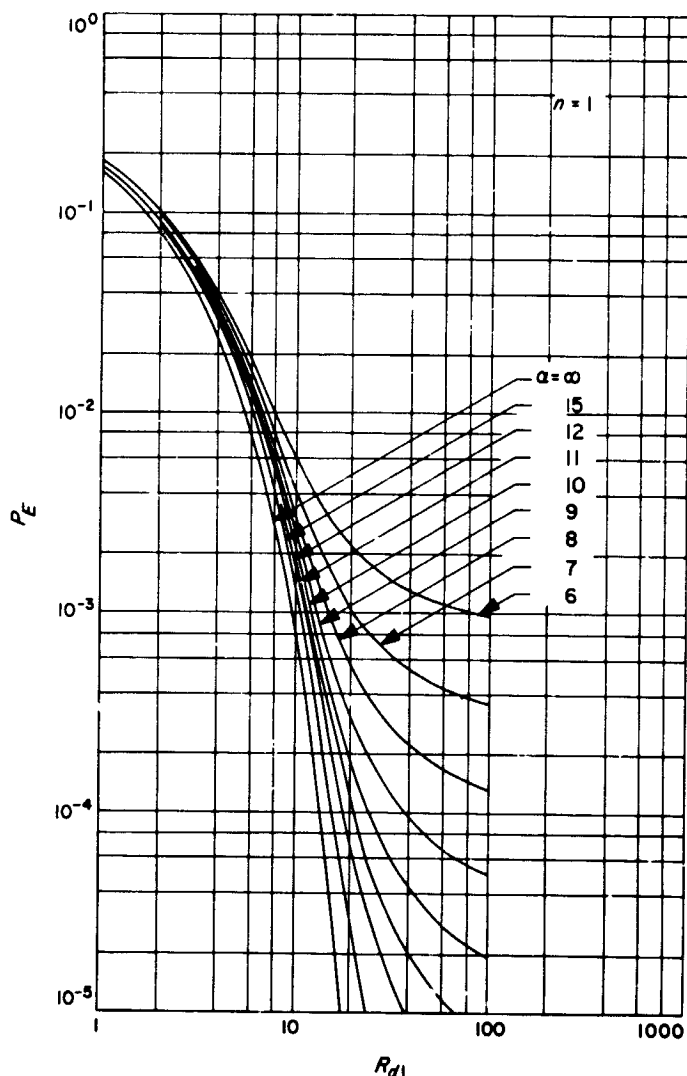


Fig. 13. Error probability vs the SNR R_{d1} for various values of the parameter α

may be evaluated following the procedure given in Ref. 14. In fact, it is easy to show that the average bit error probability is given by, Ref. 14,

$$P_E(2) = \int_{-\pi}^{\pi} p(\phi_2) \operatorname{Erfc} [((1 - m_2^2) R_2)^{1/2} \cos \phi_2] d\phi_2 \quad (20)$$

This equation has been plotted in Figs. 15–17 for various values of the parameters α_1 and α_2 , where $(1 - m_2^2) R_2 = R_{d2}$ is the SNR existing in the data channel. These figures show the effect of varying α_1 (which, in practice, is approximately equal to the SNR existing in the vehicle's carrier tracking loop) when the SNR in the ground receiver's carrier tracking loop α_2 is held constant. Thus,

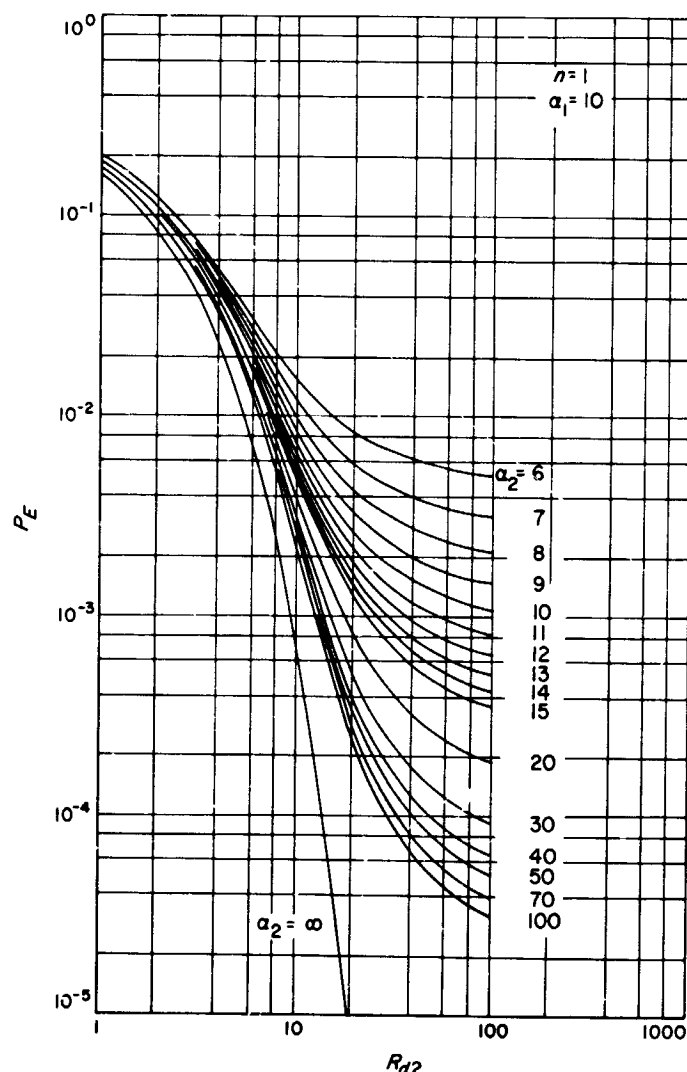


Fig. 14. Error probability vs the SNR R_{d2} for various values of the parameter α_1 with $\alpha_2 = 10$

the losses due to noisy carrier references are clearly manifested. These figures also indicate that the selection of the modulation factors m_1 and m_2 must take these into consideration. This selection is the subject of the next subsection.

5. Power Allocation and System Performance

In this subsection we treat the problem of dividing the power between the carrier component and the sidebands so as to minimize the probability of error. Simple formulas will be developed which allow the design engineer to compute the modulation factors m_n , $n = 1, 2$ without the aid of a digital computer. Finally, design curves will be given which allow one to make engineering tradeoffs and carry out the particular design.

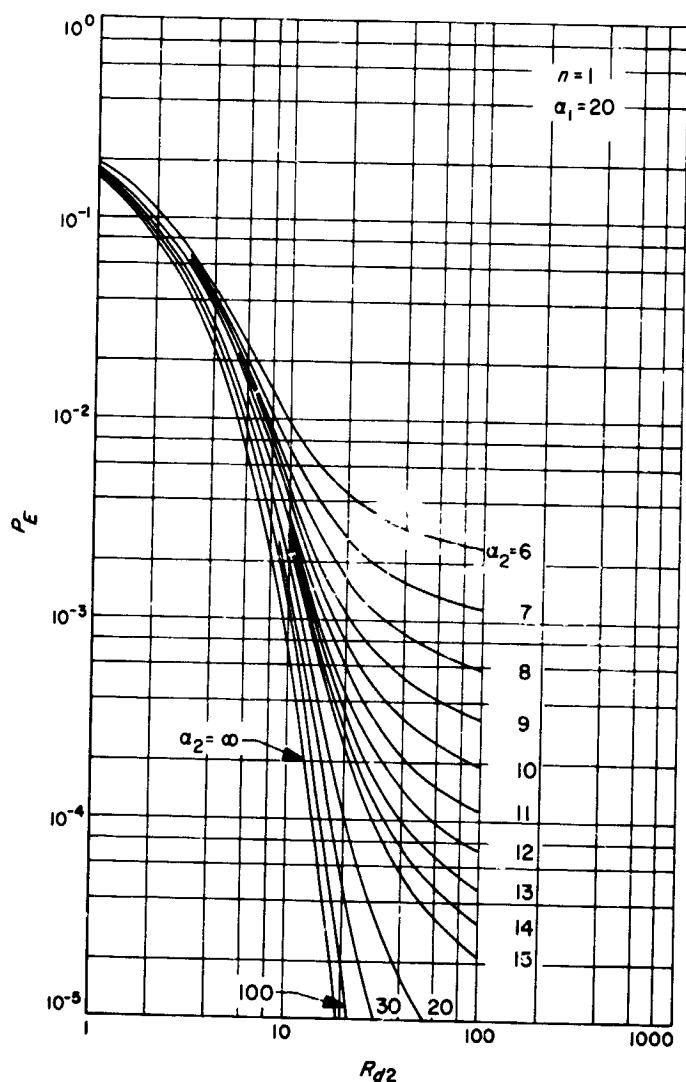


Fig. 15. Error probability vs the SNR R_{d2} for various values of the parameter α_1 with $\alpha_2 = 20$

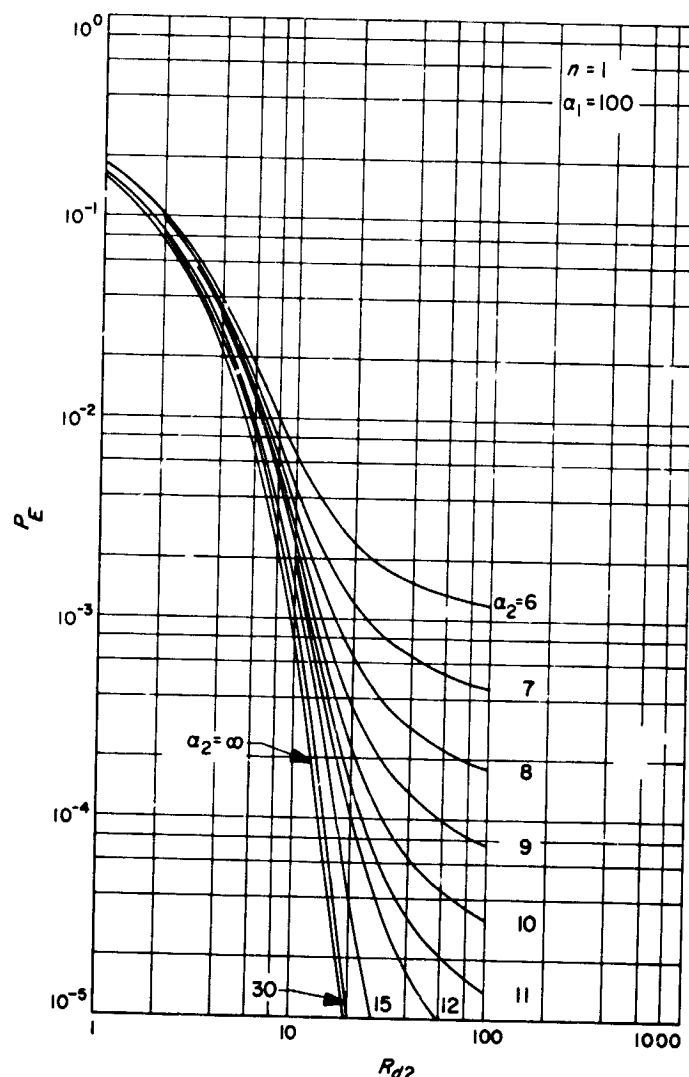


Fig. 16. Error probability vs the SNR R_{d2} for various values of the parameter α_1 with $\alpha_2 = 100$

a. Allocation of power in two-way systems. From Eq. (20) it is apparent that any attempt to find the value of m_2 , which minimizes $P_E(2)$ by the method of differentiation, immediately presents formidable difficulties; however, the surface generated by Eq. (20) has been studied on the IBM 7090 computer. The procedure used by the machine was to search for that value of m_2 which minimizes $P_E(2)$ and then evaluate $P_E(2)$. The results of these computations are illustrated in Fig. 17. This figure is a plot of the optimum value of m_2^2 , say m_{20}^2 , versus R_2 for various values of δ_2 with $\alpha_1 = 9$ db. This value of α_1 corresponds to a near threshold condition in the spacecraft's carrier tracking loop. Fig. 18 represents a plot of the system error probability versus R_2 , for various values of δ_2 , when the power is optimally divided between the carrier and sidebands. Notice from this figure that the $P_E(2)$ versus R_2

characteristic exhibits a bottoming behavior, i.e., the system exhibits an irreducible error probability. This behavior is due to the presence of additive noise on the up-link and may be eliminated by using a clean carrier reference in the vehicle or by increasing the up-link SNR to a point where the phase-jitter in the vehicle's carrier tracking loop becomes negligible.

The irreducible error probability, say $P_{E_{ir}}(2)$, may be obtained from Eq. (20) by letting N_{02} approach zero, i.e., α_2 approaches infinity, and R_2 approaches infinity. Taking the limit, we find that the integration is zero in the interval where $\cos \phi_2 \geq 0$ and becomes

$$P_{E_{ir}}(2) = \int_{\pi/2}^{\pi} \frac{\exp(\alpha_1 \cos \phi_2)}{\pi I_0(\alpha_1)} d\phi_2 \quad (21)$$

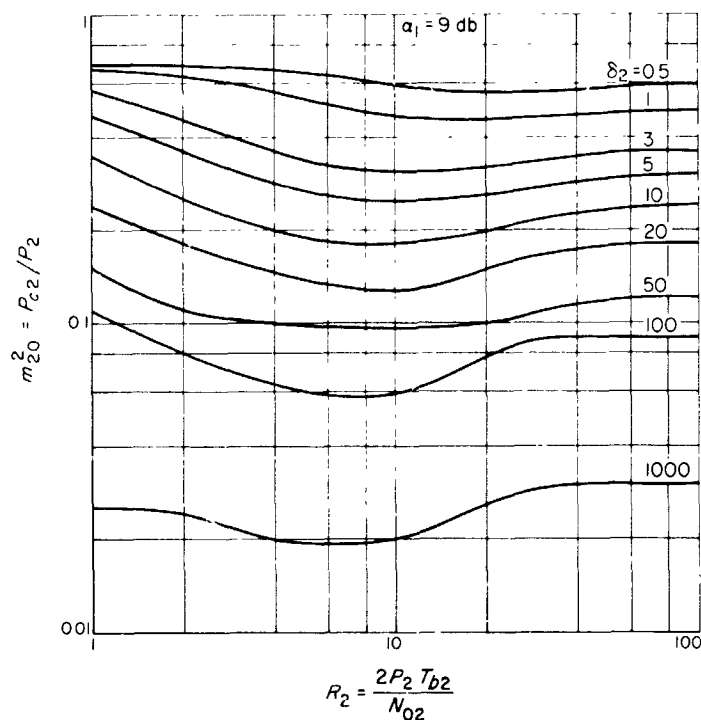


Fig. 17. Optimum modulation factor m_{20}^2 vs R_2 for various values of the parameter δ_2 with $\alpha_2 = 9$ db

where $\cos \phi_2 < 0$. This function has been studied in SPS 37-37, Vol. IV, p. 267.

In practice, it is convenient to have an approximate formula which specifies the optimum, say m_{20}^2 , value of the modulation factor as a function of the various system parameters. This approximate formula is easily obtained by assuming that the SNR existing in the individual carrier tracking loops is large enough that the linear PLL theory applies. In this case, the probability density $p(\phi)_2$ of the phase-error becomes Gaussian with variance $\sigma_{\phi_2}^2 =$

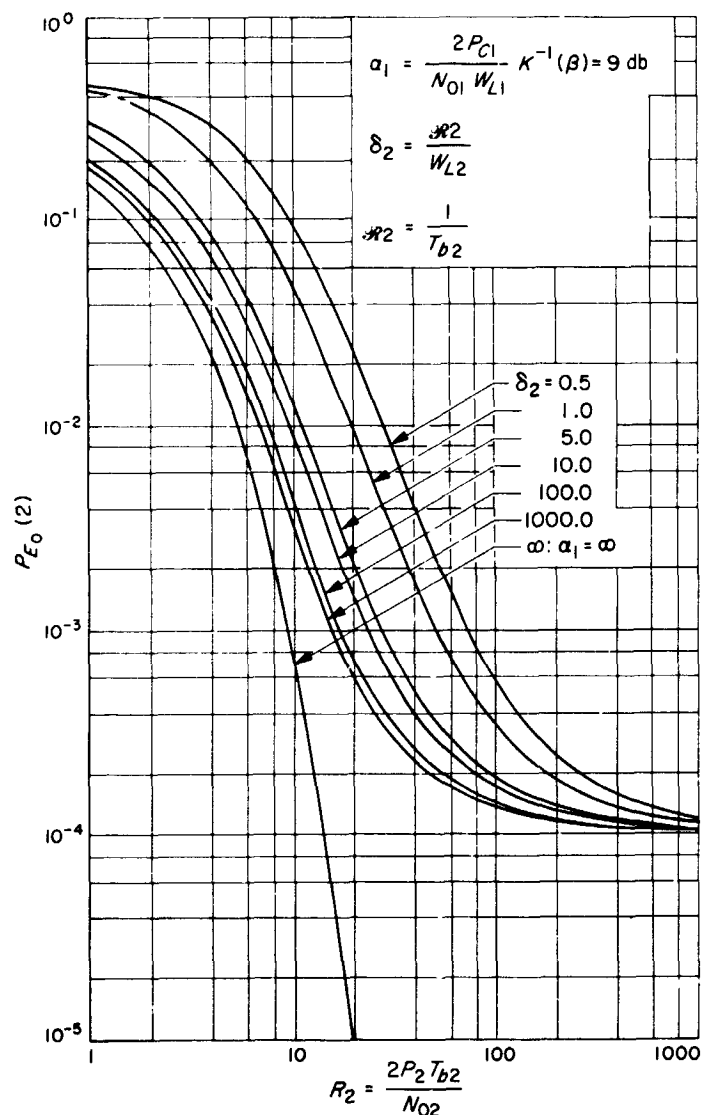


Fig. 18. Minimum error probability vs R_2 for various values δ_2 with $\alpha_1 = 9$ db

$\alpha_1^{-1} + \alpha_2^{-1}$ and the system error probability of Eq. (20) reduces to

$$P_E(2) = \frac{1}{(2\pi\sigma_{\phi_2}^2)^{1/2}} \int_{-\infty}^{\infty} \exp\left[-\frac{\phi_2^2}{2\sigma_{\phi_2}^2}\right] \text{Erfc} \left[\left((1 - m_2^2) R_2 \right)^{1/2} \cos \phi_2 \right] d\phi_2 \quad (22)$$

Differentiating Eq. (22) with respect to m_2 and equating the result to zero yields

$$\begin{aligned} E \left[\exp \left\{ -\frac{R_2(1 - m_2^2)}{2} \cos^2 \left(\frac{y}{(\alpha_1^{-1} + m_2^2 \delta_2 R_2)^{1/2}} \right) \right\} \right. \\ \cdot \frac{m_2 \delta_2 R_2 (1 - m_2^2)^{1/2}}{(\alpha_1^{-1} + m_2^2 \delta_2 R_2)^{1/2}} y \sin \left(\frac{y}{(\alpha_1^{-1} + m_2^2 \delta_2 R_2)^{1/2}} \right) \\ \left. - \left(\frac{m_2^2}{1 - m_2^2} \right)^{1/2} \cos \left(\frac{y}{(\alpha_1^{-1} + m_2^2 \delta_2 R_2)^{1/2}} \right) \right] = 0 \end{aligned} \quad (23)$$

where $E(\cdot)$ denotes the mathematical expectation of the quantity in the parenthesis, and y is a random variable which is normal $(0, 1)$. Carrying out this expectation and solving for the value of m_i^2 which produces the minimum, gives

$$m_{i0}^2 = \frac{\delta_2(R_2 - 1) - \alpha_1^{-1}(1 + 2\delta_2)}{2\delta_2 R_2(1 + \delta_2)} + \frac{\{[\delta_2(R_2 - 1) - \alpha_1^{-1}(1 + 2\delta_2)]^2 + 4\delta_2(1 + \delta_2)[\delta_2 R_2 + \alpha_1^{-1}(R_2 - \alpha_1^{-1})]\}^{1/2}}{2\delta_2 R_2(1 + \delta_2)} \quad (24)$$

This approximate formula, of course, is good in the region where the linear PLL theory applies.

b. Power allocation in one-way systems. The mathematical model, which we have established for two-way systems, reduces to the mathematical model for one-way systems if one allows B_{L2} to approach zero and replaces all subscripts on parameters which possess a two by one. In this case, the expression for the average error probability becomes

$$P_E(1) = \int_{-\pi}^{\pi} p(\phi_1) \operatorname{Erfc}[(1 - m_i^2)R_1]^{1/2} \cos \phi_1 d\phi_1 \quad (25)$$

where $p(\phi_1)$ is given by

$$p(\phi_1) = \frac{\exp[m_i^2 \delta_1 R_1 \cos \phi_1]}{2\pi I_0(m_i^2 \delta_1 R_1)}; \quad |\phi_1| \leq \pi \quad (26)$$

and

$$\delta_1 = \frac{\mathcal{R}_1}{\omega_{L1}} = \frac{1}{T_{b1}\omega_{L1}}; \quad R_1 = \frac{2P_1 T_{b1}}{N_{01}} \quad (27)$$

As before, the value of m_i^2 which minimizes $P_E(1)$ in Eq. (25) has been found by use of the IBM 7090 computer. This value, say m_{i0}^2 , is plotted in Fig. 19 versus R_1 for various values of δ_1 . The error probability corresponding to this minimum, say $P_{E0}(1)$, is illustrated in Fig. 20 versus R_1 for various values of δ_1 .

Again an approximate formula which specifies the optimum m_{i0}^2 as a function of the various system parameters is of interest to the design engineer. This approximate formula may be obtained from Eq. (24) by letting α_1 approach infinity and replacing all parameters with sub-

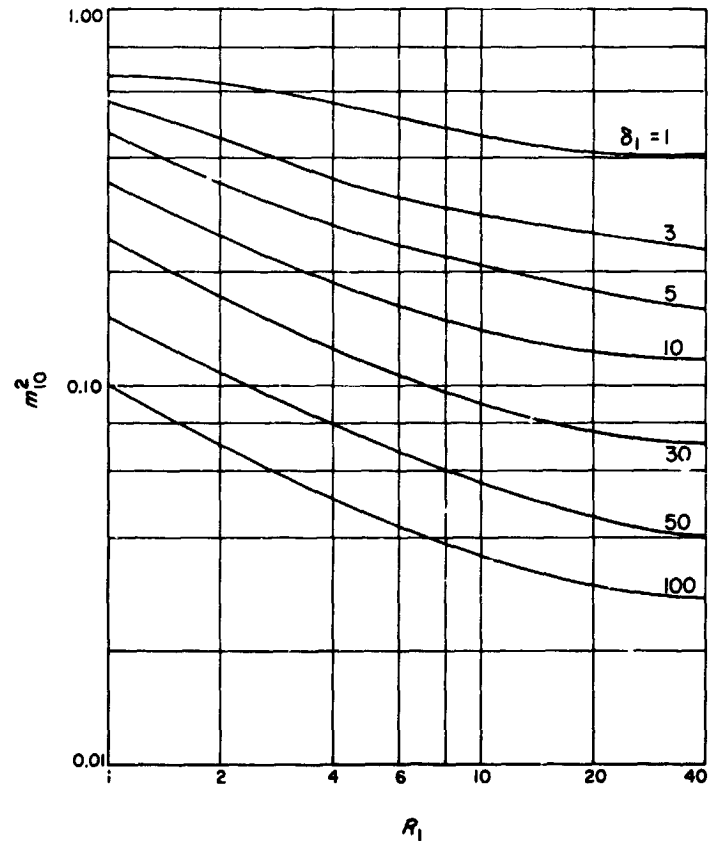


Fig. 19. Optimum modulation factor m_{i0}^2 vs R_1 for various values of the parameter δ_1

scripts two by ones. Thus, from Eq. (24) we have with $\alpha_1 = \infty$,

$$m_{i0}^2 = \frac{(R_1 - 1) + ((R_1 - 1)^2 + 4R_1(1 + \delta_1))^{1/2}}{2R_1(1 + \delta_1)} \quad (28)$$

This approximate formula is plotted in Fig. 21 versus R_1 for various values of δ_1 and may be used to compare with the exact results given in Fig. 16.

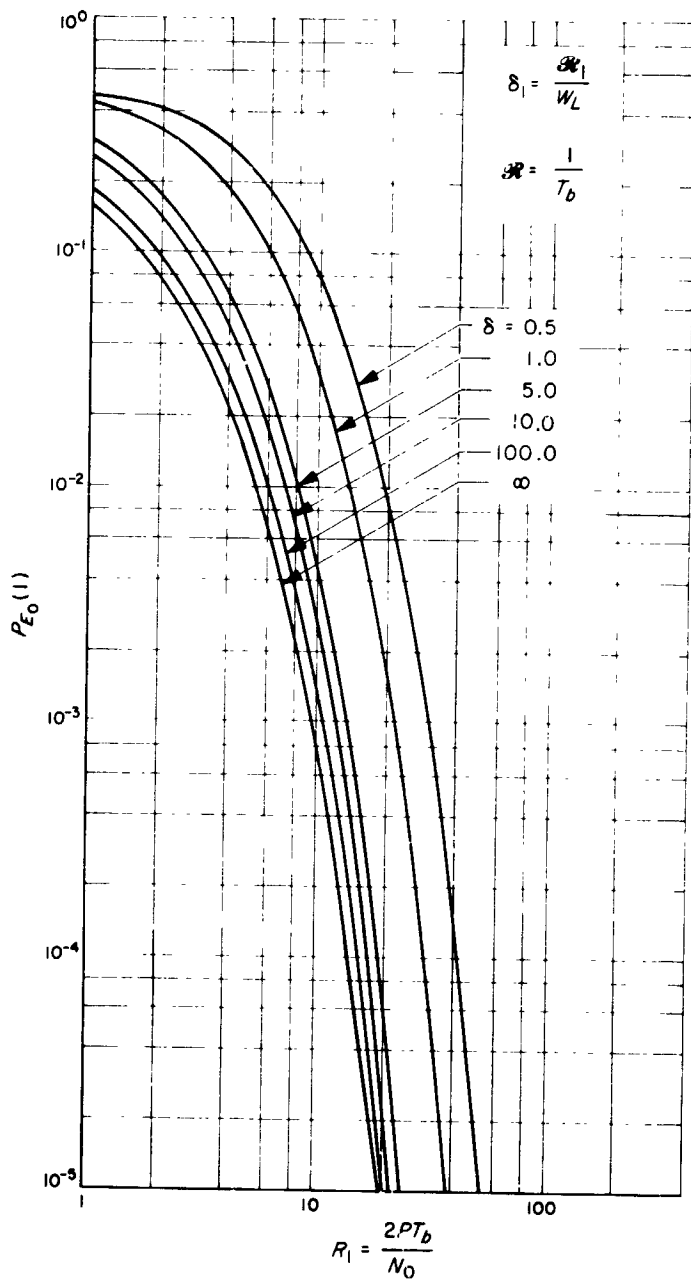


Fig. 20. Minimum error probability vs R_1 for various values of the parameter δ_1

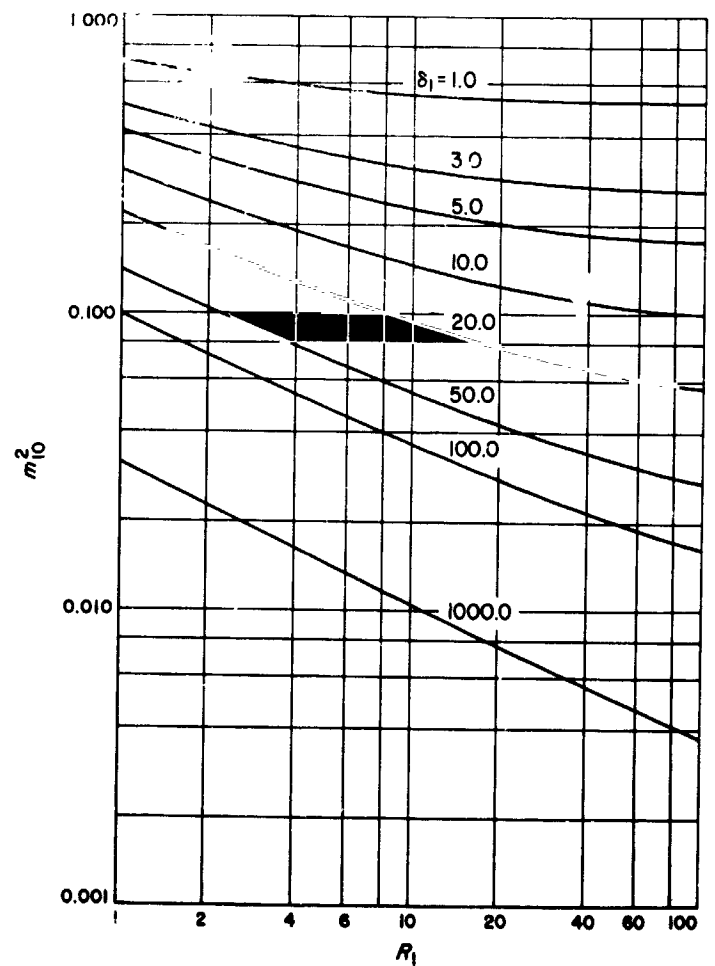


Fig. 21. Optimum modulation factor m_{10}^2 vs R_1 for various values of the parameter δ_1 (approximate formula used)

C. Combinatorial Communications: On The Number Of Information Bits In Certain

Cyclic Codes, R. McEliece

1. Introduction

Any binary group code can be described by its generator matrix whose rows may be thought of as a basis for the vector space which constitutes the code. It is always important to know the number of information bits in such a code. However, if the code is defined only by its generator matrix, determining the number of bits is difficult, because it is equivalent to finding the rank of the generator matrix. However, when the code is cyclic, it frequently happens that the generator matrix is what is called a *circulant*, and in this case it is possible to greatly simplify the task of finding the number of information bits. In subsection 2 an algebraic method for calculating the rank of a circulant will be developed, and in subsection 3 an application to the so-called quadratic residue codes will be given.

2. Algebraic Theory

Definition: An $n \times n$ matrix with entries in a field F which has the form

$$A = \begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_n \\ a_n & a_1 & a_2 & \cdots & a_{n-1} \\ a_{n-1} & a_n & a_1 & \cdots & a_{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_2 & a_3 & a_4 & \cdots & a_1 \end{bmatrix}$$

is called a *circulant* matrix.

For our applications, it will be necessary to compute the rank of a circulant matrix. The following theorem is useful:

Theorem 1: Let $F[x]$ be the ring of polynomials with coefficients from the field F . Define the polynomial $A(x) = a_1 + a_2x + a_3x^2 + \cdots + a_nx^{n-1}$. Let $r(x) = \gcd(x^n - 1, A(x))$ in $F[x]$. Then the rank of A is equal to $n - \deg(r(x))$, where A is the circulant whose first row is $(a_1, a_2, \dots, a_n)$.

Proof: In $F[x]$ let I be the principal ideal generated by the polynomial $x^n - 1$, and denote by $F_n[x]$ the quotient ring $F[x]/I$. Since $F[x]$ is a principal ideal domain, so is

$F_n[x]$. The elements in $F_n[x]$ are regarded as polynomials modulo $x^n - 1$.

Let us view $F_n[x]$ merely as a vector space of dimension n over F . Then the elements $A(x), x \cdot A(x), \dots, x^{n-1} A(x)$ generate a subspace $\mathcal{A}$. Furthermore, the dimension of $\mathcal{A}$ is precisely the rank of the matrix A . But $\mathcal{A}$ is actually an ideal in the ring $F_n[x]$. This is because the given basis for $\mathcal{A}$ is only permuted when multiplied by the element x , and x generates the whole ring $F_n[x]$. Hence $\mathcal{A}$ is the principal ideal $(A(x))$. We now need the following:

Lemma 1: If $P \in F_n[x]$, then $(P) = (\gcd(P, x^n - 1))$.

Proof: We return temporarily to $F[x]$, where the ideal structure is clearer. Two polynomials P and Q generate principal ideals (P) and (Q) , and $(P) \supseteq (Q)$ if and only if P divides Q . We suppose that Q is the kernel of a ring homomorphism $\theta: F[x] \rightarrow F[x]/(Q)$. Then the usual sort of calculation shows that $\theta(P) = P \cup Q/Q$. But in $F[x]$, $P \cup Q$ is principal and is generated by $\gcd(P, Q)$. Our lemma is the case $Q = x^n - 1$.

Lemma 1 allows us to complete the proof of Theorem 1, for it shows that $(A(x)) = (\gcd(A(x), x^n - 1))$. Suppose $\gcd(A(x), x^n - 1) = B(x)$. Then $B(x)$ divides $x^n - 1$. It is clear that the codimension of (B) in $F_n(x)$ is the degree d of B , since $1, x, \dots, x^{n-1}$ are linearly independent (mod B), but there can be no larger set. Thus, the dimension of (B) is $n - d$, as asserted. Theorem 1 is proved.

3. An Application

If p is a prime, the quadratic residue codes we shall be interested in can be described by their generator matrices Q_p , which are circulants over $GF[2]$. These matrices are, therefore, completely described by their first rows $(a_0, a_1, \dots, a_{p-1})$. Using the familiar Legendre symbol (a/p) , we describe the first row:

$$a_0 = 0$$

$$a_k = \begin{cases} 1 & \text{if } \left(\frac{k}{p}\right) = +1 \\ 0 & \text{if } \left(\frac{k}{p}\right) = -1 \end{cases}$$

The code is a cyclic linear code with block length p , and the number of information symbols is the rank of the matrix Q_p . Since the assignment $a_0 = 0$ was arbitrary, we shall be interested in not only the rank of Q , but also Q' .

which is also a circulant but has $a_0 = 1$. We now define the polynomials

$$Q(x) = Q_p(x) = \sum_{k=0}^{p-1} a_k x^k$$

In view of the lemma of subsection 2, we shall be interested in finding the degree of the polynomials $(Q(x), x^p + 1)$ and $(Q(x) + 1, x^p + 1)$ for various odd primes p .

Let $P(x) = x^{p-1} + x^{p-2} + \cdots + x + 1$. Then $x^p + 1 = (x + 1)P(x)$ over $GF(2)$. It is easy to determine whether or not $x + 1$ is a factor of $Q_p(x)$; if the number of nonzero coefficients is even, it is; if the number is odd, it is not. It is, therefore, sufficient to compute the degree of $(Q(x), P(x))$ and $(Q(x) + 1, P(x))$. As a final preliminary, set $N(x) = P(x) + Q(x) + 1$; i.e., $N(x)$ indicates the non-residues of p .

Lemma 2: Let a be a residue $(\text{mod } p)$. Then $(\text{mod } x^p + 1)$

$$Q(x^a) = \begin{cases} Q(x) & \text{if } \left(\frac{a}{p}\right) = +1 \\ N(x) & \text{if } \left(\frac{a}{p}\right) = -1 \end{cases}$$

$$N(x^a) = \begin{cases} N(x) & \text{if } \left(\frac{a}{p}\right) = +1 \\ Q(x) & \text{if } \left(\frac{a}{p}\right) = -1 \end{cases}$$

Proof: The polynomial

$$Q(x^a) \pmod{x^p + 1} = \sum_{(k/p) = +1} x^{ka \pmod{p}}$$

But as k runs through the quadratic residues, ka either runs through the quadratic residues or through the non-residues, according to whether $(a/p) = +1$ or -1 . The proof for $N(x)$ is the same.

We are now in a position to compute the degrees of $(Q(x), P(x))$ and $(Q(x) + 1, P(x))$, but we must first distinguish between two cases.

Case I: $(2/p) = -1$; i.e., $p \equiv \pm 3 \pmod{8}$. By Lemma 2, $N^2(x) = N(x^2) \equiv Q(x) \pmod{x^p + 1}$, and $Q^2(x) \equiv N(x)$

$\times \pmod{x^p + 1}$. We may replace the modulus $x^p + 1$ by $P(x)$ in both cases, and obtain:

$$\begin{aligned} (Q(x), P(x)) &= (Q(x), Q(x) + N(x) + 1) \\ &= (Q(x), N(x) + 1) \\ &= (Q(x), Q^2(x) + 1) \\ &= \text{constant} \end{aligned}$$

$$\begin{aligned} (Q(x) + 1, P(x)) &= (Q(x) + 1, Q(x) + N(x) + 1) \\ &= (Q(x) + 1, N(x)) \\ &= (Q(x) + 1, Q^2(x)) \\ &= \text{constant} \end{aligned}$$

Here in both cases the degree is zero, and so the rank of the matrix Q is either p or $p - 1$, depending upon whether $p \equiv +1$ or $-1 \pmod{4}$. (The rank of Q' is $p - 1$ or p .)

Case II: $(2/p) = +1$; i.e., $p \equiv \pm 1 \pmod{8}$. By Lemma 2, $Q^2(x) \equiv Q(x) \pmod{x^p + 1}$. Consequently, if F is a splitting field for $x^p + 1$, we see that $Q(\delta) = 0$ or 1 for all $\delta \in F$. Thus, each root of $P(x)$ is either a root of $Q(x)$ or of $Q(x) + 1$. Therefore,

$$\begin{aligned} P(x) &= (P(x), Q(x))(P(x), Q(x) + 1) \pmod{x^p + 1} \\ &= (P(x), Q(x))(P(x), N(x)) \pmod{x^p + 1} \end{aligned}$$

Now let e be any nonresidue of p , and suppose that δ is a p th root of 1 in F with $Q(\delta) = 0$. Then by Lemma 2, $N(\delta^e) = Q(\delta) = 0$. Similarly, $N(\delta) = 0$ implies $Q(\delta^e) = 0$. Thus, there are as many p th roots of 1 with $Q(\delta) = 0$ as there are with $Q(\delta) = 1$. Hence

$$\text{degree}(P(x), Q(x)) = \text{degree}(P(x), Q(x) + 1) = \frac{p-1}{2}$$

So in this case the rank of the matrix Q is either $(p + 1)/2$ or $(p - 1)/2$, according as $p \equiv 1 \pmod{4}$ or $p \equiv -1 \pmod{4}$. The rank of Q' is either $(p - 1)/2$ or $(p + 1)/2$ in this case.

In both of these cases, the codes corresponding to the higher dimensions (p and $(p + 1)/2$) may be obtained from those of lower dimension ($p - 1$ and $(p - 1)/2$) by the

addition of the all-ones vector to the code word. We state these results in a theorem:

Theorem 2: Let p be an odd prime. We define a *quadratic residue code* of length p in terms of its generator matrix Q , which is a circulant with first row $(a_0, a_1, \dots, a_{p-1})$, where

$$a_0 = \begin{cases} 1 & \text{if } p \equiv -1 \pmod{4} \\ 0 & \text{if } p \equiv +1 \pmod{4} \end{cases}$$

$$a_k = \begin{cases} 1 & \text{if } \left(\frac{k}{p}\right) = +1 \\ 0 & \text{if } \left(\frac{k}{p}\right) = -1 \end{cases} \quad k = 1, 2, \dots, p-1$$

Then the number of information bits in the code is $p-1$ if $p \equiv \pm 3 \pmod{8}$ and $(p-1)/2$ if $p \equiv \pm 1 \pmod{8}$. We may increase the number of bits by 1 in each case by adjoining the all -1's vector to the code.

4. Conclusion

In subsection III we applied Theorem 1 to the quadratic residue codes. It should be emphasized, however, that while it may not often happen that we will be able to calculate code dimensions in a theoretical way, Theorem 1 is still of use. For Theorem 1 reduces the difficult general problem of calculating the dimension of a cyclic code to the computationally simpler problem of determining the greatest common divisor of two polynomials. This is, of course, easily done by means of Euclid's algorithm, which is very efficient and is very easy to program on a digital computer.

D. Combinatorial Communications: A Rational Algorithm for Marsh's Cubic Transformation, S. W. Golomb^a

1. Introduction

Given an irreducible polynomial $f(x)$ of degree n over $GF(2)$, it is frequently desired to generate others of the same type. The two transformations

$$T: f(x) \rightarrow f(x+1)$$

^aPrepared under contract 951076 with the University of Southern California, Department of Electrical Engineering, Los Angeles, California.

and

$$U: f(x) \rightarrow x^n f(1/x)$$

have the property that both irreducibility and degree are preserved. Also, the relationship of the roots before and after is clear. ($T: \alpha \rightarrow \alpha + 1$ and $U: \alpha \rightarrow \alpha^{-1}$.) However, T and U together generate a group of only six transformations, which thus severely limit the number of new polynomials obtainable from a given one in this way.

In Ref. 19, March introduces the "cubic transformation"

$$M: f(x) \rightarrow f(x^{1/3}) f(\omega x^{1/3}) f(\omega^2 x^{1/3}) = f^*(x)$$

where $\omega^3 = 1$. It is easily seen that the roots of $f^*(x)$ are the cubes of the roots of $f(x)$. In particular, for odd degree n , $2^n - 1$ is not divisible by 3, and the transformation M preserves not only irreducibility but also the degree of primitivity of the roots. In a variety of cases, iteration of M enables one to generate *all* irreducible polynomials of degree n from a given one. (These degrees include $n = 3, 5, 7, 13, 17$, and 19 .)

The purpose of this note is to describe a "rational" algorithm for effecting the transformation M , which is useful in preparing tables[†] (Ref. 20). The procedure is rational in the sense that ω and ω^2 do not appear in the final result $f^*(x)$ nor in the intermediate computations.

2. The Algorithm

Divide the exponents of the terms in $f(x)$ into three classes: A, B, and C, according to the residue class of the exponent modulo 3. We produce the set of exponents for $f^*(x)$ from those for $f(x)$ by the following 3 steps:

- (1) Copy the exponents of $f(x)$.
- (2) Adjoin all numbers $(2u_1 + u_2)/3$ where u_1 and u_2 are distinct exponents of $f(x)$ in the same residue class modulo 3.
- (3) Adjoin all numbers $(a + b + c)/3$ where $a \in A$, $b \in B$, and $c \in C$.

Any exponent for $f^*(x)$ which is produced an *even* number of times by these operations must be discarded; if produced an *odd* number of times, it should be retained

[†]S. W. Golomb, L. R. Welch, and A. W. Hale's memorandum *On the Factorization of Trinomials Over GF(2)*, Jet Propulsion Laboratory, Pasadena, California.

(once). If any of the three categories A, B, C is empty, then step (3) is vacuous. If a category has less than two members, it does not contribute to step (2).

Example 1. Let $f(x) = x^0 + x^2 + 1$. Then the categories are

A	B	C
0		2, 5

To produce $f^*(x)$, we follow the three steps:

	A	B	C	
step 1	0		2, 5	
step 2	3	4		$\frac{2 \times 2 + 5}{3}$, $\frac{2 \times 5 + 2}{3}$
step 3				
mod 2 sum	0, 3	4	2, 5	vacuous

Thus $f^*(x)$ has the exponents 0, 2, 3, 4, 5 and

$$f^*(x) = x^0 + x^2 + x^3 + x^4 + x^5 + 1.$$

Example 2. We iterate the transformation, this time starting with $f(x) = x^0 + x^2 + x^3 + x^4 + 1$. To form $f^{**}(x)$, we follow the three steps:

	A	B	C	
step 1	0, 3	4	2, 5	
step 2	3	1, 4	2	$\frac{2 \cdot 0 + 3}{3}$, $\frac{2 \cdot 3 + 0}{3}$, $\frac{2 \cdot 2 + 5}{3}$, $\frac{2 \cdot 5 + 2}{3}$
step 3	3, 3	4	2	$\frac{0 + 4 + 2}{3}$, $\frac{0 + 4 + 5}{3}$, $\frac{3 + 4 + 2}{3}$, $\frac{3 + 4 + 5}{3}$
mod 2 sum	0	1, 4	2, 5	

Thus, $f^{**}(x) = x^0 + x^1 + x^2 + x + 1$.

The reader is invited to verify $f^{***}(x) = x^0 + x^1 + 1$.

3. Proof of Algorithm

We wish to show that $f^*(y^3) = f(y)f(\omega y)f(\omega^2 y)$ can be obtained in the manner just described, where $y = x^{1/3}$. Write $f(y) = f_0(y) + f_1(y) + f_2(y)$, where $f_i(y)$ contains precisely those terms of $f(y)$ with the exponent congruent to i modulo 3. Then

$$f(\omega y) = f_0(y) + \omega f_1(y) + \omega^2 f_2(y)$$

and

$$f(\omega^2 y) = f_0(y) + \omega^2 f_1(y) + \omega f_2(y)$$

Thus,

$$\begin{aligned}
 f(y^3) &= (f_0 + f_1 + f_2)(f_0 + \omega f_1 + \omega^2 f_2)(f_0 + \omega^2 f_1 + \omega f_2) \\
 &= (f_0^3 + f_1^3 + f_2^3) + (1 + \omega + \omega^2)(f_0 f_1^2 + f_1 f_2^2 + f_2 f_0^2) \\
 &\quad + f_0^2 f_1 + f_1^2 f_2 + f_2^2 f_0 + \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} f_0 f_1 f_2 \\
 &= (f_0^3 + f_1^3 + f_2^3) + f_0 f_1 f_2.
 \end{aligned}$$

since $1 + \omega + \omega^2 = 0$, while the determinant is 1. (The matrix is nonsingular by linear independence of the rows, and the determinant is rational by symmetry in ω and ω^2 .)

The exponents in f_0 , f_1 , and f_2 are those in the classes A, B, and C, respectively. The exponents of f_1^3 are: (1) the triples of the exponents of f_1 , and (2) the sums $2u_1 + u_2$, where u_1 and u_2 are distinct exponents of f_1 . The exponents of $f_0 f_1 f_2$ are: (3) all sums of the form $a + b + c$, with $a \in A$, $b \in B$, and $c \in C$. Allowing for $y^3 = x$, these are the three steps in the algorithm for finding $f^*(x)$ from $f(x)$.

E. Combinatorial Communications: On Enumerative Equivalence of Group Elements, S. W. Golomb* and A. W. Hales*

1. Introduction

Let G be a finite group (of order $|G|$) operating on a finite set S . By a well-known formula of Burnside (Ref. 21), widely exploited since the appearance of Polya's paper (Ref. 22), the number C of orbits (equivalence classes of elements in S under the operations of G) is given by

$$C = \frac{1}{|G|} \sum_{g \in G} I(g) \quad (1)$$

where $I(g)$ is the number of fixed points of S under the group element g . It is well known (Ref. 23) that, considering S as a representation of G , $I(g)$ is a group character. Thus, if g_1 and g_2 are conjugate elements of G , $I(g_1) = I(g_2)$ for all representations S .

*Prepared under JPL contract 951076 with the University of Southern California, Department of Electrical Engineering, Los Angeles, California.

*Consultant from the University of California, Los Angeles, California.

The purpose of this article is to characterize the circumstances under which two group elements, g_1 and g_2 , of the abstract group G satisfy $I(g_1) = I(g_2)$ for every representation S of G . Two group elements with this property will be called (weakly) *enumeratively equivalent* (this is clearly an equivalence relation on G), and in applying Eq. (1) it suffices to compute $I(g)$ only once for each enumerative equivalence class in G .

We define two group elements, g_1 and g_2 , of the abstract group G to be *strongly enumeratively equivalent* if they have the same set of fixed points, no matter what representation of G is considered. We call two group elements, g_1 and g_2 , *related* (and say g_1 is a *relative* of g_2) if they generate the same cyclic subgroup of G , i.e., if each is a power of the other. The characterization theorems for strong weak enumerative equivalence are as follows:

Theorem 1. Two elements, g_1 and g_2 , of an abstract group G are strongly enumeratively equivalent if and only if g_1 is a relative of g_2 .

Theorem 2. Two elements, g_1 and g_2 , of an abstract finite group G are weakly enumeratively equivalent if and only if g_1 is a conjugate of a relative of g_2 .

Theorem 2 is no longer true if the word "finite" is omitted. We shall indicate the appropriate modifications in this case. Finally, we shall discuss those groups G in which " g_1 a relative of g_2 " implies " g_1 a conjugate of g_2 ," and vice versa.

2. An Example

In Fig. 22, the points a, b, c are the vertices of an equilateral triangle, and the line df is perpendicular to the triangle, with e as the midpoint of both the segment df and the triangle abc . On the set $S = \{a, b, c, d, e, f\}$, we have a group of operators $G = \{A, B, C, D, E, F\}$, where A, B , and C are 180-deg rotations of S around the indicated medians of the triangle, D and F are the ± 120 -deg

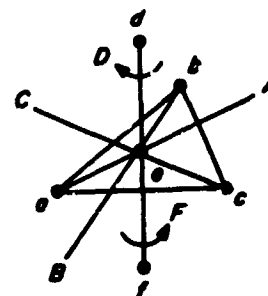


Fig. 22. The dinedral group

rotations of S about the perpendicular line df , and E is the identity operator. (Abstractly, G is the same as the dihedral group of the equilateral triangle and the symmetric group on three letters.) The fixed points are listed and counted in Table 2.

Table 2. Fixed points

Group element	Fixed points	$I(g)$
E	a, b, c, d, e, f	6
A	a, e	2
B	b, e	2
C	c, e	2
D	d, e, f	3
F	d, e, f	3

Formula (1) says that $1/6 \sum I(g)$, which is 3, should equal the number of orbits. It is clear that the orbits are $\{e\}$, $\{a, b, c\}$, and $\{d, f\}$. Since D and F are related, by Theorem 1 they must have the same fixed points (viz., d, e , and f). The operators A, B , and C are not related and have different sets of fixed points. However, since they are conjugate, by Theorem 2 they all have the same $I(g)$.

By Theorems 1 and 2 respectively, the strong enumerative equivalence classes in G are $\{E\}$, $\{A\}$, $\{B\}$, $\{C\}$, $\{D, F\}$ and the weak enumerative equivalence classes are $\{E\}$, $\{A, B, C\}$, and $\{D, F\}$. The example verifies that for this group, the classes are certainly no larger than allowed by the theorems.

3. Strong Enumerative Equivalence

Lemma 1. If g_1 is a power of g_2 , then every fixed point of g_2 is also a fixed point of g_1 .

Proof: Let s_0 be any fixed point of g_2 , so that $g_2(s_0) = s_0$, and let $g_1 = g_2^a$. Then

$$g_1(s_0) = g_2^a(s_0) = g_2 g_2 \cdots g_2(s_0) = s_0.$$

Lemma 2. If g_1 and g_2 are related elements of the group G , then they have the same set of fixed points in any representation S of G .

Proof: g_1 and g_2 are related if and only if each is a power of the other. Then by Lemma 1, if they are related, they have the same fixed points.

Lemma 3. If g_1 and g_2 are nonrelated elements of G , there exists a representation S in which g_1 and g_2 do not have the same set of fixed points.

Proof: Construct S as follows: Let all the cyclic subgroups of G , and all their left cosets, be the elements of S . Let G operate on S by left multiplication. (The effect of a group element is to permute a subgroup and its cosets. Hence, S is a valid representation of G .)

If g_1 and g_2 are nonrelated elements of G , they do not generate the same cyclic subgroup. Hence, there is a cyclic subgroup H containing one (say g_1) but not the other. As an element of S , H is then a fixed point of g_1 but not of g_2 .

Theorem 1. Two elements, g_1 and g_2 of an abstract group G , are strongly enumeratively equivalent if and only if g_1 is a relative of g_2 .

Proof: The two halves of this theorem are Lemmas 2 and 3, respectively.

4. Weak Enumerative Equivalence

We need several lemmas.

Lemma 4. If g_1 is a conjugate of a power of g_2 , then $I(g_1) \geq I(g_2)$ for every representation S of G .

Proof: By Lemma 1 of the previous section, it suffices to show that the function I is constant on conjugate classes of G . But this is proved in (Ref. 23) Theorem 3.

Lemma 5. If g_1 is not a conjugate of a power of g_2 , then there is a representation S of G in which $I(g_1) < I(g_2)$.

Proof: Let H be the cyclic subgroup of G generated by g_2 ; let S consist of H and its left cosets; and let G operate on S by left multiplication. Then $I(g_2) \geq 1$, because g_2 leaves H fixed. On the other hand $I(g_1) = 0$, because if g_1 had a fixed point in S , say mH , we would have

$$g_1(mH) = mH$$

$$(m^{-1}g_1m) = H$$

implying that $m^{-1}g_1m$, a conjugate of g_1 , is in H , and hence is a power of g_2 . This contradiction proves the lemma.

Lemma 6. Two elements, g_1 and g_2 , of an abstract group G are (weakly) enumeratively equivalent if and only if each is a conjugate of a power of the other.

Proof: This follows immediately from Lemmas 4 and 5.

Lemma 7. Let g_1 and g_2 be elements of finite order in a group G . Then the following are equivalent:

- (1) Each of g_1, g_2 is a conjugate of a power of the other.
- (2) g_1 is a conjugate of a relative of g_2 .

Proof: It is clear that assertion (2) implies (1). To show that (1) implies (2), we observe that condition (1) implies both that (0 is the order function) $0(g_1) \leq 0(g_2)$ and $0(g_1) \geq 0(g_2)$. Hence, g_1 and g_2 have the same order. We know that $g_1 = x^{-1} g_2^n x$ for some x . Here g_1 and g_2^n have the same order. Hence, g_2 and g_2^n have the same order and, therefore, generate the same cyclic subgroup, i.e., g_2^n is a relative of g_2 . This completes the proof.

Theorem 2. Two elements, g_1 and g_2 , of an abstract finite group G are (weakly) enumeratively equivalent if and only if g_1 is a conjugate of a relative of g_2 .

Proof: Since every element of a finite group is of finite order, the theorem follows from Lemmas 6 and 7.

The following example shows that the finiteness hypothesis in Lemma 7 (and hence in Theorem 2) is necessary.

Example: Let G be the split extension of $Q \times Q$ (when Q is the rationals under $+$) by an infinite cyclic group generated by x , with $x^{-1}(a, b)x = (2b, 2a)$. Then $(1, 0)$ is conjugate to the "power" $(0, 2)$ of $(0, 1)$, and $(0, 1)$ is conjugate to the "power" $(2, 0)$ of $(1, 0)$, but $(1, 0)$ is *not* a conjugate of a relative of $(0, 1)$.

5. Considerations of Implication

In the light of the previous results, it is interesting to ask the following two questions:

- (A) For which groups G does " g_1 is a conjugate of g_2 " imply " g_1 is a relative of g_2 "?
- (B) For which groups G does " g_1 is a relative of g_2 " imply " g_1 is a conjugate of g_2 "?

Question A is easy to answer.

Theorem 3. Let G be an abstract group. Then the following are equivalent:

- (1) g_1 is a conjugate of g_2 implies g_1 is a relative of g_2 .
- (2) All subgroups of G are normal.

Proof: Assume (1). Then if H is a subgroup of G , h is in H ; and g is in G . We have $g^{-1}hg$ is a relative of h and, hence, is also in H . Thus, H is normal. Conversely, suppose (2) is

true. If g_1 is a conjugate of g_2 , then, since the cyclic subgroup generated by g_2 is normal, g_1 must be a power of g_2 . Similarly, g_2 is a power of g_1 , and the proof is complete.

Groups in which every subgroup is normal are called *Hamiltonian groups*, and have been completely classified (Ref. 24). The result is this: a group is Hamiltonian if and only if it is either abelian, or the direct product of the quaternion group of order 8 by an abelian group of exponent two by an abelian group in which every element has finite odd order. This completes the answer to question A.

Question B is much more difficult to answer, and will not be dealt with here. We only point out the following facts, without proof: no group of finite odd order satisfies the condition of question B; any abelian group of exponent two does satisfy the condition; the quaternion group satisfies the condition; and all symmetric groups satisfy the condition.

F. Information Processing: Arithmetic Decoding of Cyclic Codes, II, G. Solomon

1. Summary

In SPS 37-42, Vol. IV, pp. 205-208, we introduced simplified mechanizable arithmetic decoding of maximal length shift register codes and cyclic Reed-Mueller Codes. The number of computations was proportional to the number of information bits. This article extends these procedures for more general BCH cyclic codes.

2. Main Result

Let k be even. Consider the BCH code generated via linear recursion by the polynomial

$$f(x) = (x + 1)f_1(x)f_5(x)$$

where f_1, f_5 are the irreducible polynomials of α and α^5 over $GF(2)$, α primitive. For these codes of length $2^k - 1$, and usual dimension $2k + 1$ (only exception: $k = 4$) we have a very simple decoding procedure which will correct $\lfloor (2^k - 1)/6 \rfloor - 1$ errors:

Example.

(15, 7) BCH code will correct 2 errors with this procedure.

(63, 13) BCH code will correct 10 errors with this procedure.

(255, 17) BCH code will correct 42 errors with this procedure.

This arithmetic procedure will correct, in general, slightly fewer errors than the usual BCH error correcting procedures, but is much simpler in mechanization and concept.

3. Method

Let $a = (a_i)_{i=0,1,\dots,2^k-2}$ be the received vector.

- (1) Perform the permutation $\phi(a)$ by mapping $x \rightarrow x+1$, $x \in GF(2^k)$, as in SPS 37-42, Vol. IV, pp. 205-208.
- (2) Let $T\bar{a}$ be the cyclic shift of $\bar{a}$ by $(2^k-1)/3$ stages to the right. Perform $T\phi(a)$, $T^2\phi(a)$.
- (3) Compute the vector sum $\phi(a) + T\phi(a) + T^2\phi(a) = b$.
- (4) Compute the weight ω of b over all coordinates but the 0th. Let $\omega' = \omega/3$.
- (5) Add as mod 0 (ordinary arithmetic) to ω' .
- (6) If $\omega' < (2^k-1)/(3 \cdot 2)$, decode a_0 as 0. Otherwise, decode a_0 as 1.
- (7) Cyclically shift the received vector $\bar{a}$ one stage to the right to obtain $(a_1, a_2, \dots, a_0)$.
- (8) Perform (1)-(6) on shifted vector. Decode a_1 .
- (9) Continue until all information bits are decoded.

4. Proof

For the codes described above, the vectors are given by $\{g(\alpha^i); i=0,1,\dots,2^k-2\}$ where (SPS 37-42, Vol. IV)

$$g(x) = c_0 + Tr cx + Tr dx^5$$

α primitive, $c_0 \in GF(2)$, $c \in GF(2^k)$, $d \in GF(2^l)$ where $l|k$ and 2^l-1 is the order of α^5 .

The coordinates $1 + \alpha^i, 1 + \alpha^{[2^k-1/3]+i}, 1 + \alpha^{2[(2^k-1)/3]+i} = \beta_i, \beta_i', \beta_i''$ say, $i=1, \dots, (2^k-1)/3-1$ are distinct in $GF(2^k)$ and take on 2^k-1 values; also, $(\beta_i-1)^3 = \alpha^{3i}$, etc. It can be shown that

$$\begin{aligned} g(\beta^i) + g(\beta^{i'}) + g(\beta^{i''}) \\ = c_0 + Tr c(1) + Tr d(1) = a_0, \quad \text{if correct} \end{aligned}$$

We then have $[(2^k-1)/3]-1$ determinations of a_0 in addition to the value of a_0 itself. Decode a_0 as 0 if the

majority of these determinations is 0, and as 1, if the majority is 1. Note that, since k is even, $(2^k-1)/3$ is an odd integer. Hence, a tie can not occur, and the decoding procedure always produces an answer regardless of how many errors were made.

One readily verifies that if at most $(2^k-1)/6-1$ errors have been made, the majority decision is indeed correct.

G. Information Processing: Decoding Codes Beyond the Bose-Chaudhuri Bound,

E. R. Berlekamp¹⁰

It is known from combinatorial arguments that most t -error correcting Bose-Chaudhuri-Hocquenghem (BCH) codes are capable of correcting many error patterns containing more than t errors, although no feasible general algorithms for correcting correctable error patterns are known. In certain cases, decoding words with $t+e$ errors can be reduced to the solution of some simultaneous non-linear equations in e unknowns ($e > 0$). Unfortunately, feasible methods for solving these equations are known only in a few special cases. The case of $t+1$ errors is a particular example.

1. Introduction

In a t -error correcting binary BCH code of odd block length N , positions of the code are associated with the N th roots of unity, which form a multiplicative subgroup of the nonzero elements of some finite field, $GF(2^k)$, of characteristic 2.

The code is constructed in such a way that various power-sum symmetric functions of the error locations are available as parity checks on the received word. In particular, the t -error correcting BCH code is chosen so that the power-sum symmetric functions $S_1, S_2, S_3, \dots, S_{2t}$ are available, where

$$S_j = \sum_{\text{error positions}} B_i^j$$

The decoding problem is to find the S 's given these S 's.

In order to solve these equations, one usually proceeds in two steps. The first goal is to construct the error

¹⁰Consultant from the Electrical Engineering Department, University of California, Berkeley, California.

polynomial

$$\sigma(x) = \sum \sigma_i x^i = \prod (1 - B_j x)$$

The degree of the error polynomial is equal to the number of errors, and the roots of the error polynomial are the locations of these errors. The coefficients of the error polynomial (the σ_i) are in fact the elementary symmetric functions of the error locations. These elementary symmetric functions are related to the power-sum symmetric functions by Newton's Identities. As was shown in a previous article (Ref. 25), Newton's Identities in a field of characteristic two are conveniently expressed in generating function notation by the equation

$$S(x)\sigma(x) = \text{Even}(x)$$

where $S(x)$ is the generating function of the S 's, given by $S(x) = 1 + S_1x + S_2x^2 + S_3x^3 + \dots$; $\sigma(x)$ is the error polynomial; and $\text{Even}(x)$ denotes some even polynomial in x .

2. Finding $\sigma(x)$

Typically, one is given only $1 + S_1x + S_2x^2 + \dots$, $+ S_{2t}x^{2t}$ (i.e., $S(x) \bmod x^{2t+1}$), and one wishes to find a $\sigma(x)$ such that $S \cdot \sigma = \text{Even} \bmod x^{2t}$. This is most readily done by an iterative process. We define a sequence of successive approximations, $\sigma^{(0)}, \sigma^{(1)}, \sigma^{(2)}, \dots$, and an auxiliary sequence $\tau^{(0)}, \tau^{(1)}, \tau^{(2)}, \dots$, as follows:

$$\sigma^{(0)} = \tau^{(0)} = 1$$

Then let $\Delta_1^{(n)}$ be the coefficient of x^{2n-1} in the power series of $S \cdot \sigma^{(n)}$. Now define

$$\sigma^{(n+1)} = \sigma^{(n)} - \Delta_1^{(n)} x \tau^{(n)}$$

$$\tau^{(n+1)} = \begin{cases} \frac{x\sigma^{(n)}}{\Delta_1^{(n)}} & \text{if } \Delta_1^{(n)} \neq 0 \text{ and if } \deg \sigma^{(n)} \leq \deg \tau^{(n)} \\ x^2 \tau^{(n)} & \text{if } \Delta_1^{(n)} = 0 \text{ or if } \deg \sigma^{(n)} > \deg \tau^{(n)} \end{cases}$$

One can readily verify that, for all n ,

$$S \cdot \sigma^{(n)} = \text{Even} \bmod x^{2n}$$

$$S \cdot \tau^{(n)} = \text{Odd} + x^{2n} \bmod x^{2n+1}$$

$$\deg \sigma^{(n)} + \deg \tau^{(n)} = 2n$$

$$\sigma^{(n)}(0) = 1$$

By further arguments, it was shown by Berlekamp (Ref. 25) that if $\rho(x)$ is any polynomial whatever $\neq \sigma^{(n)}(x)$, and if either $\deg \rho < \deg \sigma^{(n)}$ or if $\deg \sigma^{(n)} \leq \deg \rho \leq n$, then

$$S \cdot \rho \neq \text{Even} \bmod x^{2n}$$

Consequently, if there are no more than t errors, this iterative procedure terminates with the correct error polynomial, $\sigma^{(t)} = \sigma$.

We can express the $(n+1)$ st, $(n+2)$ nd, $\dots$, $(n+k)$ th approximations in terms of the n th approximation by the formulas

$$\sigma^{(n+k)} = \hat{f}^{(k,n)} \sigma^{(n)} + \tilde{f}^{(k,n)} \tau^{(n)}$$

$$\tau^{(n+k)} = \tilde{g}^{(k,n)} \sigma^{(n)} + \hat{g}^{(k,n)} \tau^{(n)}$$

Here $\hat{f}^{(k,n)}$ and $\tilde{f}^{(k,n)}$ represent the even and odd parts of the polynomial $f^{(k,n)}$; $\hat{g}^{(k,n)}$ and $\tilde{g}^{(k,n)}$ represent the even and odd parts of the polynomial $g^{(k,n)}$. From the previous definition of the iterative algorithm, one can readily verify that the polynomials f and g must satisfy

$$f^{(0,n)} = g^{(0,n)} = 1$$

$$f^{(k+1,n)} = f^{(k,n)} - \Delta_1^{(n+k)} \times g^{(k,n)}$$

$$g^{(k+1,n)} = \begin{cases} \frac{x f^{(k,n)}}{\Delta_1^{(n+k)}} & \text{if } \Delta_1^{(n+k)} \neq 0 \text{ and } \deg \sigma^{(n+k)} \leq \deg \tau^{(n+k)} \\ x^2 g^{(k,n)} & \text{if } \Delta_1^{(n+k)} = 0 \text{ or if } \deg \sigma^{(n+k)} > \deg \tau^{(n+k)} \end{cases}$$

In the common case where $\deg \sigma^{(n)} = \deg \tau^{(n)} = n$, we have $\deg \sigma^{(n+k)} = n + \deg f^{(k,n)}$; $\deg \tau^{(n+k)} = n + \deg g^{(k,n)}$. If $\sigma^{(n+K)} = \sigma$, then for all $k \geq K$, $f^{(k,n)} = f^{(K,n)}$. Therefore, we can consider the polynomial $f^{(\infty,n)}$, with the obvious definition.

The branch in the iterative algorithm depends on the scalar

$$\begin{aligned} \Delta_1^{(n)} &= \left[\sum_{i=0}^{\deg(\sigma^{(n)})} S_{2n+1-i} \sigma_i^{(n)} \right] \\ &= S_{2n+1} - S_{2n+1}^{(n)} \end{aligned}$$

"Throughout this article, we let $\sim$ and $\wedge$ denote the odd and even parts of a polynomial. Notice that $\sim$ is the graph of an odd sin wave and $\wedge$ is the graph of an even triangular wave.

where we have set

$$S_{2n+1}^{(n)} = \left[\sum_{i=1}^{\deg(\sigma^{(n)})} S_{2n+1-i} \sigma_i^{(n)} \right]$$

Evidently,

$$\sigma^{(n+1)} = \sigma^{(n)} \text{ iff } \Delta_1^{(n)} = 0 \text{ iff } S_{2n+1} = S_{2n+1}^{(n)}$$

For this reason, we call $S_{2n+1}^{(n)}$ the *anticipated value* of S_{2n+1} . More generally, define the anticipated values of further power-sum symmetric functions by the iterative equations

$$S_0^{(n)} = 1$$

$$S_{2k}^{(n)} = (S_k^{(n)})^2$$

$$S_{2k+1}^{(n)} = \deg \left[\sum_{i=1}^{\sigma^{(n)}} S_{2k+1-i} \sigma_i^{(n)} \right]$$

For $k \leq 2n$, $S_k^{(n)} = S_k$. Evidently, the polynomial

$$S^{(n)}(x) = \sum_{i=0}^{\infty} S_i^{(n)} x^i$$

is the unique solution of the equations

$$\hat{S}^{(n)}(x) = S^{(n)}(x^2)$$

$$S^{(n)}(x) \sigma^{(n)}(x) = \text{Even}(x)$$

For this reason, we view $S^{(n)}(x)$ as an estimate of $S(x)$, based only on $S_1, S_2, \dots, S_{2n}$. If there are no more than n errors, then this estimate must be correct. However, if there are additional errors, then the estimated power-sum generating polynomial will differ from the true generating polynomial. Since $S^{(n)}(x) = S(x) \bmod x^{2n}$, we measure the difference by the polynomial

$$\Delta^{(n)}(x) = (S(x) - S^{(n)}(x))/x^{2n}$$

$\Delta_1^{(n)}$ is thus seen to be the first coefficient of a polynomial which represents the difference between the actual power-sum generating function and the n th approximation to it. Breaking $\Delta^{(n)}(x)$ up into even and odd parts gives

$$\Delta^{(n)} = \tilde{\Delta}^{(n)} + \hat{\Delta}^{(n)}$$

Evidently, $\Delta^{(n)} = 0 \bmod x^{2n}$. Finally, since

$$S^{(n)}(x) \tau^{(n)}(x) = x^{2n} \bmod x^{2n+1}$$

we define

$$A^{(n)} = \frac{S^{(n)} \tau^{(n)}}{x^{2n}}$$

3. Expression for $\Delta^{(n)}$

We now give an expression for $\Delta^{(n)}$ in terms of $\sigma^{(n)}, \tau^{(n)}, S^{(n)}, A^{(n)}$, and $f^{(\infty, n)}$. If we are given $S_1, S_2, S_3, \dots, S_{2n}$, then we can compute $\sigma^{(n)}, \tau^{(n)}, \dots$, and $A^{(n)}$. The equations we are about to give will relate the two unknown polynomials $f^{(\infty, n)}$ and $\Delta^{(n)} = \Delta_1 x + \Delta_2 x^2 + \dots$. Details are omitted.

The coefficients Δ_i of x^i in $\Delta^{(n)}(x)$ are given as

$$\Delta_1 = f_1$$

$$\Delta_3 = f_1(A_2 + f_1 \tau_1 + \sigma_2 + f_2) + f_3$$

$$\begin{aligned} \Delta_5 = & f_1(A_4 + A_2(f_1 \tau_1 + \sigma_2 + f_2) + f_1 \tau_3 + f_3 \tau_1 \\ & + \sigma_4 + f_2 \sigma_2 + f_4 + (f_1 \tau_1 + \sigma_2 + f_2)^2 \\ & + f_3(A_2 + f_1 \tau_1 + \sigma_2 + f_2) + f_5 \end{aligned}$$

In general, Δ_{2k-1} is given by an expression of k th degree in f_1 , $(k-1)$ st degree in f_2 and f_3 , $(k-2)$ nd degree in f_4 and f_5 , $\dots$, and 1st degree in f_{2k-2} and f_{2k-1} .

4. Decoding More than t Errors

Let us consider how these expressions can be used to decode more than t errors in a t -error-correcting Bose-Chaudhuri-Hocquenghem code. We first compute $\sigma^{(1)}, \tau^{(1)}, \sigma^{(2)}, \tau^{(2)}, \dots, \sigma^{(t)}, \tau^{(t)}$. Then we compute the first several coefficients of $S^{(t)}$ and of $A^{(t)}$. We are then in a position to apply some of the above formulas with $n = t$.

We next consider the S_k , for $k > 2t$. For certain values of k , this power-sum symmetric function will be a known power of one of the given power-sum symmetric functions:

$$S_k = S_i^j; \quad i < k$$

whenever

$$i \cdot 2^j = k \bmod N$$

The values of k which are expressible in this way depend critically on the specific parameters of the code, the block length N and the error-correction capability t . For the perfect Hamming codes ($t = 1$ and N one less than a power of two), there are no odd k which are expressible in the above manner. In general, however, several such k are. For example, for the 5-error-correcting binary BCH code of block length 63, we have S_1, S_3, S_5, S_7 and S_9 given. S_{11}, S_{13} and S_{15} are unknown, but $S_{17} = S_1^{16}$, $S_{19} = S_3^{16}$ (which is known); $S_{21} = S_5^{16}$; S_{23} is unknown, and $S_{25} = S_7^{16}$ (which is unknown), $\dots$, etc.

In general, these relationships may be most readily determined by examining the binary representations of these numbers, as suggested by Mann (Ref. 26). Since multiplication of j by 2 mod $(2^k - 1)$ is equivalent to a cyclic shift of the k -digit binary expansion of j , S_i is a power of S_j in $GF(2^k)$ if and only if the k -digit binary expansions of i and j are equal except for a cyclic shift.

For every known S_j , j odd, $2t < j < 4t$, we also know $\Delta_{j-2t}^{(t)}$. For each such known S_j , we therefore have an algebraic equation relating the unknown coefficients of the polynomial $f^{(\infty, n)}$. If $\deg \sigma^{(t)} = t$ and there are actually $t + e$ errors, then $\deg f^{(\infty, t)} = e$. Thus, we will have several simultaneous algebraic equations in e unknowns. If we could solve these equations, then we could first determine the polynomial $f^{(\infty, t)}$ and then the error polynomial. Of course, we do not generally know the value of e , but the objective is clearly to solve these equations with a polynomial f of as small degree as possible.

For example, if $\Delta_k^{(t)} = 0$ for all k for which it is known, then the polynomial $f(x) = 1$ solves all of the equations. In this case, of course, the received word lies in a coset containing no more than t errors and the error polynomial is given by $\sigma^{(t)}$.

If instead, $\Delta_k^{(t)} \neq 0$ for some value or values of k , then there must be more than t errors, and $\sigma^{(t)}$ is definitely not the error polynomial. For large values of t , this in itself is well worth knowing, because it enables the decoder to avoid going through a search over the N th roots of unity to attempt to find the roots of $\sigma^{(t)}$.

In order to correct the additional errors, one must solve the equations for the coefficients of the polynomial f . If there are only $t + 1$ errors, then the algebraic equations contain only one unknown, and the situation is relatively hopeful. If this equation has degree ≤ 4 , then it can be quickly solved (without any search) by the methods of Berlekamp, Rumsey and Solomon (SPS 37-39, Vol. IV, pp. 219-226). For example, we saw that in the 5-error correcting code of block length 63, the decoder knows S_1, S_3, S_5, S_7, S_9 , from which he can compute $\sigma^{(1)}, \tau^{(1)}, \dots, \sigma^{(5)}, \tau^{(5)}, S^{(5)}$ and $A^{(5)}$. Knowledge of $S_{17} = S_5^{16}$ enables one to compute $\Delta_7^{(5)}$, thereby obtaining a quartic equation for $f_1^{(\infty, 5)}$. Solution of this quartic enables the decoder to determine $f(x)$, and then the error polynomial $\sigma(x)$, assuming there were no more than 6 errors. If one or more of these equations is linear in some unknown, then that unknown can be eliminated by substitution. For example, if Δ_3 is known, then we can find the error polynomial for a pattern of $(t + 2)$ errors with the solution of a single algebraic equation for f_1 . The equation for Δ_3 , which is linear in f_2 , can be used to eliminate f_2 from the expression for the next known Δ_i , which is algebraic in f_1 and f_2 .

In some cases, the equations generated by this method appear to be unnecessarily complicated. For example, consider a two-error-correcting BCH code. The error polynomial can be shown to be given by

$$\sigma(x) = \begin{cases} 1 \text{ if } S_1 = 0 \text{ and } R_3 = S_3 + S_1^3 = 0 & (0 \text{ errors}) \\ 1 + S_1 x \text{ if } S_1 \neq 0 \text{ and } R_3 = S_3 + S_1^3 = 0 & (1 \text{ error}) \\ 1 + S_1 x + \frac{R_3}{S_1} x^2 \text{ if } S_1 \neq 0, R_3 \neq 0 \text{ and } \text{Tr}\left(\frac{R_3}{S_1^3}\right) = 0 & (2 \text{ errors}) \\ (1 + (S_1 + \xi)x) \left(1 + \xi x + \left(\frac{R_3}{\xi} + S_1^2 + S_1 \xi\right)x^2\right) & \\ \text{if } R_3 \neq 0 \text{ and } \text{Tr}\left(\frac{R_3}{S_1^3}\right) \neq 0; \text{Tr}\left(\frac{R_3}{\xi^3}\right) = 0 & (3 \text{ errors}) \end{cases}$$

5. Three-Error Case

The zero and one-error cases can be verified immediately. The polynomial for the two-error case follows directly from the iterative algorithm. This polynomial has two roots in the field if and only if $\text{Trace}(R_3/S_1^3) = 0$. (For proof, see SPS 37-39, Vol. IV, pp. 219-226.)

In order to verify the three-error case, we must check that the given expression for $\sigma(x)$ satisfies Newton's identities and that it has three distinct roots in the field. By the iterative algorithm a polynomial of fourth degree satisfies Newton's identities when $R_3 \neq 0$ if and only if it is of the form

$$\sigma^{(3)}(x) = 1 + S_1x + \left(\frac{R_3}{S_1} + \frac{\Delta_1}{R_1}\right)x^2 + \left(\frac{\Delta_1 S_1}{R_1}\right)x^3$$

The assumed polynomial was

$$1 + (S_1 + \xi)x \left(1 + \xi x + \left(\frac{R_1}{\xi} + S_1^2 + S_1\xi\right)x^2\right)$$

These two expressions agree if we set

$$\Delta_1 = \frac{R_1}{S_1} (S_1 + \xi) \left(\frac{R_1}{S_1} + S_1^2 + S_1\xi\right)$$

Finally, we must check that the claimed polynomial actually has three roots in the field. The condition for this to occur is that

$$\text{Tr}\left(\frac{R_3}{\xi^3}\right) = 0$$

If the minimum weight member of a particular coset is not unique, then there is little gain in decoding that coset, for the *a posteriori* probability of error will be greater than 1/2, no matter how the coset is decoded. For this reason, it is of interest to inquire whether or not the triple error pattern decoded by the above method is unique. The answer depends upon the field. In $GF(2^3)$, there are only three nonzero elements with trace 0; each of them has a unique cube root. Any of the three may be selected as a_0 , but, for any given syndrome, the resulting triple error pattern will be the same. The choice of a_0 will affect only those of the three errors denoted by $(S_1 + \xi)$. Since $S_3 = S_1^3$ in $GF(2^3)$, it is evident that this double error correcting BCH code can in fact correct three errors in all cases. Since this code is actually the

trivial code consisting only of two codewords, all zeroes and all ones, this is no surprise.

In $GF(2^4)$, there are 7 nonzero elements with trace zero. Three of these are suitable choices for a_1 , and three are suitable choices for a_2 , but only one of them is a suitable choice for a_0 . Thus, in $GF(2^4)$ we must have $a_0 = 1$; there is no other possibility. Hence, if R_3 is a cube, then there are only three possible values of ξ , namely the three cube roots of R_3 . The error pattern is represented by $S_1 + \xi_i$, for $i = 1, 2, 3$; with $\xi^3 = R_3$. Thus, in $GF(2^4)$, cosets with R_3 , a perfect cube, have unique coset leaders. Other cosets of weight three do not.

In all other fields, coset leaders are never unique. In $GF(2^{k+1})$, this is obvious, since there are $2^{2k} - 1$ nonzero elements with trace 0, and any of them may be chosen as a_0 ; a_1 and a_2 need not be used at all. In $GF(2^{2k})$, we begin by choosing $a_0 = 1$. Since $(2^{2k-1} - 1)$ nonzero field elements have trace zero, and only $(2^{2k-1} - 1)/3$ have multiplicative order divisible by three, it is clear that there are at least $(2^{2k} - 4)/6$ elements with trace 0 and multiplicative order not divisible by three. If one of them provides a suitable choice for a_1 , then we may set $a_2 = a_1^2$, or, conversely, if one of them is a suitable choice for a_2 , we might set $a_1 = a_2^2$. This shows that there must be an equal number of choices for a_1 and a_2 , and hence, at least $(2^{2k-2} - 1)/3$ choices for each.

Finally, we must show that there are multiple choices for the constant a_0 . If α is a primitive element of $GF(2^{2k})$, then it can be shown that $a_0 = \alpha^{(2^{2k-1}-1)}$ represents such a choice, as does $a_0 = 1$. The proof of this fact is omitted. Thus, there is little value in decoding cosets of weight three in two-error-correcting binary BCH codes, except in the case when the block length is 15.

6. Conclusion

The method introduced here for correcting more than t errors seems to be advantageous chiefly at moderate-to-low information rates (large t , small difference between S_{2t-1} and the next known S_i , and correspondingly small degree of the algebraic equations). The method outlined by Gorenstein, Peterson, and Zierler (Ref. 27) for the special case $t = 2$, and implemented along the lines suggested in (SPS 37-39, Vol. IV, pp. 219-226) has obvious advantages when $t = 2$. At intermediate rates, the method suggested in (Ref. 25) appears preferable.

Our method is seen to be very efficient for correcting one additional error, particularly when S_{2t+3} , S_{2t+5} , or S_{2t+7} is known, since in these cases the algebraic equation

for f_1 has degree 2, 3, or 4, respectively. Unfortunately, this method does not appear to be very feasible for correcting more than one additional error, unless someone devises a good algorithm for solving simultaneous algebraic equations in several unknowns in a finite field.

H. Information Processing: Signal and Noise in Nonlinear Devices, C. A. Greenhall¹²

This article presents a definition of the signal and noise portions of the output of a rather general nonlinear device. We are able to write down the sample functions of the output signal and noise processes in a particularly simple way. This leads to a formula for the signal output of a hard limiter and thence to a convenient integral expression for the output signal amplitude of a hard bandpass limiter. The input signal can be both amplitude and phase modulated. This integral expression was obtained in the special case of phase modulation by Tausworthe in SPS 37-35, Vol. IV, pp. 307-309. In this article we show the relationship between our method and his.

1. Definition of Output Signal and Noise

Suppose the input to the device in question is

$$x(t) = s(t) + n(t) \quad (-\infty < t < \infty) \quad (1)$$

where the signal $s(t)$ and noise $n(t)$ are sample functions of independent real stationary processes $S(t, \xi_s)$ and $N(t, \xi_n)$, respectively. The ξ_s and ξ_n are sample "points" (functions) of independent sample spaces Ω_s and Ω_n , which have probability measures P_s and P_n . Assume that these processes have finite variance:

$$\begin{aligned} E(s^2(t)) &= \int_{\Omega_s} S^2(t, \xi_s) P_s(d\xi_s) < \infty \\ E(n^2(t)) &= \int_{\Omega_n} N^2(t, \xi_n) P_n(d\xi_n) < \infty \end{aligned} \quad (2)$$

The output $y(t)$ ($-\infty < t < \infty$) of the device is to be a sample function of a process $Y(t, \xi_s, \xi_n)$ dependent on the input process $x(t)$. We require that the device be time invariant; in other words, if the input is $x(t - \delta)$ then the output is $y(t - \delta)$. This guarantees that Y is stationary.

¹²National Research Council postdoctoral resident research associate supported by NASA.

We also ask that

$$E(y^2(t)) = \int_{\Omega_s} \int_{\Omega_n} Y^2(t, \xi_s, \xi_n) P_s(d\xi_s) P_n(d\xi_n) < \infty \quad (3)$$

The fundamental problem is to determine what is meant by the "signal" and "noise" portions of $y(t)$. We write $y(t) = s_y(t) + n_y(t)$, and propose the following conditions on this decomposition:

- (i) $s_y(t)$ is to be the sample function of a stationary process $S_y(t, \xi_s)$ of finite variance on the original signal sample space Ω_s .
- (ii) $n_y(t) = y(t) - s_y(t)$ is to be uncorrelated with all random variables in the space M of signal random variables of finite variance, i.e., with all random variables $f(\xi_s)$ on Ω_s such that $E(f^2) < \infty$. Thus

$$E(n_y(t)f) = E(n_y(t))E(f) \quad (4)$$

for all f in M .

Conditions (i) and (ii) imply that

$$\begin{aligned} E[y(t_1)y(t_2)] &= E[s_y(t_1)s_y(t_2)] + E[n_y(t_1)n_y(t_2)] \\ &\quad + 2E(n_y)E(s_y) \end{aligned} \quad (5)$$

and hence the power spectrum of y is, except possibly for a dc term, the sum of the power spectra of s_y and n_y . This condition is implied in Davenport's paper (Ref. 28) on the bandpass limiter, in which he separates the signal and noise contributions to the power spectrum of the output.

To see how far conditions (i) and (ii) determine s_y and n_y , we write Eq. (4) as

$$E[(n_y - E(n_y))f] = 0 \quad (6)$$

for all f in M . Then write $y(t)$ as

$$y(t) = [s_y(t) + E(n_y)] + [n_y(t) - E(n_y)]$$

Since $E(n_y)$ is just a constant, it belongs to M . Therefore, $s_y(t) + E(n_y)$ is in M by (i), and $n_y(t) - E(n_y)$ is orthogonal to M by Eq. (6). Thus, $s_y(t) + E(n_y)$ is the projection $p(t) = P(t, \xi_s)$ of the random variable $y(t)$ onto M , the "signal space." An embryonic form of this idea appeared in SPS 37-23, Vol. IV, pp. 160-164, in which the signal portion was defined as the projection

of $y(t)$ onto the subspace generated by just a single random variable in M .

Furthermore, the random variables

$$\begin{aligned}s_y(t) &= p(t) + c \\ n_y(t) &= y(t) - s_y(t)\end{aligned}\quad (7)$$

where c is any constant, satisfy (i) and (ii). Thus the output signal and noise are determined within constants.

The projection $p(t)$ can be written down explicitly. We know that $p(t)$ is an integrable random variable in Ω_s satisfying $E(p(t)f) = E(y(t)f)$ for all bounded measurable f on Ω_s . This implies that with probability one on Ω_s ,

$$P(t, \xi_s) = E(y(t)|S)(\xi_s) = \int_{\Omega_n} Y(t, \xi_s, \xi_n) P_n(d\xi_n) \quad (8)$$

the conditional expectation of $y(t)$ with respect to the original signal process $S(t, \xi_s)$. It can be verified that P is stationary.

We now set $c = 0$ in Eq. (7) and adopt our definitions of output signal and noise

$$\begin{aligned}s_y(t) &= E(y(t)|S) \\ n_y(t) &= y(t) - s_y(t)\end{aligned}\quad (9)$$

In summary, the signal portion of the output at time t is obtained by fixing the input signal and averaging the output at time t over all possible noise inputs belonging to the noise process N .

The definition Eq. (9) may also be of use for nonstationary input signals. If the processes are not stationary,

then conditions (i) and (ii) yield that

$$s_y(t) = E(y(t)|S) + c(t)$$

where $c(t)$ is an arbitrary deterministic function of time. Some further condition (like $c = \text{constant}$) is needed to define s_y well enough.

2. Special Classes of Nonlinear Devices

We apply the definitions Eq. (9) of output signal and noise to the following two classes of devices.

a. Linear filter. Let

$$y(t) = (Hx)(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau \quad (-\infty < t < \infty) \quad (10)$$

where $x = s + n$ as in Eq. (1) and

$$\int_{-\infty}^{\infty} (1+t^2)h^2(t)dt < \infty \quad (11)$$

The condition Eq. (11) on the impulse response h of the filter ensures that with probability one the integral Eq. (10) exists for all t and that $E(y^2) < \infty$, given that $E(x^2) < \infty$. We will show that

$$s_y(t) = (Hs)(t), \quad n_y(t) = (Hn)(t) \quad (12)$$

where s_y and n_y are defined by Eq. (9). The condition Eq. (11) and $E(s^2) < \infty$ ensures that $(Hs)(t)$ is a random variable on Ω_s with finite variance, so all we have to do is verify the projection property

$$E[(Hs)(t)f] = E[y(t)f] \quad (13)$$

for all random variables f on Ω_s of finite variance. Thus

$$\begin{aligned}E\left[f \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau\right] &= \int_{-\infty}^{\infty} h(t-\tau)E[f(s(t)+n(t))]d\tau \\ &= \int_{-\infty}^{\infty} h(t-\tau)E[fs(t)]d\tau = E\left[f \int_{-\infty}^{\infty} h(t-\tau)s(\tau)d\tau\right]\end{aligned}\quad (14)$$

which is Eq. (13). With the definition Eq. (9), then, a linear filter does not mix the signal and noise.

This property extends further. Suppose we follow the nonlinear device of subsection 1 by a filter H . Thus the combined output is $z(t) = (Hy)(t) = (Hs_u)(t) + (Hn_u)(t)$. If we replace s and n in Eq. (14) by s_u and n_u , and note that $E(fn_u(t)) = 0$ by definition, we see that

$$s_{uu}(t) = Hs_u(t), \quad n_{uu}(t) = Hn_u(t) \quad (15)$$

The decomposition Eq. (9) is not affected by passing through a linear filter.

b. Zero-memory device. Here we let

$$y(t) = F(x(t)) \quad (16)$$

where F , the characteristic of the device, is a real-valued function such that

$$E[y^2(t)] = \int_{\Omega_s} \int_{\Omega_n} F^2(S(t, \xi_s) + N(t, \xi_n)) P_s(d\xi_s) P_n(d\xi_n) < \infty$$

According to Eqs. (8) and (9),

$$\begin{aligned} s_u(t) &= \int_{\Omega_s} F(s(t) + N(t, \xi_n)) P_n(d\xi_n) \\ &= \int_{-\infty}^{\infty} F(s(t) + n) p(n) dn = G(s(t)) \end{aligned} \quad (17)$$

where p is the probability density of $n(t)$. As far as the signal is concerned, the device acts like another device with characteristic G , which, of course, depends on the noise distribution $p(n)dn$. We will call G the *signal characteristic* of the device.

Now suppose the input signal and noise to this device F are narrow-band about a center frequency ω_0 . Let the signal have the form $s(t) = V(t) \sin(\omega_0 t + \theta(t))$, $V(t) \geq 0$, where $V \sin \theta$ and $V \cos \theta$ are narrow-band about zero fre-

quency with bandwidths small compared with ω_0 . Also, assume that the random variables $V(t)$ and $\theta(t)$, t fixed, are independent, that $\theta(t)$ is uniformly distributed in $[0, 2\pi]$, and that the distribution of $V(t)$ is independent of t . Then

$$\begin{aligned} E[G^2(s(t))] &= E[G^2(V(t) \sin(\omega_0 t + \theta(t)))] \\ &= E\left[\frac{1}{2\pi} \int_0^{2\pi} G^2(V(t) \sin \phi) d\phi\right] < \infty \end{aligned} \quad (18)$$

Hence with probability one, we can expand $G(V(t) \sin \phi)$ in a Fourier series:

$$G(V(t) \sin \phi) = \sum_{k=-\infty}^{\infty} c_k(V(t)) e^{ik\phi} \quad (19)$$

in $L^2(0, 2\pi)$, where the Fourier coefficients c_k are given by

$$c_k(V) = \frac{1}{2\pi} \int_0^{2\pi} G(V \sin \phi) e^{-ik\phi} d\phi \quad (20)$$

The same change of variable as was used in Eq. (18) will give that

$$G(s(t)) = \sum_{k=-\infty}^{\infty} c_k(V(t)) e^{ik(\omega_0 t + \theta(t))} \quad (21)$$

in $L^2(\Omega_s)$. The convergence in Eq. (21) is uniform in t because the distribution of $V(t)$ is independent of t , by assumption. The terms in Eq. (21) are the signal components in the narrow frequency zones about each $\pm k\omega_0$ ($k = 1, 2, \dots$). We can find a (nonrealizable) filter H satisfying Eq. (11) whose complex transfer function is 1 in the k th zone and 0 in all other zones (by making the transfer function smooth enough). If such a filter passes the k th harmonic unchanged and annihilates all the others, then by Eq. (15) the signal output of the device consisting of the zero-memory device followed by H is

$$2 \operatorname{Re} [c_k(V(t)) e^{ik(\omega_0 t + \theta(t))}] \quad (22)$$

In the first zone, the phase modulation $\theta(t)$ passes through unchanged.

3. Bandpass Limiter

An example of a zero-memory device is the ideal bandpass limiter, where

$$\begin{aligned} F(x) &= 1 & (x \geq 0) \\ &= -1 & (x < 0) \end{aligned} \quad (23)$$

(See Eq. 16.) Henceforth the input noise $n(t)$ will be a stationary Gaussian process, with $E(n) = 0$, $E(n^2) = \sigma^2$. We easily calculate from Eq. (17) that the signal at the output of the hard limiter is

$$\begin{aligned} s_y(t) &= G(s(t)) = g\left(\frac{s(t)}{\sigma}\right) \\ g(x) &= \left(\frac{2}{\pi}\right)^{1/2} \int_0^x e^{-\frac{1}{2}z^2} dz \end{aligned} \quad (24)$$

(See Fig. 23.) For large signal-to-noise ratios the signal characteristic G is itself like a hard limiter. For small signal-to-noise ratios, G is almost linear, i.e.,

$$G(s) \approx \left(\frac{2}{\pi}\right)^{1/2} \frac{s}{\sigma} \quad (25)$$

Consider now the case of narrow-band signal and noise inputs. The harmonic expansion Eq. (21) of $s_y(t)$ can be written

$$s_y(t) = \sum_{k=1}^{\infty} b_k(v(t)) \sin(k\omega_n t + k\theta(t)), \quad v(t) = \frac{V(t)}{\sigma} \quad (26)$$

$$b_k(v) = \frac{1}{\pi} \int_{-\pi}^{\pi} g(v \sin \phi) \sin k\phi \, d\phi \quad (k = 1, 2, \dots) \quad (27)$$

if k is even then $b_k = 0$.

Tausworthe (SPS 37-35, Vol. IV, pp. 307-309) obtained the expression Eq. (27) for the signal amplitude in the k th zone, in the absence of amplitude modulation. It can be put in the form of a hypergeometric function, Davenport's original form (Ref. 28). Here is a Bessel function form of b_k : integrate Eq. (27) by parts to obtain

$$\begin{aligned} b_k(v) &= \left(\frac{2}{\pi}\right)^{1/2} \frac{v}{k\pi} \int_{-\pi}^{\pi} e^{-\frac{1}{2}v^2 \sin^2 \phi} \cos k\phi \cos \phi \, d\phi \\ &= \left(\frac{2}{\pi}\right)^{1/2} \frac{v}{k\pi} \int_{-\pi}^{\pi} e^{-\frac{1}{2}v^2(1 - \cos 2\phi)} \frac{1}{2} [\cos(k-1)\phi + \cos(k+1)\phi] \, d\phi \\ &= \left(\frac{2}{\pi}\right)^{1/2} \frac{v}{k} e^{-\frac{1}{2}v^2} \left[\frac{1}{4\pi} \int_{-\pi}^{\pi} e^{\frac{1}{2}v^2 \cos \theta} \cos\left(\frac{k-1}{2}\theta\right) d\theta + \frac{1}{4\pi} \int_{-\pi}^{\pi} e^{\frac{1}{2}v^2 \cos \theta} \cos\left(\frac{k+1}{2}\theta\right) d\theta \right] \\ &= \left(\frac{2}{\pi}\right)^{1/2} \frac{v}{k} e^{-\frac{1}{2}v^2} \left[I_{\frac{1}{2}(k-1)}\left(\frac{1}{4}v^2\right) + I_{\frac{1}{2}(k+1)}\left(\frac{1}{4}v^2\right) \right] \quad (k \text{ odd}) \end{aligned} \quad (28)$$

where the I_m are modified Bessel functions of the first kind.

The signal power in the k th zone is

$$\frac{1}{2} E \left[b_k^2 \left(\frac{V(t)}{\sigma} \right) \right]$$

From this we can compute signal-to-noise ratio in this zone, and the total power there is $8/(\pi k)^2$ (SPS 37-35,

Vol. IV, pp. 307-309), and the signal and noise are uncorrelated.

4. Comparison with Tausworthe's Treatment

We will briefly review Tausworthe's 1965 (SPS 37-35, Vol. IV, pp. 307-309) derivation of the signal amplitude Eq. (27) in the k th zone of the output of a hard limiter. He assumes a signal

$$s(t) = (2)^{1/2} \sin(\omega_n t + \theta(t)) \quad (29)$$

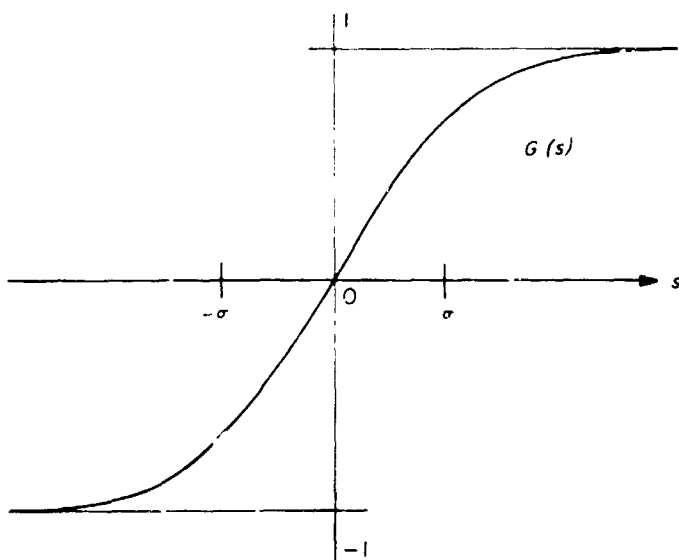


Fig. 23. Signal characteristic of a hard limiter in Gaussian noise

where $\theta(t)$ is uniformly distributed. The noise is narrow band stationary Gaussian. The output of the hard limiter is written

$$y(t) = \sum_{k=1, \text{ odd}}^{\infty} y_k(t)$$

$$y_k(t) = S_k(t) + N_k(t), S_k(t) = E(y(t) w_k(t)) w_k(t)$$

$$w_k(t) = (2)^{1/2} \sin(k \omega_c t + k \theta(t)) \quad (30)$$

where y_k , S_k , N_k are, respectively, the total output, signal, and noise in the k th zone. He then puts $E(y(t) w_k(t))$ into the form of Eq. (27).

To see why our methods agree, first we remark that

$$E(y_k(t) w_l(t)) = 0 \quad (k \neq l; k, l = 1, 2, \dots) \quad (31)$$

This can be shown by writing $y_k(t)$ in the form $A(t) \sin(k \omega_c t + k \psi(t))$ (SPS 37-35, Vol. IV, pp. 307-309, section 3), and observing that the distribution of $\psi(t)$ given $\theta(t)$ is symmetric about $\theta(t)$. The calculations are omitted.

Summing over all zones, the output signal is

$$S(t) = \sum_{k=1}^{\infty} E(y(t) w_k(t)) w_k(t) \quad (32)$$

where we have used Eq. (31) to justify including the even harmonics. Thus $S(t)$, t fixed, is just the projection of the random variable $y(t)$ onto the subspace generated by $w_1(t), w_2(t), \dots$. By the same maneuver as was used in Eq. (16) it can be shown that this subspace is just the set of random variables $f(s(t))$ depending only on $s(t)$ such that $E[f^2(s(t))] < \infty$. However, our $s_y(t)$ in Eq. (26) also belongs to this subspace and is the projection of $y(t)$ onto the larger subspace of random variables which depend on the whole signal process. Therefore, $S(t) = s_y(t)$; i.e., our methods give the same signal component of the output when $V(t)$ is constant.

I. Data Compression Techniques: Use of Six and Eight Quantiles to Test Hypotheses in Data-Compressed Experiments, I. Eisenberger

1. Introduction

Theoretical analyses relative to the use of sample quantiles for data compression of space telemetry have been given in three previous JPL Technical Reports, Refs. 29-31. The first of these reports, Ref. 29, deals with the problem of estimating the parameters of a normal distribution using up to twenty sample quantiles, and also describes two goodness-of-fit tests, each using four sample quantiles. The second and third reports, Refs. 30 and 31 are concerned with hypothesis testing and the estimation of the correlation coefficient ρ of a bivariate normal distribution, using up to four sample quantiles. A forthcoming technical report, *Tests of Hypotheses and Estimation of the Correlation Coefficient Using Quantiles, III*, will extend most of the results given in Refs. 30 and 31 to six and eight sample quantiles. The purpose of this article is to give the hypotheses and assumptions relating to the tests and the assumptions relating to estimating ρ . The derivation and statement of the test statistics and estimators are given in the report.

2. Review of Quantiles

To define a quantile, consider a sample of n independent values $x_1, x_2, \dots, x_n$ taken from a distribution of continuous type with distribution function $G(x)$ and density function $g(x)$. The p th quantile, or the quantile of order p of the distribution or population, denoted by ζ_p , is defined as the root of the equation $G(\zeta) = p$; that is

$$p = \int_{-\infty}^{\zeta} dG(x) = \int_{-\infty}^{\zeta} g(x) dx$$

The corresponding *sample* quantile z_p is defined as follows: If the sample values are arranged in nondecreasing order of magnitude

$$x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$$

then $x_{(i)}$ is called the i th order statistic and

$$z_p = x_{[np]+1}$$

where $[np]$ is the greatest integer $\leq np$.

If $g(x)$ is differentiable in some neighborhood of each quantile value considered, it has been shown (Ref. 32) that the joint distribution of any number of sample quantiles is asymptotically normal as $n \rightarrow \infty$ and that, asymptotically,

$$\begin{aligned} E(z_p) &= \zeta_p \\ \text{Var}(z_p) &= \frac{p(1-p)}{n g^2(\zeta_p)} \\ \rho_{12} &= \left[\frac{p_1(1-p_2)}{p_2(1-p_1)} \right]^{1/2} \end{aligned}$$

where ρ_{12} is the correlation between z_{p_1} and z_{p_2} , $p_1 < p_2$. The statement " $g(x) = N(\mu, \sigma)$ " will mean that the random variable under consideration is normally distributed with mean μ , variance σ^2 , and has the density function $g(x)$ associated with it.

3. Hypotheses and Assumptions

For comparison purposes, the designation here of the tests will coincide with that used for the tests in the reports.

In test A, we are given a set of n independent observations from a normal population with known variance σ^2 ; the test is designed to decide whether the mean μ has a value of μ_1 or a value of μ_2 . In test $\bar{A}$, the assumption that σ^2 is known is not used.

In test B, we test whether the standard deviation σ has a value of σ_1 or a value of σ_2 . When more than one quantile is used, it is not necessary to assume that μ is known.

In tests D, $\bar{D}$, and $\bar{E}$, it is assumed that we are given sets of independent samples taken from two independent normal populations with means μ_1 and μ_2 and variances

σ_1^2 and σ_2^2 . In test D we assume that $\sigma = \sigma_1 = \sigma_2$ is known and μ_1 is unknown; we test whether $\mu_2 = \mu_1$ or $\mu_2 = \mu_1 + \theta$, $\theta \neq 0$. In test $\bar{D}$, the assumption that σ is known is not used. In test $\bar{E}$ we assume that both $\mu = \mu_1 = \mu_2$ and σ_1 are unknown and test whether $\sigma_2 = \sigma_1$ or $\sigma_2 = \theta \sigma_1$, $\theta > 0$.

In tests F and $\bar{F}$, we are given n independent pairs of observations $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ taken from two normally distributed populations where $g_1(x) = N(\mu_1, \sigma_1)$ and $g_2(y) = N(\mu_2, \sigma_2)$. In test F we assume that $\mu_1, \mu_2, \sigma_1, \sigma_2$ are known and test whether $\rho = 0$ or $\rho \neq 0$. In test $\bar{F}$ we assume that both $\mu = \mu_1 = \mu_2$ and $\sigma = \sigma_1 = \sigma_2$ are unknown and again test whether $\rho = 0$ or $\rho \neq 0$.

In estimating ρ , it will first be assumed that the assumptions of test F hold. This estimator will be denoted by $\hat{\rho}_1$. For the second estimator $\hat{\rho}_2$ it will be assumed that $\mu = \mu_1 = \mu_2$ is unknown and that σ_1 and σ_2 are known.

Table 3 summarizes the above hypotheses and assumptions.

Table 3. Hypotheses and assumptions relating to the tests, and assumptions relating to estimating μ_1 and ρ_2

Test	Null hypothesis	Alternative hypothesis	Assumptions
A	$g(x) = N(\mu_1, \sigma)$	$g(x) = N(\mu_2, \sigma)$	σ known
$\bar{A}$	$g(x) = N(\mu, \sigma)$	$g(x) = N(\mu, \sigma)$	σ unknown
B	$g(x) = N(\mu, \sigma_1)$	$g(x) = N(\mu, \sigma_2)$	μ unknown
D	$g_1(x) = N(\mu, \sigma)$	$g_1(x) = N(\mu, \sigma)$	x and y independent; σ known, μ unknown.
$\bar{D}$	$g_2(y) = N(\mu, \sigma)$	$g_2(y) = N(\mu + \theta, \sigma)$ $\theta \neq 0$	x and y independent; μ and σ unknown.
$\bar{E}$	$g_1(x) = N(\mu, \sigma)$ $g_2(y) = N(\mu, \sigma)$	$g_1(x) = N(\mu, \sigma)$ $g_2(y) = N(\mu, \theta \sigma)$ $\theta > 0$	x and y independent; μ and σ unknown.
F	$g_1(x) = N(\mu_1, \sigma_1)$ $g_2(y) = N(\mu_2, \sigma_2)$ $\rho = 0$	$g_1(x) = N(\mu_1, \sigma_1)$ $g_2(y) = N(\mu_2, \sigma_2)$ $\rho \neq 0$	$\mu_1, \mu_2, \sigma_1, \sigma_2$ known
$\bar{F}$			$\mu_1 = \mu_2 = \mu$ and $\sigma = \sigma_1 = \sigma_2$ unknown
	Estimating ρ_1		$\mu_1, \mu_2, \sigma_1, \sigma_2$ known
	Estimating ρ_2		$\mu_1 = \mu_2 = \mu$ unknown σ_1 and σ_2 known

J. Astrometrics: A New Method For Extracting The Reflectivity Distribution From Planetary Radar Data, S. Zohar

1. Introduction

The present approach to the extraction of the planetary reflectivity distribution from the range gated reflection

off the observed planet is based on the spectrum of this signal. In practice, the spectrum is computed from the signal's measured autocorrelation. A recently derived result (SPS 37-43, Vol. IV, pp. 330-338) makes it possible to extract the information directly from the autocorrelation function. Computationally, this method is markedly different from the spectral approach, being based on a different set of assumptions. There are indications that the new method will more closely approximate reality. However, this is difficult to substantiate on theoretical grounds; only test case comparisons between the two methods will tell which is better. In the meantime, the main importance of this new method lies simply in its providing a different path to the desired reflectivity distribution. Thus, any feature or details which show up in both processing schemes could be considered with a high degree of confidence as representing physical reality.

2. Formulation of the Problem

Let the reflectivity distribution be subjected to a parallel projection along the line of sight onto a plane perpendicular to this line. We introduce the coordinates θ, ν in the projection plane as shown in Fig. 24. These are associated with range and doppler shift, respectively. The reflectivity distribution can now be expressed as a two dimensional Fourier series in these two variables.¹³

$$\hat{F}(\theta, \nu) = \sum_{n, r=-\infty}^{\infty} b_{nr} e^{i2(\pi n \nu + r \theta)}$$

with

$$b_{-n, r} = b_{nr}; \quad b_{n, -r} = b_{nr}. \quad (1)$$

We have shown (SPS 37-43, Vol. IV, pp. 330-338) that the Nyquist rate samples of the (normalized) range-gated autocorrelation function of the reflected signal (ρ_k) depend on the b_{nr} Fourier coefficients as follows:

$$\rho_k = h_0 \frac{W_r}{W} \sum_{k, n, r=-\infty}^{\infty} a_k b_{nr} \cos [2\pi(n+k)\nu_c] \times J_{k+r}[\pi(n+k)\hat{\nu}]; \quad (2)$$

¹³Strictly speaking, $\hat{F}(\theta, \nu)$ is not the reflectivity function defined in SPS 37-43, Vol. IV, pp. 330-338; it is the so-called "feature function." The distinction between the two, however, is not important here; hence, we use somewhat loosely the term "reflectivity distribution." For details, as well as the justification of Eq. (1), see SPS 37-43, Vol. IV, pp. 330-338.

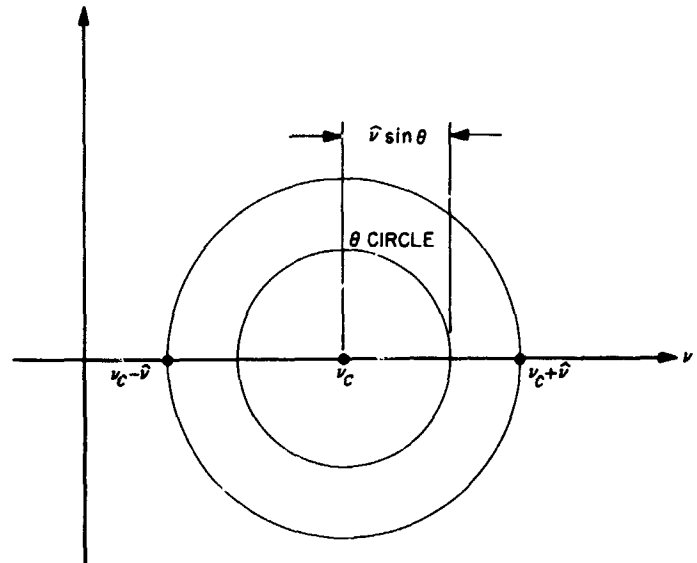


Fig. 24. Coordinates for the reflectivity distribution

$\hat{\nu}$ appearing in the Bessel function argument is half the normalized limb-to-limb bandwidth; ν_c is the normalized frequency of the subradar point (Fig. 24). Here h_0, a_k are computable from the range-gate and backscatter functions, while W_r and W are, respectively, computable and measurable powers in the radar system.

We have also shown that for a featureless "grey planet" Eq. (2) leads to the much simpler samples g_k given by:¹⁴

$$g_k = \cos(2\pi k \nu_c) \sum_{k=0}^{\infty} \epsilon_k a_k J_k(\pi k \hat{\nu}). \quad (3)$$

Eqs. (2) and (3) are the basis for the new extraction procedure.

Our purpose is to find the reflectivity distribution $\hat{F}(\theta, \nu)$, given the correlation coefficients ρ_k . Equivalently, our problem can be referred to as the inversion of Eq. (2); namely, given the set ρ_k , find the set b_{nr} . It should be realized from the outset that if only the first I samples are available, then there is no unique $\hat{F}(\theta, \nu)$ that fits them; rather, there is a large family of possible solutions, all of which share the first I correlation coefficients. We can narrow the range of possible functions to a single one only by introducing specific assumptions to compensate for the missing data. What assumptions should one make? Here we have only some simple guidelines, such as the requirement that the assumptions should be plausible and that the solutions based on different assumptions should not be

¹⁴ ϵ_k is Neumann's symbol ($\epsilon_k = 2$, except $\epsilon_0 = 1$).

drastically different. In the final analysis, of course, the assumptions should lead to manageable mathematics.

It is with this background in mind that we approach the inversion of Eq. (2). Our fundamental assumption is that all b_{nr} 's vanish for n, r greater than some specific bounds consistent with the amount of data available. The basic justification for this is that $\hat{F}(\theta, \nu)$, being a function representing some aspect of physical reality, is not expected to have jumps or discontinuities. Its Fourier coefficients b_{nr} should, therefore, decay to zero with increasing n, r .

The effect of this assumption on Eq. (2) is to replace the infinite summations over n, r with finite ones so that our problem is essentially reduced to the solution of a finite set of linear equations. A possible approach to the problem is to obtain the autocorrelation for several adjacent zones covering the region of interest and then process the data of all the zones simultaneously. Such a procedure does actually lead to a reasonable estimate of $\hat{F}(\theta, \nu)$, and we shall return to it later on. However, the amount of computation involved in such a scheme is prohibitive and seems to rule this approach out as a practical solution.

An alternative approach which does lead to a practical scheme is to use sufficiently narrow gates so that $\hat{F}(\theta, \nu)$ could be assumed independent of θ over the zone. This results in the processing of each zone independently of the others, leading to a drastic reduction in computations. We consider this approach in detail in the next subsection.

3. The New Extraction Method

Our initial task is to obtain the special case of Eq. (2) for the constraint that $\hat{F}(\theta, \nu)$ be independent of θ . This constraint is satisfied by assigning the following form to b_{nr} :

$$b_{nr} = b_{n0}\delta_{r0} \equiv b_n\delta_{r0}.$$

Substitution of this in Eq. (2) leads to¹⁵

$$\begin{aligned} \rho_k &= h_0 \frac{W_r}{W} \sum_{n=-\infty}^{\infty} b_n \cos [2\pi(k+n)\nu_c] \\ &\times \sum_{k=0}^{\infty} \varepsilon_k a_k J_k^2 [\pi(k+n)\nu]. \end{aligned}$$

¹⁵The vanishing of the high order b_n 's will be handled later on.

This, however, can be expressed in terms of the correlation coefficients of the corresponding "grey planet" Eq. (3), yielding

$$\rho_k = h_0 \frac{W_r}{W} \sum_{n=-\infty}^{\infty} g_{k-n} b_n. \quad (4)$$

The computation of the g -parameters has been discussed in detail (SPS 37-43, Vol. IV, pp. 330-338). Of the remaining parameters, W_r and h_0 can be computed; whereas W , the signal power, can be measured. We see, therefore, that if the ρ_k 's are measured, the only unknowns in Eq. (4) are the b_n 's. In practice, however, we never get the autocorrelation of the received signal directly. The receiver, from whose output we try to obtain the ρ_k 's, will usually have a nonflat frequency response, as well as its own noise in the frequency band of interest. Under these conditions, the measured autocorrelation is not the signal autocorrelation appearing in Eq. (4). It is, however, simply related to it. Thus, it has been shown (SPS 37-31, Vol. IV, pp. 315-320) that ρ_k is related to measured parameters by the following equation:

$$\beta m_i - n_i = \alpha \sum_{k=-\infty}^{\infty} n_{i-k} \rho_k \quad (5)$$

where

n_i = i th normalized correlation coefficient of the receiver output when fed with white noise;

m_i = i th normalized correlation coefficient of the receiver output when a signal is received;

$$\alpha = \frac{W}{W_N} = \frac{\text{signal power at receiver input}}{\text{noise power at receiver input}};$$

$$\beta = \frac{W_M}{W_N} = \frac{\text{signal plus noise power at receiver output}}{\text{noise power at receiver output}}.$$

Obviously, the equation relating the unknown b_n 's to measured entities is obtained by substituting Eq. (4) in Eq. (5). This yields

$$v_i \equiv \frac{1}{h_0} \cdot \frac{W_N}{W_r} (\beta m_i - n_i) = \sum_{k, j=-\infty}^{\infty} n_{i-k} g_{k-j} b_j. \quad (6)$$

We pause here briefly to point out an interesting aspect of Eq. (6). The basic fact leading Eq. (5) is that for band-

limited signals, the effect of a filter on the autocorrelation can always be represented in the form

$$y_i = \sum_{k=-\infty}^{\infty} p_{i-k} x_k$$

where

- x_k = autocorrelation coefficient of input signal;
- y_i = autocorrelation coefficient of output signal;
- p_i = a set of parameters characterizing the filter and derivable from its more familiar transfer function.

In view of this, we may rephrase Eq. (6)

$$v_i = \sum_{k=-\infty}^{\infty} n_{i-k} \left(\sum_{j=-\infty}^{\infty} g_{k-j} b_j \right)$$

and take the attitude that the v_i 's are the autocorrelation coefficients of a signal obtained by passing the "feature signal" (the "signal" whose correlation coefficients are the b_j 's) through two filters in cascade: the "spherical filter" (parameters g_i) and the filter representing the receiver (parameters n_i) (Fig. 25).

From this point of view it is quite clear that obtaining the feature function is essentially identical with the more familiar problem of obtaining a receiver input autocorrelation from its measured output autocorrelation. The only difference is that we have here a "receiver" with a more complicated (but computable) transfer function.

We return now to the solution of Eq. (6). Invoking our assumption concerning the vanishing of the high order b_j 's, we set

$$b_j = 0 \quad \text{for } |j| > J. \quad (7)$$

Hence

$$v_i = \sum_{j=-J}^J \left\{ \sum_{k=-\infty}^{\infty} n_{i-k} g_{k-j} \right\} b_j = \sum_{j=-J}^J S_{ij} b_j \quad (8)$$

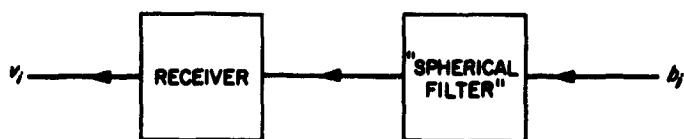


Fig. 25. The system effect on the b_i 's

where

$$S_{ij} = \sum_{k=-\infty}^{\infty} n_{i-k} g_{k-j}. \quad (9)$$

From Eq. (8) it is obvious that to solve for the b_j 's we must have all v_i for $|i| \leq J$. The solution would then involve the inversion of the matrix S (whose ij element is S_{ij}) of order $(2J + 1)$. We turn our attention now to this matrix.

Usually, only a finite number of n_k 's is available. Hence, as before, we compensate for our incomplete information by setting the unavailable n_k 's to zero. Conceptually, however, the situation here is different from the previous one. The transfer function of the receiver could, in principle, be approximated quite well by a rational frequency function. From the finite set of parameters of this rational function, one could compute the infinite set of n_k 's. This would then require the computation of an infinite number of g_i 's. Thus, the motivation for the truncation of the k summation in Eq. (9) is not so much a lack of data as it is a desire to obtain a practical computation scheme.

The truncation is effected by setting

$$n_k = 0 \quad \text{for } |k| > K.^{16}$$

Hence

$$S_{ij} = \sum_{k=i-K}^{i+K} n_{i-k} g_{k-j}. \quad (10)$$

We proceed now to prove that S is a symmetric Toeplitz matrix.¹⁷ This is simply effected by setting $k - j = q$ in Eq. (10). Thus

$$S_{ij} = \sum_{q=(i-j)-K}^{(i-j)+K} n_{(i-j)-q} g_q = S_{i-j}. \quad (11)$$

We see that the dependence of S_{ij} on i, j is only through the difference $(i - j)$. Hence, the matrix S is a Toeplitz matrix.

¹⁶Present JPL practice is to use the same equipment to obtain both m_i and n_i . Hence $K = J$.

¹⁷A matrix μ whose ij element μ_{ij} is a function of $i - j$ rather than i, j separately will have one and the same element along every diagonal, superdiagonal or subdiagonal. It is known as a Toeplitz matrix.

To prove symmetry we rewrite Eq. (11) as follows

$$S_k = \sum_{q=k-K}^{k+K} n_{k-q} g_q.$$

Hence

$$S_k = \sum_{q=-(k+K)}^{k+L} n_{k-q} g_q.$$

The evenness of the autocorrelation functions $n(\tau), g(\tau)$ now yields

$$S_{-k} = \sum_{j=k-K}^{k+K} n_{k-j} g_j = S_k.$$

We conclude that to solve Eq. (8) and obtain the b 's we have to invert a symmetric Toeplitz matrix of order $(2J+1)$. This is readily accomplished even for large J by the method of W. F. Trench (Ref. 33). Preliminary analysis indicates, for example, that the complete solution of Eq. (3) for $J=50$ would take about 4 sec on the IBM 7094.

We turn now briefly to the general case mentioned earlier in which we do not neglect the θ variation of the feature function across the observed zone. Instead of having to deal with one autocorrelation function, we now consider $(Z+1)$ such functions corresponding to $(Z+1)$ different zones. To distinguish among these we attach an extra index to ρ, h, a, u . Thus, we have for zone z

$$\rho_{kz}, h_{0z}, a_{kz}, W_z.$$

With this terminology, Eq. (2) becomes

$$\rho_{kz} = h_{0z} \frac{W_T}{W_z} \sum_{k, n, r=-\infty}^{\infty} b_{nr} a_{kz} \cos[2\pi(n+k)v_c] J_{k+r}[\pi(n+k)\hat{v}]$$

($z = 0, 1, \dots, Z$).

(12)

In analogy with the g_m -parameters introduced in the special case, we introduce now the Y_{mrz} -parameters defined as follows:

$$\begin{aligned} Y_{mrz} &= h_{0z} \frac{W_T}{W_z} \cos(2\pi m v_c) \sum_{k=-\infty}^{\infty} a_{kz} J_{k+r}(\pi m \hat{v}) \\ &= \frac{1}{2} h_{0z} \frac{W_T}{W_z} \cos(2\pi m v_c) \sum_{k=0}^{\infty} \varepsilon_k a_{kz} [J_{r+k}^2(\pi m \hat{v}) + J_{r-k}^2(\pi m \hat{v})]. \end{aligned}$$

Applying this to Eq. (12) we get

$$\rho_{kz} = \sum_{n, r=-\infty}^{\infty} Y_{k-n, r, z} b_{nr}.$$

We note that Y_{mrz} is an even function of r . Combining this with the symmetry properties of b_{nr} (SPS 37-43, Vol. IV, pp. 330-338), we get

$$\rho_{kz} = \frac{1}{2} \sum_{n, r=0}^{\infty} \varepsilon_n \varepsilon_r (Y_{k-n, r, z} + Y_{k+n, r, z}) b_{nr}.$$

Finally, in analogy with Eq. (7) we assume

$$b_{nr} = 0 \quad \text{for } |n| > I \text{ or } |r| > Z,$$

where I is the number of available correlation coefficients in each of the $Z + 1$ zones. This leads to

$$\rho_{kz} = \frac{1}{2} \sum_{n=-Z}^{r-Z} \epsilon_n \epsilon_r (Y_{k-n,r,z} + Y_{k+n,r,z}) b_{nr}. \quad (13)$$

This equation can be solved by the simple artifice of arranging b_{nr} and ρ_{kz} as column matrices. Specifically, let us introduce the following two column matrices of order $I + 1$

$$\hat{\rho}_z = \begin{bmatrix} \rho_{0,z} \\ \rho_{1,z} \\ \vdots \\ \rho_{I,z} \end{bmatrix}; \quad \hat{b}_r = \epsilon_r \begin{bmatrix} b_{0,r} \\ b_{1,r} \\ \vdots \\ b_{I,r} \end{bmatrix}$$

and the square matrix $\hat{Y}_{rz}$ of order $(I + 1)$ whose k, n element is given by

$$(\hat{Y}_{rz})_{kn} = \frac{\epsilon_n}{2} (Y_{k-n,r,z} + Y_{k+n,r,z}).$$

Eq. (13) can now be expressed in terms of these matrices as follows:

$$\begin{bmatrix} \hat{\rho}_0 \\ \hat{\rho}_1 \\ \vdots \\ \hat{\rho}_z \end{bmatrix} = \begin{bmatrix} \hat{Y}_{00} & \hat{Y}_{01} & \cdots & \hat{Y}_{0z} \\ \hat{Y}_{10} & \hat{Y}_{11} & \cdots & \hat{Y}_{1z} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{Y}_{z0} & \hat{Y}_{z1} & \cdots & \hat{Y}_{zz} \end{bmatrix} \begin{bmatrix} \hat{b}_0 \\ \hat{b}_1 \\ \vdots \\ \hat{b}_z \end{bmatrix}$$

We see that solving for the $(I + 1)(Z + 1)$ required b_{nr} coefficients calls for the inversion of a matrix of order $(I + 1)(Z + 1)$.

No special exploitable structure is evident here and the matrix is not even symmetric. It should be pointed out in passing that there is an alternative formulation in which each submatrix turns out to be a Toeplitz matrix though of almost twice the order $(2I + 1)$. However, it is not obvious how such a structure could be exploited to simplify the inversion.

In the present JPL system, a typical case would involve $I = 50$, $Z = 8$. Hence, the matrix to be inverted is of order 459. It is doubtful whether such an inversion should even be attempted in view of the accumulation of errors and prohibitive cost.

Attacking the same problem with the previous scheme, we would treat each of the 9 zones separately (neglecting the θ variation inside a zone) and end up by inverting 9 symmetric Toeplitz matrices of order 101. Solving all 9 matrix equations would take about 33 sec on the IBM 7094.

K. Astrometrics: Elimination of Charged Electron Effects from Ranging Data, P. E. Reichley

1. Summary

The effect of charged electrons on ranging data in the ionosphere and interplanetary medium plays an important role in the growing need for greater tracking accuracies. We present in this article a method of obtaining the range change from tracking point to tracking point with effects due only to the troposphere. The effects due to the charged electron distribution appear as second order effects in this method. These results are obtained under the assumption that the troposphere and charged electron distributions are spherically symmetric and that the spacecraft is being tracked with both doppler and range points (obtained by a ranging code). This range change can then be utilized to yield the true range with or without a constant bias, depending on whether the true range is known at a prior time or not. Ranging accuracies of < 1 m are possible with this method.

2. Introduction

The range to a spacecraft in geocentric polar coordinates, as determined by a range code, is given by (SPS 37-29, Vol. IV, pp. 229-239)

$$R = \int_{r_0}^r \frac{rn(r)n_g(r)dr}{(r^2n^2(r) - b^2)^{1/2}} \quad (1)$$

where $n(r)$ and $n_g(r)$ are the phase and group indices of refraction of the signal; r_0 and r_1 are the station and spacecraft radius vectors; and b is the impact parameter of the ray path.

We shall assume that the phase index of refraction has the form

$$n^2(r) = 1 + \epsilon f(r) \equiv 1 + \epsilon (f_1(r) - f_2(r)) \quad (2)$$

where $|f(r)| \leq 1$ for $r_0 \leq r_1 < \infty$ and ϵ is a positive parameter. We shall consider $f_1(r)$ to be frequency independent, corresponding to the troposphere, and $f_2(r)$ to be frequency dependent, corresponding to the ionosphere and interplanetary medium. We shall consider frequencies at L-band (900 Mhz) or higher, in which case $\max f_1(r) > \max f_2(r)$ and $\epsilon = 2\alpha$ where α is the ground level refractivity (SPS 37-43, Vol. IV, pp. 314-321). If we neglect absorption and external magnetic fields the group index is given by (SPS 37-43, Vol. IV, pp. 314-321)

$$n_g(r) = \frac{1 + \epsilon f_1(r)}{n(r)} \quad (3)$$

The range to a spacecraft in geocentric polar coordinates as determined by doppler is given by (SPS 37-29, Vol. IV, pp. 229-239)

$$R_d = \int_{r_0}^{r_1} \frac{m^2(r) dr}{(r^2 n^2(r) - b^2)^{1/2}} \quad (4)$$

and the range rate determined by doppler is simply

$$\dot{R}_d = \frac{dR_d}{dt} \quad (5)$$

A perturbation solution for the impact parameter is given by (SPS 37-43, Vol. IV, pp. 314-321)

$$b = b_0 + \epsilon b_1 + \epsilon^2 b_2 + O(\epsilon^3) \quad (6)$$

where

$$\begin{aligned} b_0 &= \frac{r_0 r_1 \sin \theta}{(r_0^2 + r_1^2 - 2r_0 r_1 \cos \theta)^{1/2}} \\ b_1 &= \frac{1}{2} \frac{(r_1^2 - b_0^2)^{1/2} (r_0^2 - b_0^2)^{1/2}}{(r_1^2 - b_0^2)^{1/2} - (r_0^2 - b_0^2)^{1/2}} b_0 \int_{r_0}^{r_1} \frac{r f(r) dr}{(r^2 - b_0^2)^{3/2}} \\ b_2 &= - \frac{(r_1^2 - b_0^2)^{1/2} (r_0^2 - b_0^2)^{1/2}}{(r_1^2 - b_0^2)^{1/2} - (r_0^2 - b_0^2)^{1/2}} \left[\frac{1}{2} b_0 b_1^2 \left(\frac{1}{(r_0^2 - b_0^2)^{3/2}} - \frac{1}{(r_1^2 - b_0^2)^{3/2}} \right) \right. \\ &\quad \left. - \frac{1}{2} b_1 \int_{r_0}^{r_1} \frac{r f(r) dr}{(r^2 - b_0^2)^{3/2}} + \frac{3}{8} b_0 \int_{r_0}^{r_1} \frac{r^3 f^2(r) dr}{(r^2 - b_0^2)^{3/2}} - \frac{3}{2} b_0^2 b_1 \int_{r_0}^{r_1} \frac{r f(r) dr}{(r^2 - b_0^2)^{3/2}} \right] \end{aligned} \quad (7)$$

where θ is the angular separation between r_0 and r_1 .

3. Elimination of Charged Electron Effects to First Order

In ranging a spacecraft we shall assume that at some time t_0 the range to the spacecraft is obtained by means of a range code; then the doppler shift is obtained and counted incrementally (in 1-sec intervals for example) until some time t_1 , at which time the range is obtained by means of the range code. This process is then repeated from time t_1 to time t_2 , from time t_2 to time t_3 , etc.

Since we are incrementally counting doppler, this is equivalent to integrating Eq. (5) from time t_i to time t_{i+1} .

Hence, the range change due to doppler count from time t_i to time t_{i+1} , from Eqs. (4) and (5), is

$$\Delta_i R_d \equiv \int_{t_i}^{t_{i+1}} \dot{R}_d dt = R_d(t_{i+1}) - R_d(t_i) \quad (8)$$

The range change due to range code ranging from time t_i to t_{i+1} , is, from Eq. (1),

$$\Delta_i R = R(t_{i+1}) - R(t_i) \quad (9)$$

A perturbation solution for $R(t_i)$ is (SPS 37-43, Vol. IV, pp. 314-321)

$$R(t_i) = R_s(t_i) + \frac{1}{2} \epsilon \int_{r_0}^{r_1(t_i)} \frac{r(f_1(r) + f_2(r)) dr}{(r^2 - b_0^2(t_i))^{1/2}} - \frac{1}{8} \epsilon^2 \left[\int_{r_0}^{r_1(t_i)} \frac{r^3(f_1(r) - f_2(r))(f_1(r) + 3f_2(r)) dr}{(r^2 - b_0^2(t_i))^{3/2}} - 2b_0(t_i)b_1(t_i) \int_{r_0}^{r_1(t_i)} \frac{r(f_1(r) + 3f_2(r)) dr}{(r^2 - b_0^2(t_i))^{3/2}} \right] + O(\epsilon^3) \quad (10)$$

where $R_s(t_i)$ is the straight line or true range, and b_0 and b_1 are given by Eq. (7). We can obtain the same type of solution for $R_d(t_i)$ by substituting Eqs. (2) and (6) into Eq. (4) and expanding in powers of ϵ . This yields

$$R_d(t_i) = R_s(t_i) + \frac{1}{2} \epsilon \int_{r_0}^{r_1(t_i)} \frac{r(f_1(r) - f_2(r)) dr}{(r^2 - b_0^2(t_i))^{1/2}} + \frac{1}{8} \epsilon^2 \left[2b_0(t_i)b_1(t_i) \int_{r_0}^{r_1(t_i)} \frac{r(f_1(r) - f_2(r)) dr}{(r^2 - b_0^2(t_i))^{3/2}} - \int_{r_0}^{r_1(t_i)} \frac{r^3(f_1(r) - f_2(r))^2 dr}{(r^2 - b_0^2(t_i))^{3/2}} \right] + O(\epsilon^3) \quad (11)$$

after utilizing Eq. (7).

If we add Eqs. (8) and (9) and divide by 2, we have, from Eqs. (10) and (11),

$$\begin{aligned} \Delta_i R_i &\equiv \frac{\Delta_i R + \Delta_i R_d}{2} = R_s(t_{i+1}) - R_s(t_i) + \frac{1}{2} \epsilon \int_{r_0}^{r_1(t_{i+1})} \frac{rf_1(r) dr}{(r^2 - b_0^2(t_{i+1}))^{1/2}} \\ &\quad - \frac{1}{2} \epsilon \int_{r_0}^{r_1(t_i)} \frac{rf_1(r) dr}{(r^2 - b_0^2(t_i))^{1/2}} + \epsilon^2 \left[\frac{1}{4} b_0(t_{i+1})b_1(t_{i+1}) \int_{r_0}^{r_1(t_{i+1})} \frac{r(f_1(r) + f_2(r)) dr}{(r^2 - b_0^2(t_{i+1}))^{3/2}} - \frac{1}{4} b_0(t_i)b_1(t_i) \right. \\ &\quad \times \left. \int_{r_0}^{r_1(t_i)} \frac{r(f_1(r) + f_2(r)) dr}{(r^2 - b_0^2(t_i))^{3/2}} - \frac{1}{8} \int_{r_0}^{r_1(t_{i+1})} \frac{r^3(f_1^2(r) - f_2^2(r)) dr}{(r^2 - b_0^2(t_{i+1}))^{3/2}} + \frac{1}{8} \int_{r_0}^{r_1(t_i)} \frac{r^3(f_1^2(r) - f_2^2(r)) dr}{(r^2 - b_0^2(t_i))^{3/2}} \right] + O(\epsilon^3) \end{aligned} \quad (12)$$

The quantity $\Delta_i R_i$, as seen from Eq. (12), represents the range change due to spacecraft motion plus, to first order, the change due to the troposphere alone. The f_2 -terms have disappeared from the ϵ -term which is our main achievement.

The coefficient of ϵ^2 in Eq. (12) consists of the difference of two terms of the form

$$\frac{1}{4} b_0 b_1 \int_{r_0}^{r_1} \frac{r(f_1(r) + f_2(r)) dr}{(r^2 - b_0^2)^{3/2}} - \frac{1}{8} \int_{r_0}^{r_1} \frac{r^3(f_1^2(r) - f_2^2(r)) dr}{(r^2 - b_0^2)^{3/2}} \quad (13)$$

We can show that the above quantity multiplied by ϵ^2 is always less than 8 m for ranging in the solar system, excluding signals that pass close to the Sun. The upper limit of 8 m is obtained when the signal path grazes the horizon and the spacecraft is just outside of the troposphere. As the spacecraft moves farther away, the effect of the ionosphere and interplanetary medium reduces Eq. (13) in magnitude. Hence, we may neglect the coefficient of ϵ^2 in Eq. (12).

Since $f_1(r) = 0$ outside of the troposphere, we can write integrals of the form

$$T = - \int_{r_0}^{r_1} \frac{rf_1(r) dr}{(r^2 - b_0^2)^{1/2}} = - \int_{r_0}^{r_1} \frac{rf_1(r) dr}{(r^2 - b_0^2)^{1/2}} \quad (14)$$

where r_1 is the radius vector to the upper limit of the troposphere. Hence we can rewrite Eq. (12) (neglecting terms of $O(\epsilon^2)$ and higher)

$$\Delta_i R_i = R_s(t_{i+1}) - R_s(t_i) + \frac{1}{2} \epsilon T(t_{i+1}) - \frac{1}{2} \epsilon T(t_i) \quad (15)$$

Since it can be shown (SPS 37-43, Vol. IV, pp. 314-321) that the coefficient of ϵ^2 in Eq. (10) is of the same magnitude as the coefficient of ϵ^2 in Eq. (12), we have (neglecting terms of $O(\epsilon^2)$ and higher)

$$R(t_i) = R_s(t_i) + \frac{1}{2} \epsilon T(t_i) + \frac{1}{2} \epsilon I(t_i) \quad (16)$$

where

$$I = \int_{r_0}^{r_1} \frac{rf_2(r) dr}{(r^2 - b_0^2)^{1/2}} \quad (17)$$

If the true range to the spacecraft is known at some range point at time t_n , then the range $\mathcal{R}$ at later times is obtained by means of Eq. (15), i.e.

$$\begin{aligned} \mathcal{R}(t_n) &= R_s(t_n) + \sum_{i=0}^{n-1} \Delta_i R_i \\ &= R_s(t_n) + \frac{1}{2} \epsilon T(t_n) - \frac{1}{2} \epsilon T(t_0) \end{aligned} \quad (18)$$

We see that the true range is obtained at t_n with tropospheric corrections at t_0 and t_n . If the true range is not known, then $\mathcal{R}(t_n)$ can be obtained with a constant bias. Upon combining Eqs. (15) and (17)

$$\begin{aligned} \mathcal{R}(t_n) &= R(t_n) + \sum_{i=0}^{n-1} \Delta_i R_i \\ &= R_s(t_n) + \frac{1}{2} \epsilon T(t_n) + \frac{1}{2} \epsilon I(t_n) \end{aligned} \quad (19)$$

which yields the true range with a tropospheric correction at t_n and a constant bias of $\frac{1}{2} \epsilon I(t_n)$. This bias can be minimized by choosing t_n during the night and or when the spacecraft is at its highest elevation angle. As long as the spacecraft is outside of the influence of the troposphere, the corrections for the troposphere are not influenced by errors in the range to the spacecraft, which follows from Eq. (14).

4. Conclusions

The basic advantage of the ranging method presented in the article is that high precision ranging data free of charged electron effects can be obtained. The type of data required is precisely the type of ranging data that is obtained by tracking devices currently in use in the deep space net. The technique, which is very simple, could easily be automated at the tracking stations. Furthermore, since tracking is rarely done at low elevation angles, the error (one way) of 8 m can easily be reduced by a factor of 10; this is equivalent to tracking accuracies on the order of 10 nanosec.

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XXII. Spacecraft Telemetry and Command

TELECOMMUNICATIONS DIVISION

N67-29163

A. Photon Actuated, Solid State Switch Development, D. M. Bergens

1. Introduction

During the past few years JPL has been planning and developing a new generation of telecommunications equipment based on the use of microelectronics. Although commercially available monolithic integrated circuits are satisfactory for many digital and analog spacecraft and operational support equipment applications, certain electronic circuit functions are unique and require custom development. One such function is coupling a signal across an interface while maintaining electrical isolation at the interface.

Signal coupling with electrical isolation is a useful and sometimes indispensable function in the design of electrical or electronic equipment. This function can be achieved by converting electrical energy into another form of energy which serves as the coupling medium. In a transformer or relay, for example, magnetic flux serves as the coupling medium. Transformers and relays are used for isolation in present day equipment, but they are generally less reliable than solid state devices. Also, they are not physically compatible with microelectronic form factors. Acoustic or mechanical force waves in a crystal can serve as a coupling medium, but the transducers are not suited to the needs of microelectronics. Heat can also be

used as a coupling medium, but the response time is extremely slow.

Light rays are another means of providing isolation of electrical signals. Light coupling has the advantage of being unilateral, physically compact, and capable of great speed. Until recently, however, the use of light coupling in electronics has been limited due to deficiencies in the light source.

The discovery of the phenomenon of infrared emission from the junction of a forward biased gallium arsenide diode suggested a light source that might be suitable for microelectronic coupling. The Ga As emission spectrum peaks in the region of 9000 Å, where silicon photon sensitivity is greatest. Therefore, if the infrared emission could be efficiently coupled to a silicon phototransistor, a microelectronic, solid state isolation would be achieved.

In April 1963, the components division of the International Business Machines Corporation (IBM) was given a contract by JPL to determine if a practical photon-coupled isolation device of this type could be made. In January 1966, Texas Instruments, Incorporated (TI) was given a contract for development of another photon-actuated isolation device. The remainder of this article will cover these developments.

2. The IBM Development

The IBM contract was for the development of a photon-actuated solid state switch for the precision multiplexing of analog voltages in a spacecraft telemetry system. The 18-mo development is reported in detail in a JPL technical report (Ref. 1). The multiplex switch was an ambitious undertaking as a first effort in photon-coupling. Severe requirements were placed on the silicon detector, which also had to serve as the switch element. Assembly of the switch components on a TO-18 header and a top view of the silicon detector is shown in Fig. 1. Note that the silicon detector is a dual emitter transistor with common base and collector. This configuration was chosen to achieve optimum matching of the transistors for minimum off-set voltage, thermal balance, fewer connections, ease of assembly, and compactness. The dual emitter transistor was tailored to meet the difficult requirements of high breakdown, low offset voltage, low leakage current, low on-resistance, and low capacitance between emitters. It also had to be a good light receiver. Various combinations of single or double diffusion and single or double epitaxy were tried in making NPN and PNP detectors. The detector that gave the best trade-off in parameters was a NPN planar device with an epitaxial base. This device was used in assembled switches.

The principal problem encountered during the development was a decrease in photon emission from the Ga As

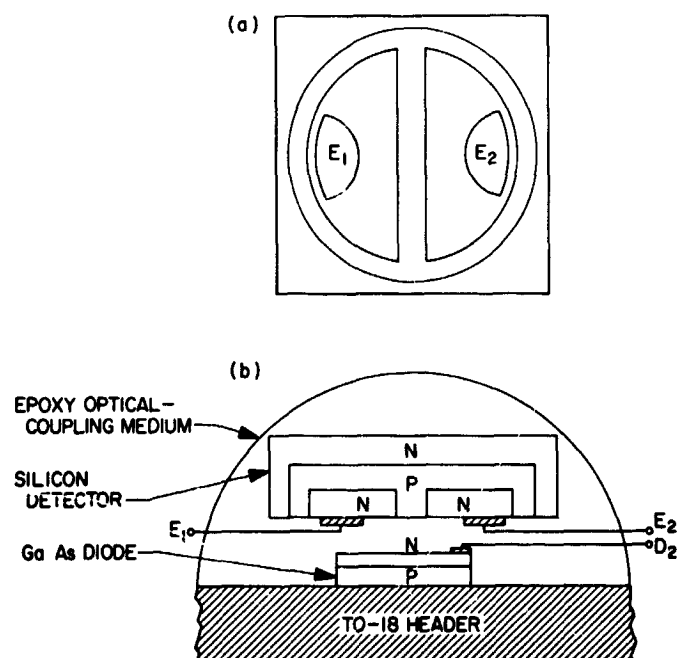


Fig. 1. Multiplex switch: (a) top view of silicon detector; (b) assembly of switch components on a TO-18 header

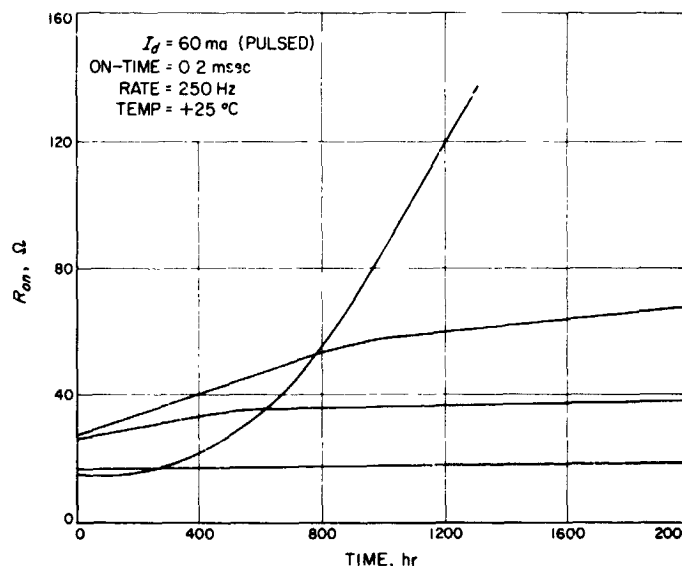


Fig. 2. Graph of on-resistance vs time for several IBM switches

diode with time. This was indicated by a gradual increase in on-resistance. Fig. 2 is a graph of on-resistance vs time for several of these switches. The curves are typical of the different degrees of degradation observed. It is apparent from the different slopes that more than one mode of degradation was occurring. This problem was first observed by IBM near the end of the contract period but was not investigated to any extent. It was theorized that the decrease in photon emission was caused by an increase in surface (leakage) current, which is a nonradiative component of the total current in the diode. Bulk current is the radiative component. It was further theorized that a planar diode would minimize this problem, but IBM did not have a suitable capability for planar diffusion in Ga As at the time.

The IBM contract ended with a successful demonstration of the parameters that could be achieved in a photon actuated switch, together with an indication of a serious degradation mode. The parameters are summarized in Table 1. They are useful parameters for many multiplexing applications. Although IBM packaged the switches in TO-18 cans for handling convenience, it was inferred that they could be packaged in conventional integrated circuit flat packages.

At the time the IBM contract ended, TI was doing a study (Ref. 2) of degradation of light output in Ga As mesa diodes as part of an investigation of optoelectronic phenomena for the Air Force Avionics Laboratory. Two degradation mechanisms had been identified and corrective measures were showing positive results. It was

Table 1. Summary of IBM switch parameters

Parameter	Condition	Value (temp range - 10° to 85°C)
R_{on}	$I_d = 60 \text{ ma (75 mw)}$	30Ω
R_{off}	$V_{EK} = \pm 5\text{v}$	$> 10^8\Omega$
I_{rev}	$V_{EK} = \pm 5\text{v}$	$< 50 \text{ na}$
$R_{isolation}$	$V_{D-T} = 35\text{v}$	$> 10^{11}\Omega$
BV_{rev}	$I_{EK} = 100 \mu\text{a}$	$> 20\text{v}$
V_{os}	$I_d = 60 \text{ ma}$	$100 \mu\text{v}$
t_{on}	$I_d = 60 \text{ ma (pulsed)}$	$< 2.5 \mu\text{sec}$
t_{off}	$R_L = 10 \text{ k}\Omega$	$< 10 \mu\text{sec}$
C_{rr}	$\text{freq} = 1.0 \text{ kHz}$	$< 6 \text{ pf}$

decided to wait until this work was complete before further work was initiated by JPL.

3. The TI Development

In the latter part of 1965 it became apparent that TI had a good handle on the Ga As diode degradation problem. They had corrected the two major degradation mechanisms in mesa diodes, thermally induced stress and increasing surface current, and then they developed a low cost planar process that produced a considerable improvement in stability over their best mesa units. These planar diodes were operated for 1000 hr at 25°C at a bias current density of 1000 amp/cm² with no degradation in light output¹ (Ref. 3). Mainly because of these results it was decided to renew JPL's development of photon isolation circuits. The multiplex switch development was not continued, however, due to the emergence of the MOS transistor as a low level voltage switch. The extremely high input impedance of the MOS provides sufficient isolation of the drive signal for the multiplexing application.

In January 1966, TI was given a contract to develop a photon actuated isolation switch for use at the outputs of a spacecraft command subsystem where complete electrical isolation is necessary for the elimination of DC ground loops. The complete device, shown in Fig. 3, consists of a diode AND gate, a current driver, and a Ga As diode optically coupled to a phototransistor. The gate and driver circuit will be a monolithic integrated circuit. All of these components will be packaged in the $\frac{1}{4} \times \frac{1}{8} \times \frac{1}{2}$ -in. TI flat package. The Ga As diode will be glassed to the phototransistor with a SeSAs glass for efficient light cou-

¹J. R. Biard, "Degradation of Quantum Efficiency in Ga As Light Emitters," 1965 IEEE Solid State Device Research Conference, Princeton, N.J., June 21, 1965.

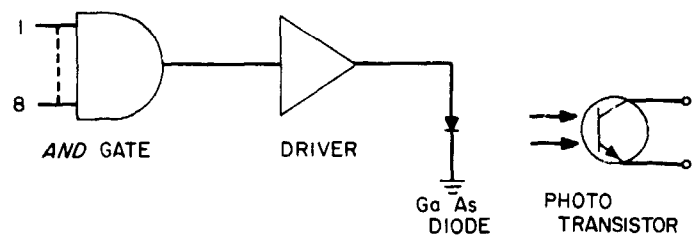
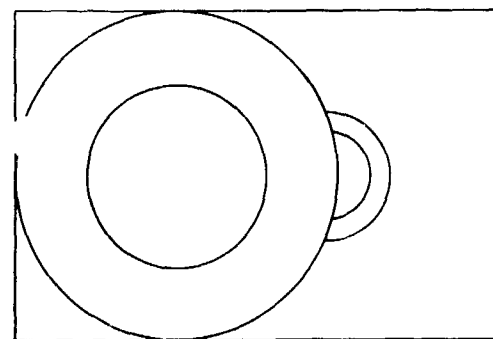


Fig. 3. Diagram of photon-actuated command isolation switch

(a)



(b)

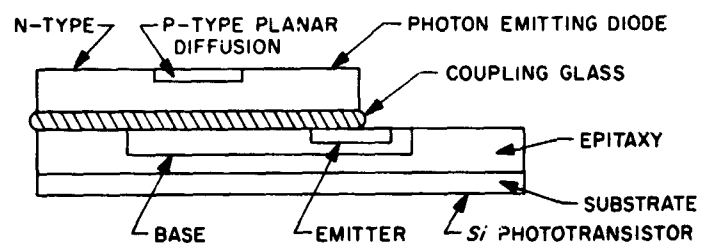


Fig. 4. Diode-transistor pair: (a) top view; (b) side view

pling. Fig. 4 is a top and side view of the diode-transistor pair.

The isolation switch is being designed to have the electrical characteristics given in Table 2. The noise transmissibility parameter is a measure of the magnitude of a high speed pulse that will pass through the phototransistor from emitter to collector when it is turned off. Fig. 5 shows the test circuit for this measurement. The other parameters are self explanatory.

Environmental requirements on the isolation switch are: operating temperature range of -20° to +100°C, +150°C storage temperature, thermal and physical shock, vibration, acceleration, and space radiation.

Table 2. Electrical characteristics

Case temperature (unless specified otherwise); -20 to +100°C					
Parameter	Symbol	Condition	Value		Unit of measure
			Min	Max	
Input voltage:					
At "1" level	V_i		3.0	6.0	v
At "0" level	V_i		0	1.0	v
Input current:					
At "1" level	I_i	$V_i = 6$ v		50	μ a
At "0" level	I_i	$V_i = 0.1$ v		-1.0	ma
Output saturation on—voltage	$V_{ce,sat}$	$V_i = 3.0$ v, $I_c = 10$ ma		0.6	v
Output on—current	I_c	$V_i = 3$ v, $V_{ce,sat} = 0.6$ v	10		ma
Output leakage off—current	I_{ceo}	$V_i = 1$ v, $V_{ce} = 20$ v, Temp = +25°C Temp = +100°C		0.1 20	μ a μ a
Output breakdown voltage	BV_{ceo}	$V_i = 0$ v, $I_c = 100$ μ a	35		v
Isolation capacitance (between output and all other terminals)	C_{iso}	freq = 1.0 kHz		10	pf
Switching times:					
turn on	t_1			10	μ sec
turn off	t_2			100	μ sec
Noise transmissibility	V_n			2.0	v
Power dissipation:					
Switch on		$V_i = 3$ v, $I_c = 0$		200	mw
Switch off		$V_i = 0$ v, $I_c = 0$		1.0	mw

results are summarized in the remainder of this report. In the second part of the development the gate and driver circuit will be fabricated as an integrated circuit, and complete isolation switches will be assembled and tested.

When the first diode-transistor pairs were tested the transistor leakage and breakdown characteristics changed drastically with elevated temperature and time. All other parameters were satisfactory. Investigation of the leakage/breakdown problem revealed the cause to be surface inversion in the base and collector regions of the phototransistor. The inversion was caused by charge migration in the coupling glass when a potential is applied between the diode and transistor. The phototransistor was successfully modified to eliminate the collector inversion by addition of a N+ guard ring around the base region and a field relief electrode overlapping the collector-base junction. The base impurity concentration was increased to reduce inversion in that region.

4. Radiation Tests

Early in the development program sample Ga As photon sources, silicon phototransistors, and digital integrated circuits from TI were submitted to an integrated proton irradiation of approximately 1×10^{10} protons/cm² at energies of 30, 60, 100 and 140 Mev. The important parameters were measured before and after irradiation, and no significant changes were found. Eleven of the first diode-transistor pairs assembled (Fig. 6) were given another proton irradiation test in which the leakage current of the phototransistor was monitored during irradiation to determine if an "off" transistor would tend to turn on. At flux rates of 4×10^5 protons/cm²/sec, eight times the maximum rate observed in the most recent period of maximum solar activity (1956-61), leakage current increases of 1 na were seen. This increase is not significant in the intended application. The 11 diode-transistor pairs received a total integrated flux of 2×10^{10} protons/cm² during the test, which is equivalent to 1 yr in a deep space trajectory during a period of maximum solar activity. Before and after measurements indicated negligible permanent change in the pair parameters. Later on the pairs will be tested for transient effects due to electron bombardment.

5. Life Test

Following the radiation test the 11 diode-transistor pairs were placed on life test at +25°C. The diode forward current bias was 50 ma DC on eight of the pairs and zero on the other three pairs. No bias was applied to the phototransistors. After 2500 hr $V_{ce,sat}$ and I_c on some of the biased units had drifted, but none were out of specification. In six

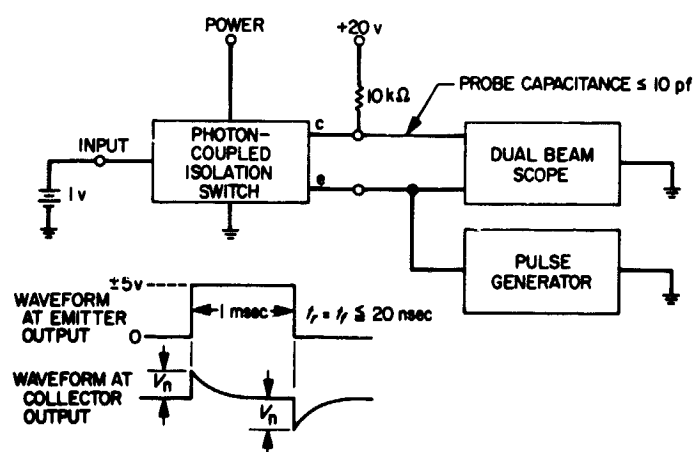


Fig. 5. Noise transmissibility test

The switch development was divided into two parts. Part one consisted of the electrical design of the gate and driver circuit, and the design and fabrication of the diode-transistor combination. Part one has been completed. The

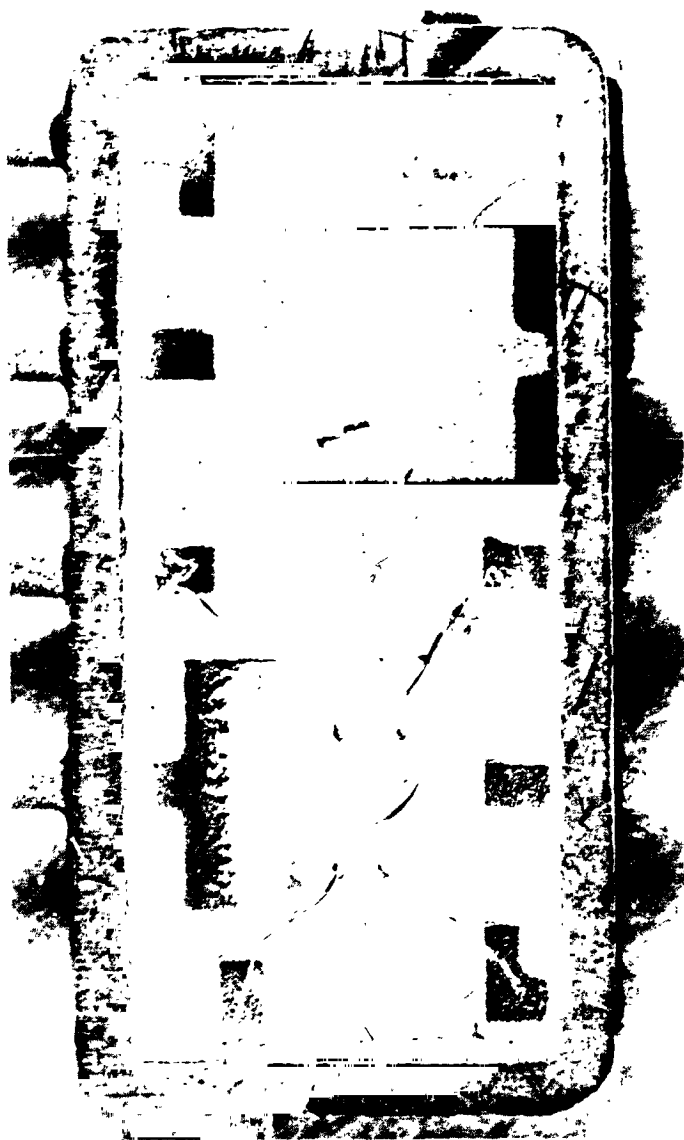


Fig. 6. Diode-transistor pair assembly

of the eight biased units the drift stopped after the first 72 hr, and they remained stable thereafter. The other two biased units continued to drift until the test was stopped at 2500 hr. The three unbiased units did not drift significantly. Considering that the Ga As diodes had not been burned in (operated with bias current for a period of time) and the phototransistors had the inversion problem mentioned above, these results were encouraging. TI is now burning in all their Ga As light sources for 100 hr to stabilize their output.

6. Current Results

Twenty-four diode-transistor pairs with burned in diodes and the modified phototransistors were recently

delivered to JPL. Their electrical characteristics are summarized in Table 3. The specified minimum and maximum values are shown with the minimum, average, and maximum measured values for comparison purposes. The average output (collector) current is greater than the input (diode) current indicating a current gain which is greater than unity. Small signal current gain (equivalent to h_{fe} in a transistor) versus collector current is given in Fig. 7 for a typical diode-transistor pair.

Twelve of the 24 units have been on life test for 500 hr with no change in parameters.

The gate and driver circuit has been designed and is presently undergoing final analysis prior to being implemented as an integrated circuit. Fig. 8 is a schematic of the circuit. All of the transistors will have the same area and geometry. In order to handle the maximum diode current of 40 ma, Q2 will be four transistors in parallel.

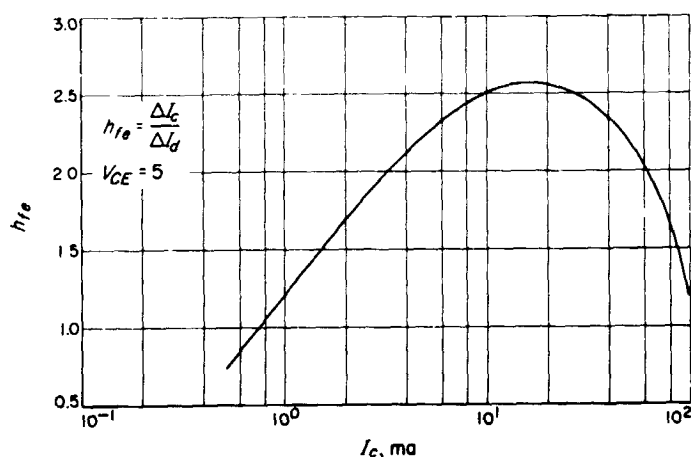


Fig. 7. Current gain of typical diode transistor pair

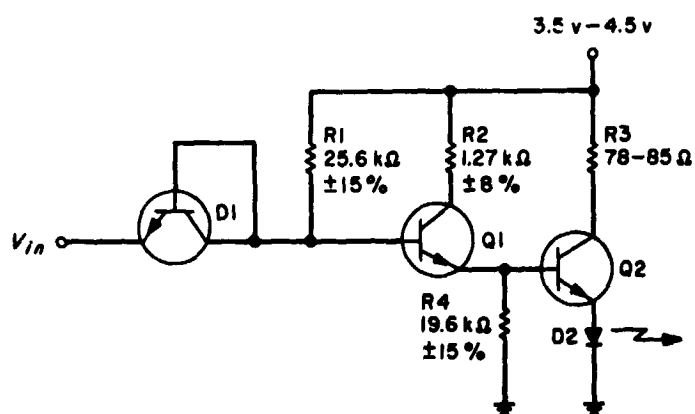


Fig. 8. Gate and driver circuit (one input shown)

Table 3. Electrical characteristics of 24 Ga As diode-Si transistor pairs
(Case temperature: +25°C unless specified otherwise)

Parameter	Symbol	Test condition	Specified value		Measured value			Unit of measure
			Min	Max	Min	Ave	Max	
Output saturation on—voltage	V_{CEK}	$I_C = 10 \text{ ma}$						
		$I_D = 24.5 \text{ ma @ } -20^\circ\text{C}$		0.6	0.08	0.097	0.14	v
		$24.0 \text{ ma @ } +25^\circ\text{C}$		0.6	0.10	0.122	0.16	v
		$22.0 \text{ ma @ } +100^\circ\text{C}$		0.6	0.14	0.177	0.24	v
Output on—current	I_C	$V_{CEK} = 0.6\text{v}$						
		$I_D = 24.5 \text{ ma @ } -20^\circ\text{C}$	10		18	36.6	48	ma
		$24.0 \text{ ma @ } +25^\circ\text{C}$	10		20	33.8	45	ma
		$22.0 \text{ ma @ } +100^\circ\text{C}$	10		16	24.7	32	ma
Output leakage off—current	I_{CKO}	$I_D = 0, V_{CEK} = +20\text{v}$						
		Temperature = +25°C		0.1	0.0001	0.0018	0.0064	μa
		+100°C		20	0.07	5.35	16.0	μa
Output breakdown voltage	BV_{CKO}	$I_D = 0, I_C = 100\mu\text{a}$						
		Temperature = +25°C	35		52	67.4	90	v
		+100°C	35		48	61.7	85	v
Isolation capacitance (between diode and transistor)	C_{iso}	Frequency = 1.0 kHz		10	1.8	2.6	6.4	pf
Switching times:		$I_D = 20 \text{ ma}, I_C = 10 \text{ ma}$						
Turn on	t_1			10	2.5	3.35	6.6	μsec
Turn off	t_2			100	17	52.7	72	μsec
Noise transmissibility (emitter to collector)	V_n	$I_D = 0, I_C = 2 \text{ ma}, V_i = +5\text{v}$		2.0	1.6	—	1.9	v
		—5v		2.0	1.5	—	2.0	v
Diode forward voltage	V_D	$I_D = 30 \text{ ma}$			1.203	—	1.256	v

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2. Biard, J. R., *Optoelectronic Functional Electronic Blocks, Interim Report No. 6*, D. D. C. No. AD610871, Texas Instruments, Inc., Dallas, Texas, February 2, 1965.
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XXIII. Spacecraft Radio

TELECOMMUNICATIONS DIVISION

N67-29164

A. The Mariner S-Band Turnaround Ranging Transponder Improvement Program, L. M. Hershey

Some results of the study being made of the S-band turnaround ranging transponder used on *Mariner C* were described in SPS 37-42, Vol. IV, pp. 183-189, and SPS 37-43, Vol. IV, pp. 374-376. This study is conducted by the Space and Re-Entry Systems Division of the Philco-Ford Corporation, formerly a part of the Western Development Laboratory (WDL) Division. In the more recent SPS article, the measured effects of temperature and signal level variations on the turnaround ranging delay through the transponder are described for the whole transponder and for the individual groupings of modules termed "six-packs."

The next step in the study is measurement of the effect on overall transponder performance of environmental variations on each module individually. To date, only the 9.56-MHz IF amplifier module has been evaluated in this manner; its effect on ranging delay is shown in Fig. 1. The IF amplifier is not in the turnaround ranging loop, but is a part of the automatic phase control (APC) loop that furnishes the reference signal for the ranging phase

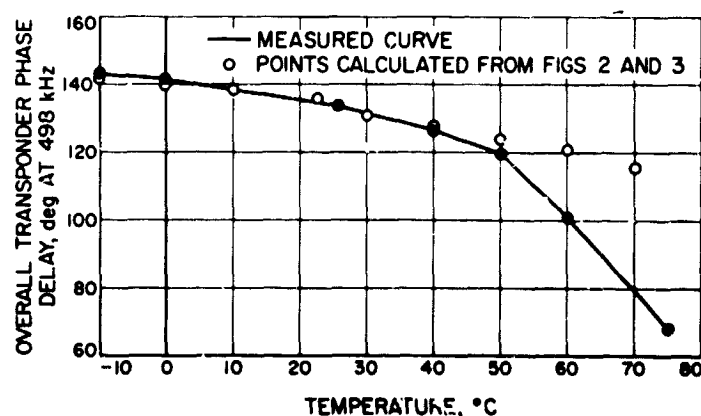


Fig. 1. Overall ranging delay variations from environmental variations on the 9.56 MHz IF amplifier

detector. It was hypothesized, therefore, that the change in the phase of the signal at the APC loop detector (caused by temperature variations), affected the ranging delay time at the ranging phase detector by changing the phase of the reference signal. To determine the validity of this hypothesis, a measurement was made of the effect of the ranging detector reference signal phase on the

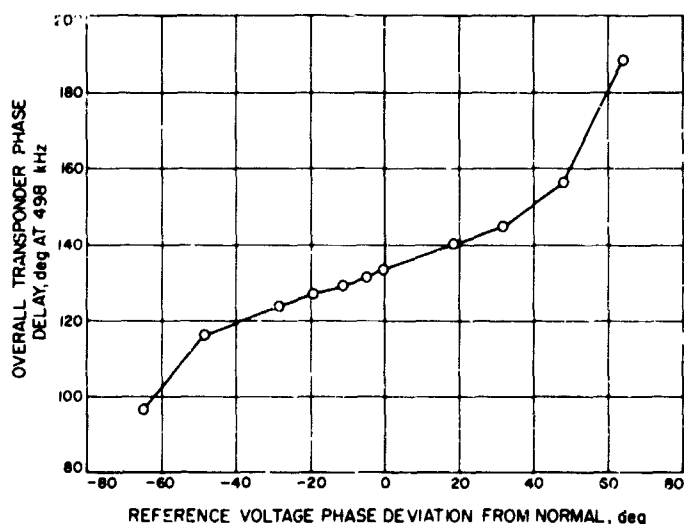


Fig. 2. Overall ranging delay vs ranging detector reference voltage phase

ranging delay through the transponder. The result, a strong effect, is shown in Fig. 2.

The hypothesis was further tested by a measurement of the RF phase shift through the IF amplifier as a function of temperature; this is shown in Fig. 3. The results of Figs. 2 and 3 were used to calculate the effect of temperature on ranging delay; the results were entered as points on Fig. 1. Agreement is shown between -10°C and 40°C , but the results deviate sharply at temperatures above this. It is tentatively concluded that the hypothesis is partially correct, but that some other mechanism is also contributing, and that the effect of the other mechanism is predominant at higher temperatures.

The effect of reference phase on the ranging delay cannot be explained by the analytical model usually used for a phase detector. Eventually, an effort should be made

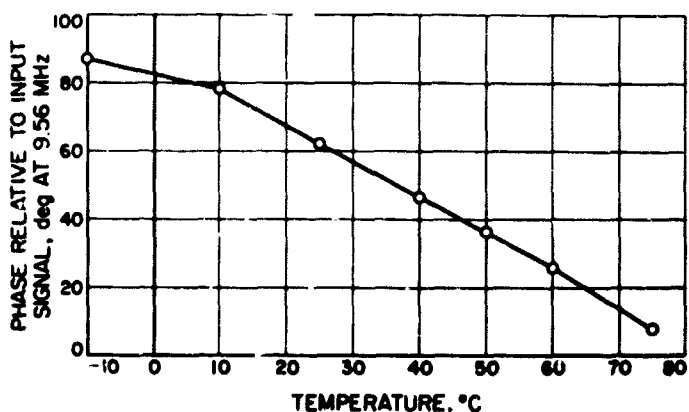


Fig. 3. IF amplifier phase variations vs temperature

to determine whether this is the result of a faulty analytical model, or deviations of the hardware from the expected performance. However, an investigation of the problem is not planned at this time.

It should be noted that the 9.56 MHz IF amplifier used in these tests exhibits phase shifts in excess of the limits specified for this module, although it had been previously tested (and passed) to these specifications. Referring to Fig. 3, the specified limit of ± 15 deg phase variation through the IF would result in a 10 deg overall ranging delay variation, which is a major contribution to the total.

The study continues with evaluation of the effect of the 9.56 MHz IF amplifier upon ranging delay, with the measurement of ranging delay where other individual modules are subjected to temperature variations, and with the measurement of doppler delay variations through the transponder at the S-band planned for the near future.

B. Spacecraft Guidance Radar Beam

Configuration Performance, R. L. Hottor

1. Introduction

This article concerns a part of the continuing effort to analyze the performance of the beam configuration of spacecraft guidance radars. These radars are used in closed loop guidance systems for lunar and planetary soft landing vehicles. Previous work concerning the radar beam-to-surface incidence angles was reported in SPS 37-43, Vol. IV, pp. 376-380. This article will show that the quality of the radar output data is affected by the geometry of the radar beam configuration, so much so that the performance of the beam configuration may be assessed by the quality of the radar output. Several radar functions make suitable criteria of quality. These are:

- (1) Altitude marking accuracy
- (2) Slant range estimation bias
- (3) Variances of the velocity and range estimates

For each of these criteria, beam geometry performance varies with changes in the beam look angle. The important criterion of estimation variance will be discussed in a future article.

The results in this article can be used to pick a beam geometry suitable for a given set of spacecraft trajectory and attitude conditions. The beam geometry chosen for

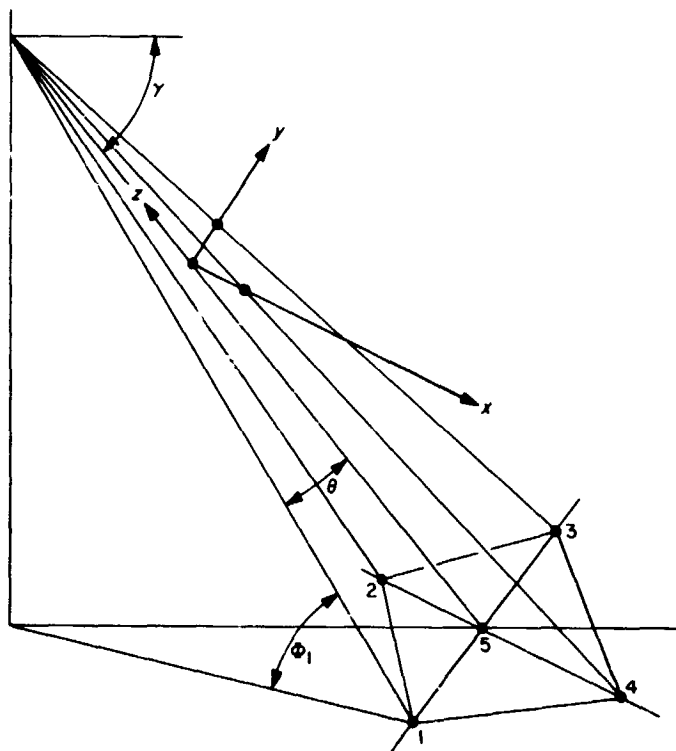


Fig. 4. Five-beam radar configuration

this work is particularly applicable to a Mars soft lander. Fig. 4 illustrates a simple five-beam configuration with four beams pointed symmetrically at an angle from the fifth. The radar measures range along beam 5, and the velocity vector in transverse and axial coordinates, as indicated in the figure. Information from each of the five beams may be combined in several ways to generate the required output information. Basically, the techniques discussed here use 3, 4, or 5 beams to perform the required function.

2. Altitude Marking Accuracy

All lunar or planetary soft landing concepts require some means to start the landing sequence. For this purpose, a range or altitude marking radar may be used to generate a signal when the surface distance in a given direction reaches a predetermined value. The signal is called a range mark. In an arbitrary radar beam configuration where true altitude may be desired to initiate the landing sequence, the beam most nearly vertical clearly gives the best approximation of true altitude. Altitude marking accuracy, therefore, is a function of vehicle attitude, trajectory incidence angle with respect to the surface, and beam configuration. How marking accuracy varies with beam configuration is the principal topic of this section.

a. Variation of marking accuracy with trajectory angles. Changes in the beam configuration chosen for this discussion (Fig. 4) involve only variations in the look angle θ . The other three important angles in the problem are the roll axis surface incidence angle γ , the beam system roll angle ϕ from the vertical plane, and the i th beam surface incidence angle ϕ_{ii} .

Altitude marking accuracy is the ratio of the altitude to the range along the steepest beam. More simply, accuracy is the sine of the beam incidence angle. Letting accuracy be $h(\theta, \phi, \gamma)$,

$$h(\theta, \phi, \gamma) = \sin \gamma \cos \theta + \cos \phi \cos \gamma \sin \theta \quad (1)$$

Two values of the roll angle ϕ are important. Beam 1 is in the vertical plane when $\phi = 0$, and, at a given trajectory angle, maximum marking accuracy occurs for this position. Minimum accuracy occurs at $\phi = \pi/4$ when two adjacent beams intersect the surface with equal angles. For a given value of the beam pointing angle, accuracy varies with the roll axis incidence angle γ . Fig. 5 demonstrates this dependence with the two roll positions, and representative pointing angles from 30 to 60 deg. Curve 3 for $h(0, 0, \gamma)$ is included in Fig. 5 for comparison. As might be expected, beam geometries with wider look angles achieve maximum marking accuracy at lower trajectory angles.

b. Optimum configuration for best marking accuracy. At this point, the best attainable accuracy with geometry that can be continuously adjusted to the optimum value should be considered. As before, the accuracy is denoted by $h(\theta, \phi, \gamma)$

$$h(\gamma, \phi, \theta) = \sin \gamma \cos \theta + \cos \phi \cos \gamma \sin \theta \quad (2)$$

Differentiating $h(\gamma, \theta, \phi)$ with respect to θ , and setting the result equal to zero,

$$h'_{\theta}(\gamma, \phi, \theta) = -\sin \gamma \sin \theta + \cos \phi \cos \gamma \cos \theta = 0 \quad (3)$$

The value of θ satisfying Eq. (3) is θ_M .

$$\begin{aligned} \tan \theta_M &= \cos \phi \cot \gamma \\ \theta_M &= \tan^{-1}(\cos \phi \cot \gamma) \end{aligned} \quad (4)$$

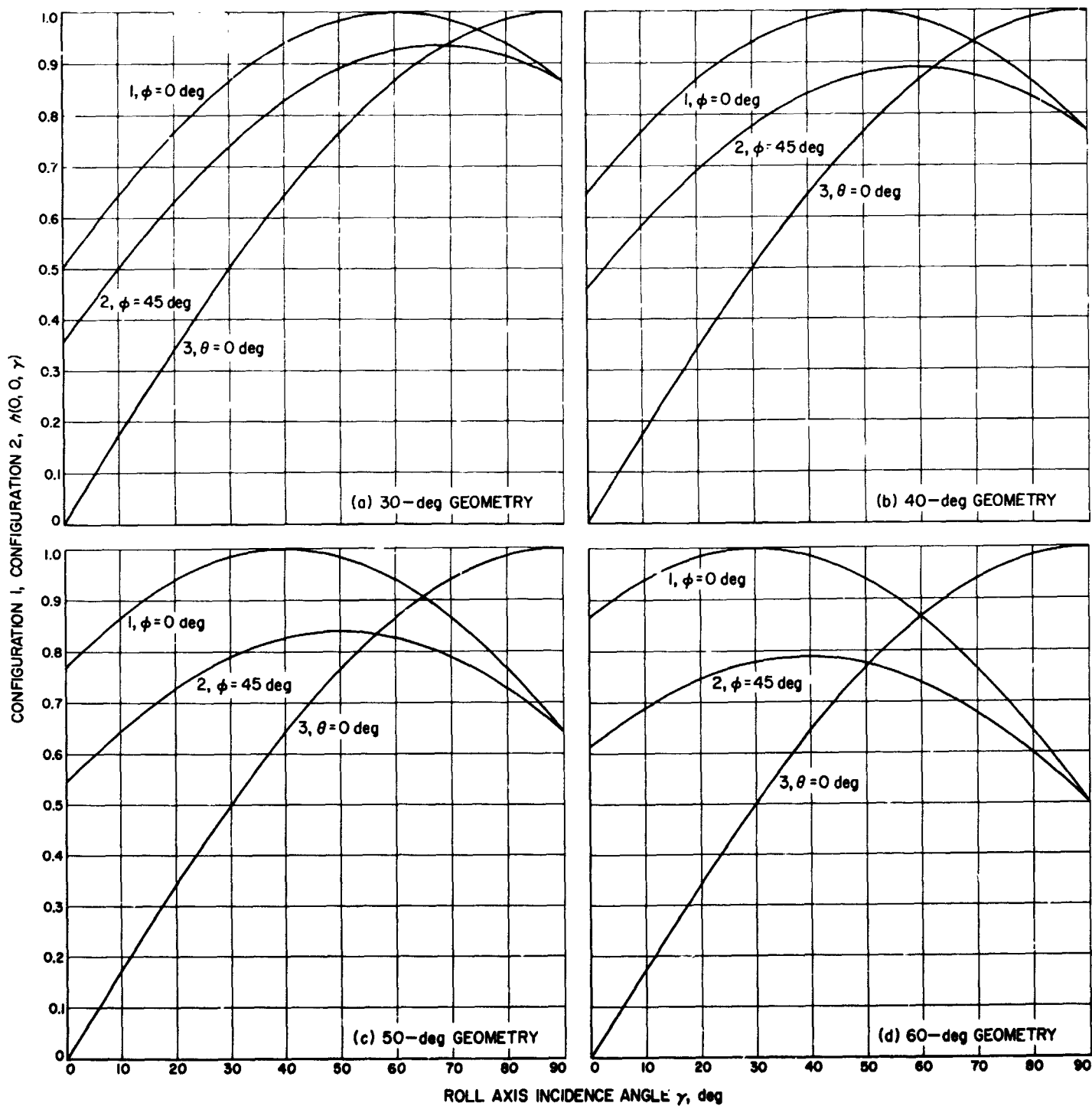


Fig. 5. Marking accuracy

Consequently, the following relations hold:

$$\cos \theta_M = \frac{1}{(1 + \cos^2 \phi \cot^2 \gamma)^{1/2}}$$

$$\sin \theta_M = \frac{\cos \phi \cot \gamma}{(1 + \cos^2 \phi \cot^2 \gamma)^{1/2}}$$

Substituting these values into Eq. (2),

$$h(\gamma, \phi, \theta_M) = \frac{\sin \gamma + \cos^2 \phi \cos \gamma \cot \gamma}{(1 + \cos^2 \phi \cot^2 \gamma)^{1/2}} \quad (5)$$

but this expression may be simplified. Multiplying numerator and denominator by $\sin \gamma$,

$$h(\gamma, \phi, \theta_M) = \frac{\sin^2 \gamma + \cos^2 \phi \cos^2 \gamma}{\sin \gamma (1 + \cos^2 \phi \cot^2 \gamma)^{1/2}}$$

$$= (\sin^2 \gamma + \cos^2 \phi \cos^2 \gamma)^{1/2} \quad (6)$$

This function is plotted in Fig. 6(a) for values of ϕ giving the extremes in roll position. Curve 3 for $h(0, 0, \gamma)$ is included again for comparison. The corresponding values of $h(\gamma, \phi, \theta_M)$ are

$$h(\gamma, 0, \theta_M) = 1.0 \quad (7)$$

$$h(\gamma, \pi/4, \theta_M) = \left(\sin^2 \gamma + \frac{\cos^2 \gamma}{2} \right)^{1/2}$$

$$= \frac{(1 + \sin^2 \gamma)^{1/2}}{(2)^{1/2}} \quad (8)$$

Values of θ_M as functions of γ are plotted in Fig. 6(b). When $\phi = 0$,

$$\theta_M = \tan^{-1}(\cot \gamma) \quad (9)$$

or

$$\theta_M = \frac{\pi}{2} - \gamma \quad (10)$$

When $\phi = \pi/4$,

$$\theta_M = \tan^{-1} \left(\frac{\cot \gamma}{(2)^{1/2}} \right) \quad (11)$$

Note that Fig. 6(a) and (b) have no bearing at all on Fig. 5(a) through (d), because maximum marking accu-

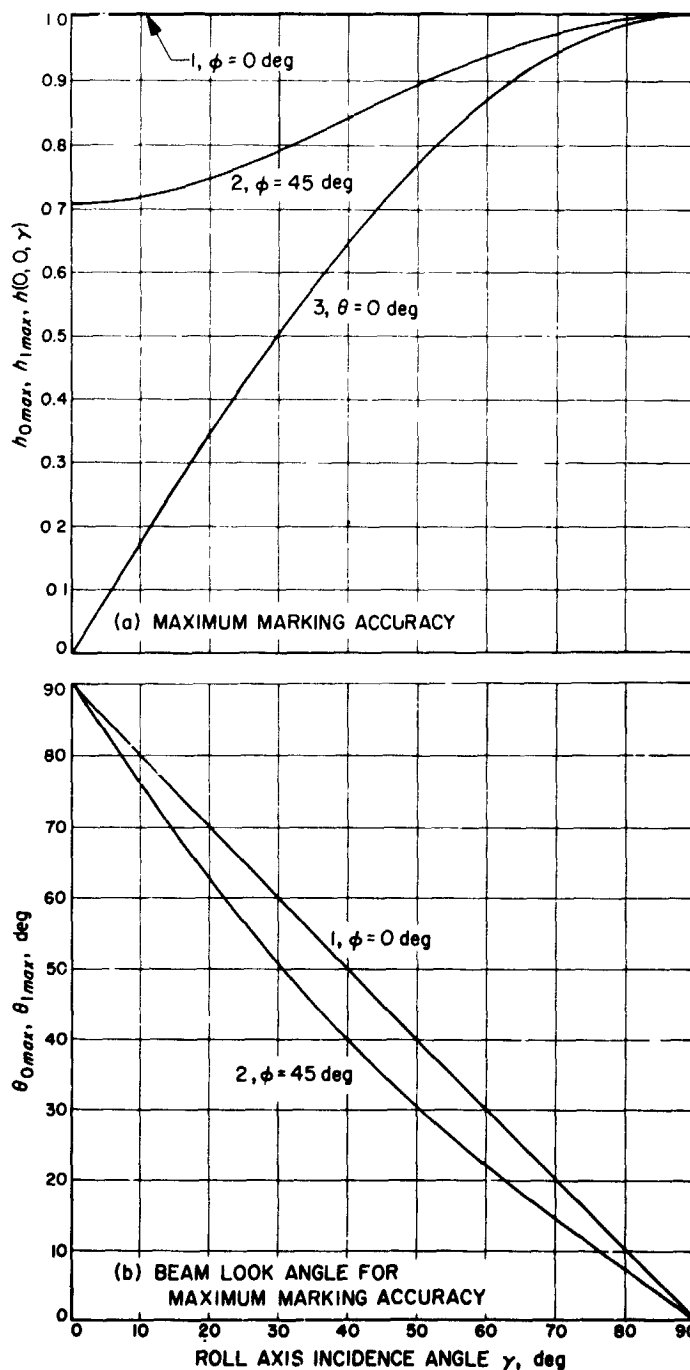


Fig. 6. Continuously adjusted beam look angle

racy is attained here by varying the geometry angle θ , whereas the maximum marking accuracy in paragraph a. occurred as the trajectory angle varied. It is not surprising, then, that these two maxima do not occur at the same trajectory angles.

3. Slant Range Bias Error

The guidance radar is also required to estimate range along the vehicle roll axis. This so-called slant range may

be measured directly along Beam 5, or by combining measurements of the other four beams. Slant range bias, as used here, refers to the error introduced by combining the range estimates of several beams. Specifically excluded are terrain bias errors that corrupt individual beam range estimates.

a. Analysis. For a three-beam configuration, slant range will be measured along the roll axis only, and, because it is measured directly, there will be no bias. For the four-beam configuration, however, range is derived from the average of the four outward pointing beams. The bias is accepted because the variance of the estimate takes

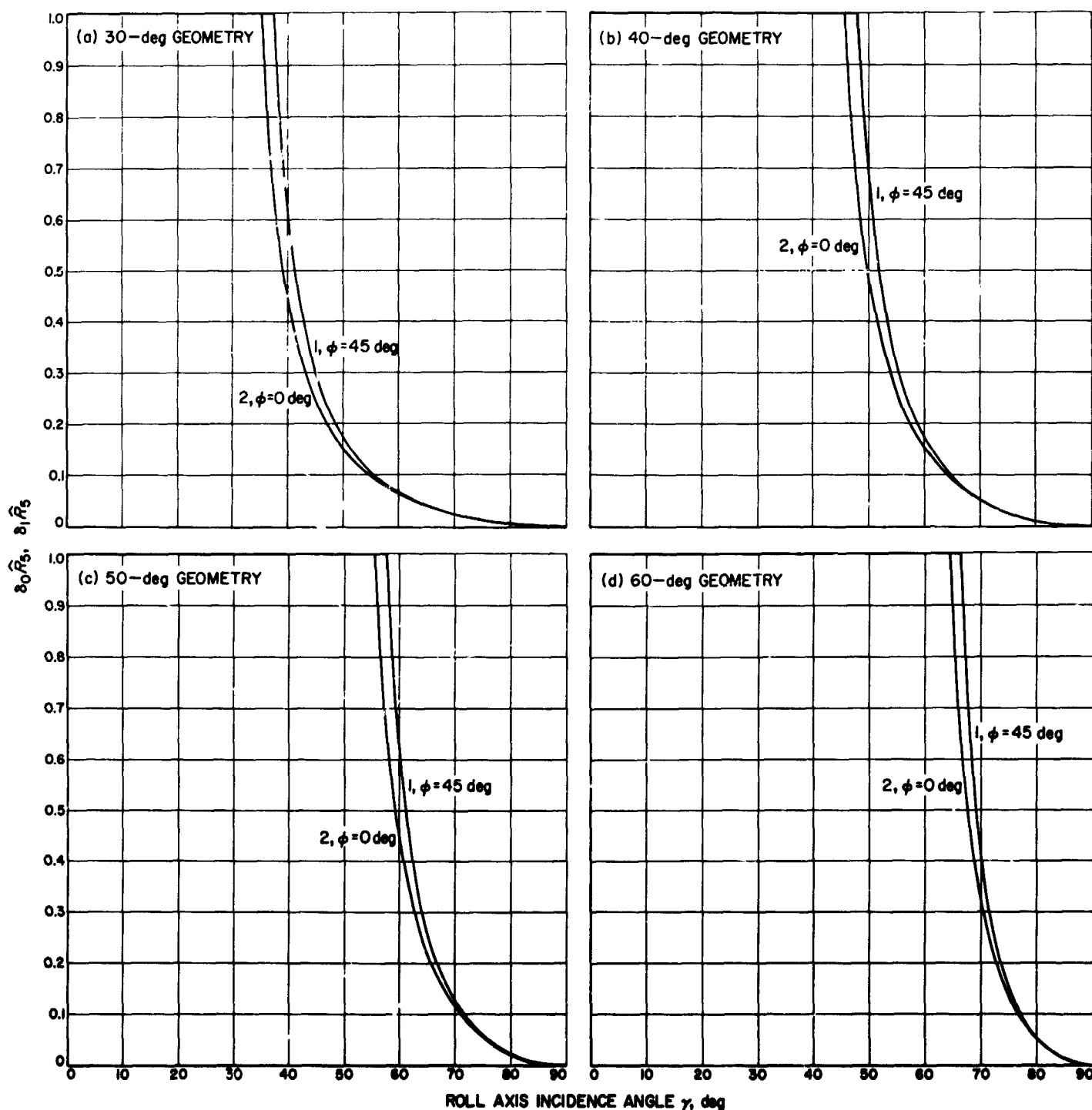


Fig. 7. Range bias for four beams

advantage of the noise averaging of four beams instead of one. A less noisy but biased range signal may be more desirable than an unbiased but noisy signal.

The range along each beam may be expressed in the terms of the vertical altitude and the sine of the beam incidence angle. Without loss of generality, unit altitude may be assumed. Eq. (12) shows an estimate of the range along the roll axis.

$$\begin{aligned}\hat{R}_s &= \frac{1}{4} (R_1 + R_2 + R_3 + R_4) \cos \theta \\ &= \frac{1}{4} \left(\frac{1}{\sin \Phi_{11}} + \frac{1}{\sin \Phi_{12}} + \frac{1}{\sin \Phi_{13}} + \frac{1}{\sin \Phi_{14}} \right) \cos \theta\end{aligned}\quad (12)$$

The bias is the difference between $\hat{R}_s$ and the true value of R_s .

$$\begin{aligned}\hat{R}_s - R_s &= \frac{1}{4} \left(\frac{1}{\sin \Phi_{11}} + \frac{1}{\sin \Phi_{12}} + \frac{1}{\sin \Phi_{13}} + \frac{1}{\sin \Phi_{14}} \right) \\ &\quad \times \cos \theta - \frac{1}{\sin \gamma}\end{aligned}\quad (13)$$

The ratio of the bias to the true value is the percentage bias:

$$\begin{aligned}\frac{\hat{R}_s - R_s}{R_s} &= \frac{1}{4} \left(\frac{1}{\sin \Phi_{11}} + \frac{1}{\sin \Phi_{12}} + \frac{1}{\sin \Phi_{13}} + \frac{1}{\sin \Phi_{14}} \right) \\ &\quad \times \cos \theta \sin \gamma - 1.0\end{aligned}\quad (14)$$

$\sin \Phi_i$, however, may be expressed in the terms of the three angles γ , θ , and ϕ , which are defined in Fig. 4.

$$\sin \Phi_{1i} = \sin \gamma \cos \theta + \cos \left[\phi + (i-1) \frac{\pi}{2} \right] \cos \gamma \sin \theta \quad (15)$$

Substituting in Eq. (14) yields

$$\begin{aligned}\delta \hat{R}_s &= \frac{1}{4} \left(\frac{1}{\sin \gamma \cos \theta + \cos \phi \cos \gamma \sin \theta} \right. \\ &\quad + \frac{1}{\sin \gamma \cos \theta + \sin \phi \cos \gamma \sin \theta} \\ &\quad + \frac{1}{\sin \gamma \cos \theta - \cos \phi \cos \gamma \sin \theta} \\ &\quad \left. + \frac{1}{\sin \gamma \cos \theta - \sin \phi \cos \gamma \sin \theta} \right) \cos \theta \sin \gamma - 1.0\end{aligned}\quad (16)$$

Combining terms yields an expression more amenable to calculation:

$$\begin{aligned}\delta \hat{R}_s &= \frac{1}{4} \left(\frac{2 \sin^2 \gamma \cos^2 \theta}{\sin^2 \gamma \cos^2 \theta - \cos^2 \phi \cos^2 \gamma \sin^2 \theta} \right. \\ &\quad \left. + \frac{2 \sin^2 \gamma \cos^2 \theta}{\sin^2 \gamma \cos^2 \theta - \sin^2 \phi \cos^2 \gamma \sin^2 \theta} \right) - 1.0\end{aligned}\quad (17)$$

Dividing through by the numerator,

$$\begin{aligned}\delta \hat{R}_s &= \frac{1}{2} \left(\frac{1}{1 - \cos^2 \phi \cot^2 \gamma \tan^2 \theta} \right. \\ &\quad \left. + \frac{1}{1 - \sin^2 \phi \cot^2 \gamma \tan^2 \theta} \right) - 1.0\end{aligned}\quad (18)$$

Combining these terms gives

$$\delta \hat{R}_s = \frac{1}{2} \left(\frac{1 - \sin^2 \phi \cot^2 \gamma \tan^2 \theta + 1 - \cos^2 \phi \cot^2 \gamma \tan^2 \theta}{1 - \cot^2 \gamma \tan^2 \theta - \sin^2 \phi \cos^2 \phi \cot^4 \gamma \tan^4 \theta} \right) - 1.0 \quad (19)$$

Finally

$$\delta \hat{R}_s = \frac{1}{2} \left(\frac{2 - \cot^2 \gamma \tan^2 \theta}{1 - \cot^2 \gamma \tan^2 \theta - \sin^2 \phi \cos^2 \phi \cot^4 \gamma \tan^4 \theta} \right) - 1.0 \quad (20)$$

b. Discussion. Evaluation of this formidable expression presents few problems for a computer. The bias $\delta \hat{R}_s$ is plotted in Fig. 7 as a function of γ , the roll axis incidence angle, for some values of θ between 30 and 60 deg. Two

extremes in roll position are illustrated by two values of ϕ , namely, 0 and $\pi/4$. The bias curve for any other roll position of the beam geometry lies between the two curves drawn. Note that the bias approaches zero as the trajectory

approaches vertical. In this region, near touchdown, accuracy is most important.

C. Multipacting in Coaxial Transmission Lines at S-Band, R. Woo

1. Introduction

Experiments have been conducted to check for multipacting in $7/8$ -in. and $1 5/8$ -in. rigid coaxial transmission lines in the frequency range of 1700 to 2400 MHz. The purpose of these experiments was twofold: (1) these frequencies and dimensions are typical RF spacecraft components, and data are needed; (2) the data can be checked against

previous data obtained at lower frequencies with a different test setup (SPS 37-41, Vol. IV, pp. 242-246) through scaling relations (SPS 37-36, Vol. IV, pp. 281-283).

2. Experimental apparatus

The experimental setup is shown in Fig. 8, and the schematic in Fig. 9. Approximately 1 kw CW of RF power was provided in the frequency range 1700 to 2400 MHz by a Philco 2000 klystron amplifier. A coaxial circulator with its isolated arm terminated in a $50\text{-}\Omega$ load was used to protect the klystron amplifier from large amounts of reflected power produced when breakdown occurs. Power level was varied by changing the drive level to the klystron amplifier. Both forward and reverse power levels



Fig. 8. Experimental setup

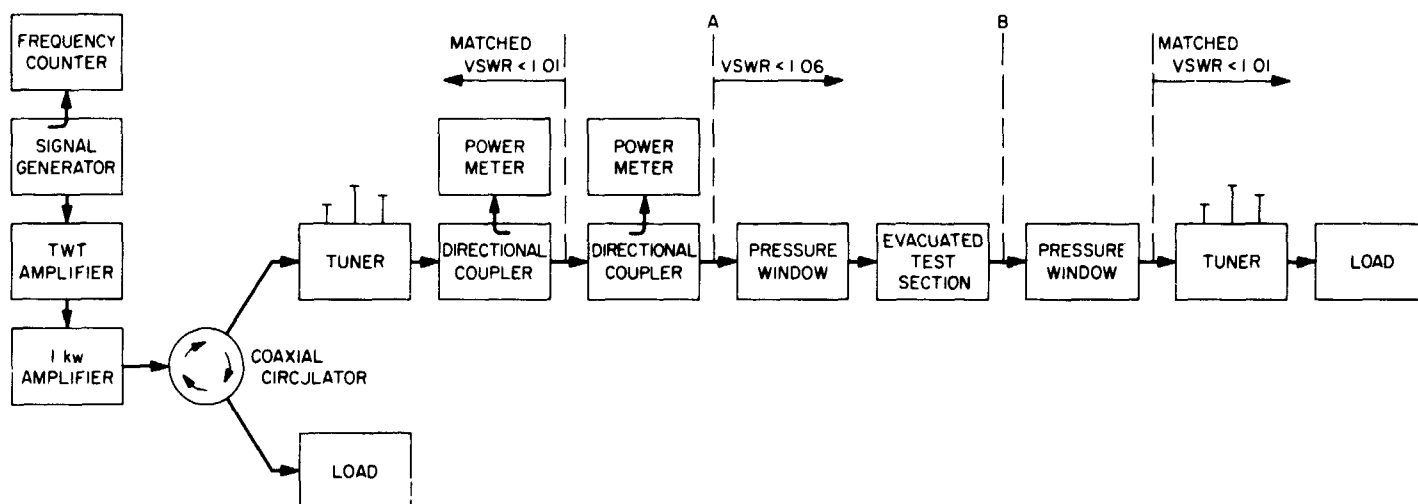


Fig. 9. Schematic of experimental setup

were monitored by Hewlett-Packard 431B power meters connected through appropriate attenuators to the coupling arms of the directional couplers. Coupling figures and attenuation losses were measured within 0.02 db with a Weinschel dual-channel insertion loss test set. Frequency was monitored with a frequency counter. Triple stub tuners were used to attain the matched conditions indicated in Fig. 9.

The vacuum system consisted of a roughing pump and an 8 l/sec vac-ion pump. Low pressures were measured with a built-in meter in the Ultek 60-055 power unit for the vac-ion pump; typical operating pressure levels were 5×10^{-6} mm of Hg. The pressure windows had mechanical seals formed between the Teflon spacers and the copper flanges (Fig. 10). Evacuation of the test section took place through an $\frac{1}{8}$ -in.-diameter hole. The test section was abrasively cleaned immediately before conducting each experiment.

3. Experimental results and discussion

In determining the voltage level in the evacuated test section, the actual power level in the test section, and not merely elsewhere in the test system, must be known. The insertion loss between the forward directional coupler and the test section must, therefore, be measured to permit corrections in the power measurement. The insertion loss measured was less than 0.01 db, making the correction factor almost negligible. Error in the calculation of the voltage level occurs if the assembly of components on the load side of the test section (point B in Fig. 9) has any appreciable VSWR. This measurement was not made because it was physically impossible. However,

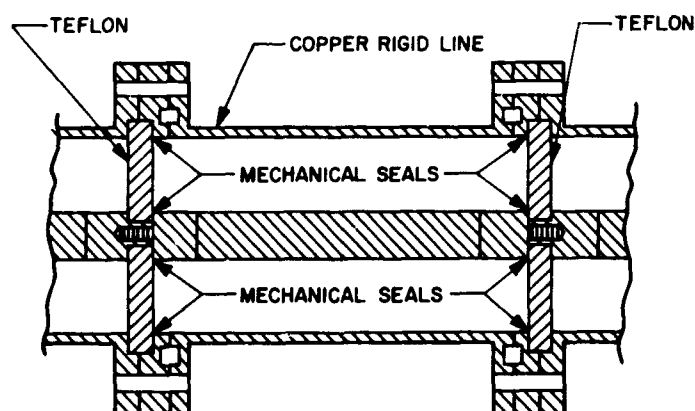


Fig. 10. Evacuated rigid line section

VSWR measurements were made at point A (Fig. 9), and found to be less than 1.06. This implies a VSWR at point B of less than 1.06. As a result, the maximum error of the indicated voltage values would be less than $\pm 3\%$. Fig. 25 of SPS 37-41, Vol. IV, p. 245 is reproduced here as Fig. 11 for reference. As shown, data were obtained for values of fd up to 500 MHz-cm. In the present series of experiments, all values of fd are greater than 958.6. At these higher values of fd , higher order multipacting modes are more likely to occur, and RF voltages considerably greater than 200 v would be necessary. Therefore, with 1 kw RF power (RF voltage is 223.6 v), multipacting was not expected. The summary of data in Table 1 substantiates these predictions.

It is common in the study of microwave breakdown in gases to irradiate the test component with an ionizing source such as Cobalt 60 to obtain more reproducible data.

Table 1. Summary of data

Coaxial rigid line size, in.	Distance between inner and outer conductors (d), cm	Frequency (f), MHz	fd, MHz-cm	Maximum power levels checked, w	Maximum rms voltage levels checked, v	Breakdown
3/8	0.5639	1700	958.6	1000	223.6	No
3/8	0.5639	2000	1127.8	1000	223.6	No
3/8	0.5639	2400	1353.3	875	209.2	No
1 1/8	1.096	1700	1863.2	1000	223.6	No
1 1/8	1.096	2000	2192.0	1000	223.6	No
1 1/8	1.096	2400	2630.4	875	209.2	No

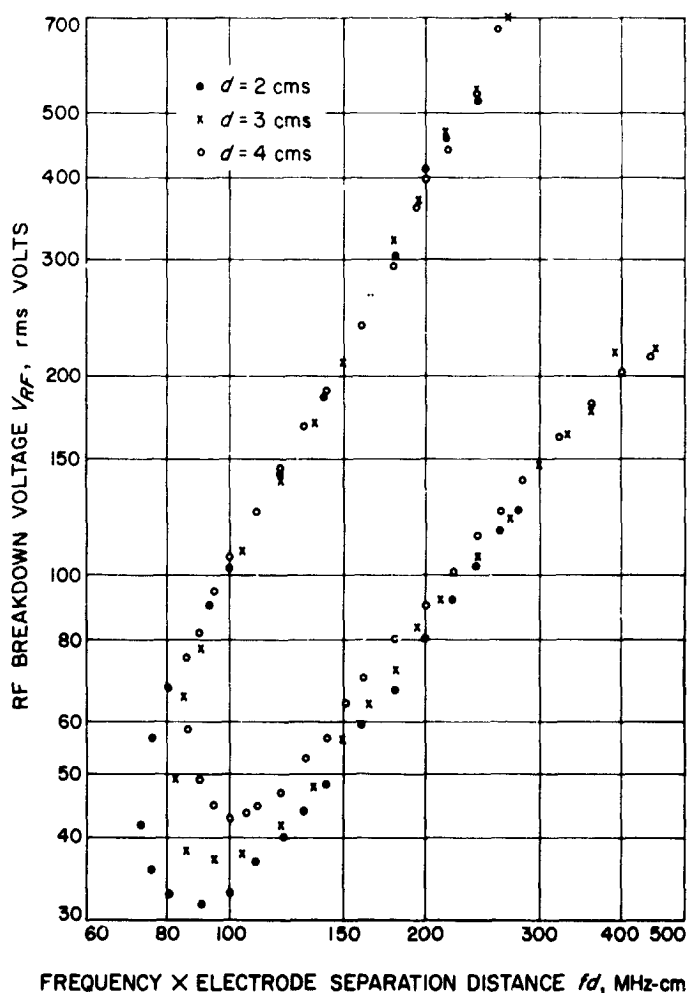


Fig. 11. Multipacting region for coaxial electrodes
d = 2, 3, and 4 cms, b/a = 2.3

In the experiments described in this article, irradiation was not employed because no data were obtained. It should be noted that irradiation is often unnecessary in CW measurements; this was the case in the multipacting experiments at lower frequencies (SPS 37-41, Vol. IV, pp. 242-246).

When the 1 kw CW RF source on order is delivered, similar experiments using this test setup will be conducted in the frequency range 150 to 1000 MHz. With the 3/8-in. and 1 1/8-in. rigid coaxial line sections, the range of values for fd will be comparable to those in Fig. 11. Thus, an invaluable check of our scaling laws can be made.

D. High Impact Antenna Study: Rectangular Cup Radiator, Part II, K. Woo

1. Introduction

In SPS 37-43, Vol. IV, pp. 386-388, the design theory of a rectangular cup radiator was discussed, and a high impact model of the antenna was described. This article presents the test results of a second high impact model. This model withstood an indirect impact of 10,000 g, and will be tested against direct impacts (refer to SPS 37-40 to 37-42, Vol. IV for other high impact antennas).

2. Antenna Design

The antenna is shown in Fig. 12. It consists of an aluminum rectangular cup, 2.250 in. $\times$ 3.150 in. $\times$ 2.475 in. deep. The cup is excited by a probe (Fig. 13) that is slanted along one of the diagonals of the cup. The probe, a brass rod 3/32 in. in diameter, is an extension of the center conductor of the input coaxial connector located at the rear of the cup. The probe is completely imbedded in Eccofoam to withstand high impacts. The foam was formed by filling the entire cup with Eccofoam FP¹ (foam-in-place liquid resin) and catalyst. Upon curing, the foam gave an all-around rigid support for the probe and the cup walls. The cured foam in this model has a density of about 16 lb/ft³, a relative dielectric constant of about 1.3, a compressive strength of about 700 psi, and an excellent adhesion to the aluminum cup walls.

¹Distributed by Emerson and Cuming, Inc.

(a) FRONT

(b) REAR

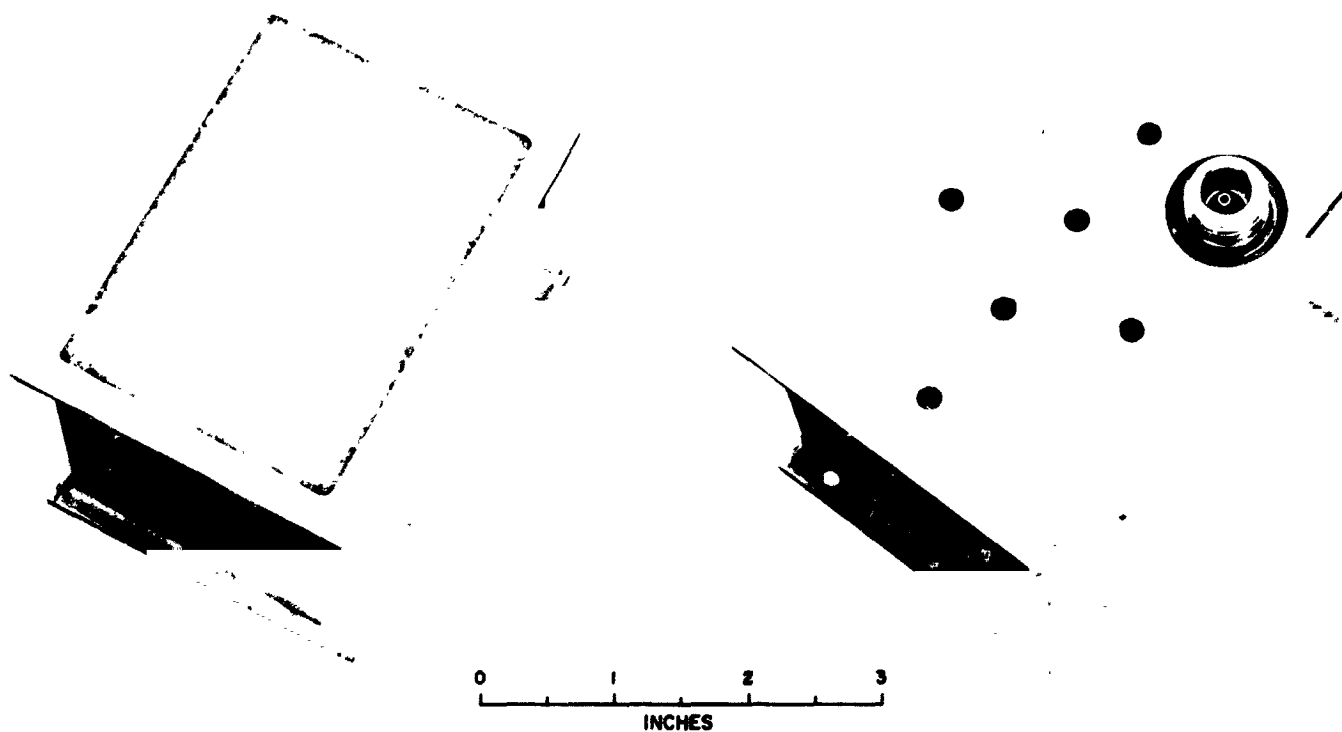


Fig. 12. High impact rectangular cup radiator

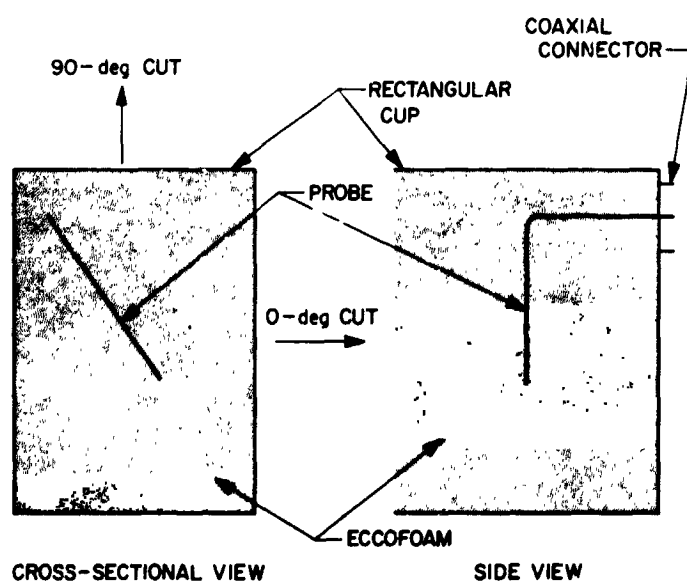


Fig. 13. Schematic of rectangular cup radiator

3. Test Results

Indirect impact test of the antenna at 10,000 g (0.6 msec duration) was performed. The antenna was mounted in a test fixture, and then the fixture was impacted against a target. The cup body was unharmed, and the foam survived the impact without noticeable cracking or separation from the cup walls. There was no appreciable change in the electrical performance of the antenna. Fig. 14 shows the radiation patterns at 2298 MHz along the 90-deg cut (Fig. 13) of the right-hand circularly polarized (RCP) and left-hand circularly polarized (LCP) components taken before and after the impact. The input VSWRs before and after the impact were, respectively, 1.7.

Electrical measurements show that the half-power beamwidth of the antenna varies from a maximum of about 88 deg (along 0-deg cut) to a minimum of about 72 deg (along 90-deg cut).

Further work on the antenna will include:

- (1) Refining the electrical performance.
- (2) Testing survivability against direct impacts.
- (3) Investigating such environmental problems as out-gassing, sterilization, and high power operation.

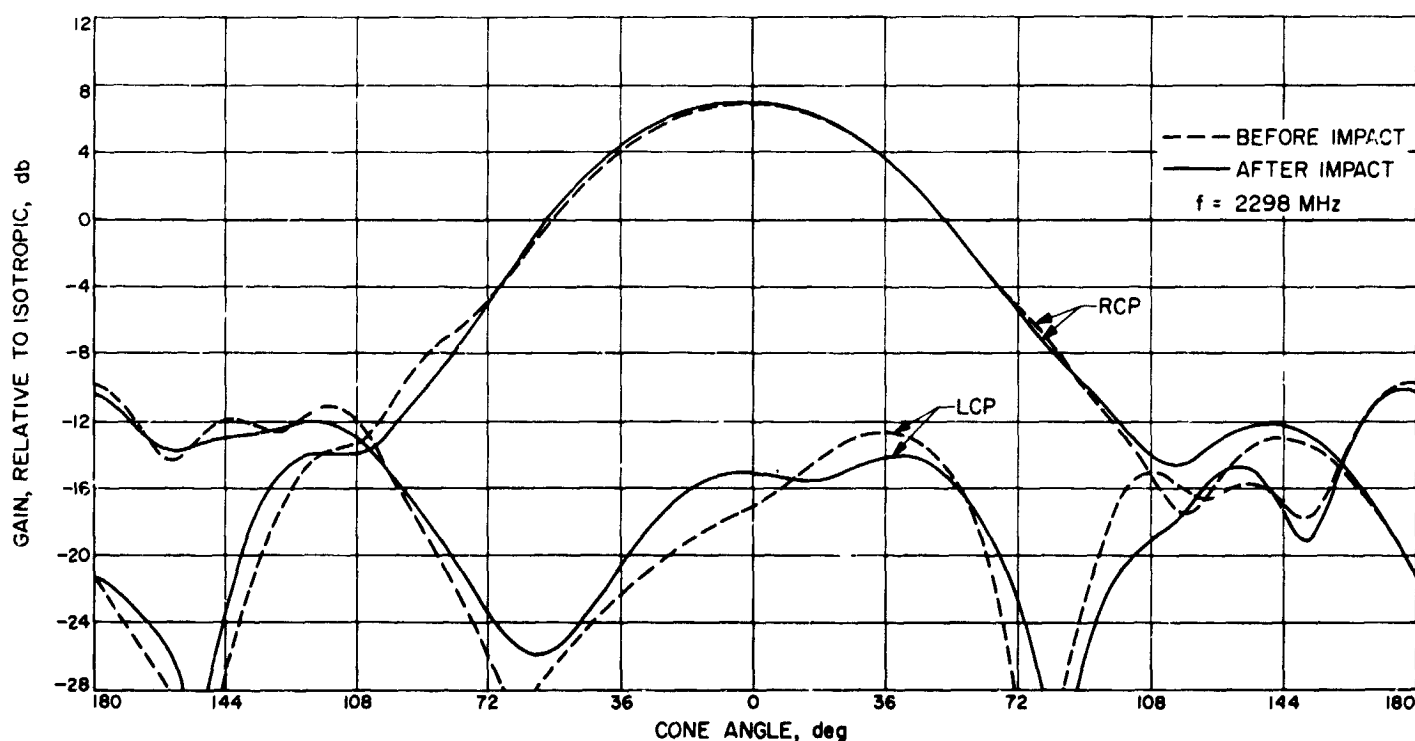


Fig. 14. Radiation patterns before and after impact, 90-deg cut